Resource Economics
Final Exam – Fall 2021

Problems 1 and 2 are brief discussions that you can type into INSPERA. If you are concise, half a page is enough. Please try not to exceed one page. Problem 3 requires calculations and can be answered on paper. It is your responsibility that the submitted document/photo/scan is legible.

1. Resource Taxation, 25%.
How does taxation of a resource or the resource extracting sector affect extraction? Write a brief verbal essay. Distinguish three different types of taxes and comment on the royalty.

Solution: Here, we have to distinguish the type of tax imposed on the resource sector. We will discuss a profit tax on the sector, a revenue tax on the sector, and a direct tax on the resource being extracted.

1. If we tax the profits of the resource sector/the resource extracing firm, we found that the extraction path is not affected. The taxing government can use the tax to appropriate the royalty without distorting the market. More precisely, this finding assumed that there are no exploration costs. Positive exploration costs can distort the amount of the resource being extracted.

2. If we tax the revenue of the resource extracting firms, we found that the tax distorts the extraction path in a way that resembles a change of the marginal extractions costs. Again, the government will appropriate some of the royalty, but this time also distort the extraction of the resource.

3. If we tax the resource directly, we found that we will generally reduce optimal extraction. We have shown this finding in our integrated assessment of climate change model where a higher social cost of carbon reduces the optimal emissions and, thus, fossil fuel extraction. In a decentralized world, the finding implies that imposing a carbon tax reduces the extractions in the carbon sector. Again, the royalty will be split between the firms and the government.[1]

   Note: If the students also discuss the green paradox that's perfect, but it was not really emphasized in the lecture material.

2. Non-Convex Ecosystem Dynamics, 25%.
Explain the concept of a Skiba point in a non-convex resource management problem. No equations required. What differs in the underlying dynamic system as opposed to

our “usual” dynamic system (not containing a Skiba point)? What are the implications for the solution set?

Solution: The Skiba point arises as part of the solution concept controlling a dynamic (eco-)system with feedbacks. These feedbacks can imply that there are multiple stable steady states of the *optimally controlled* ecosystem. Usually one corresponds to a relatively good environmental state, and another to a highly polluted environmental state that is too expensive to clean up. Depending on the initial state of the system it will be optimal to converge into one of these steady states. The Skiba point gives the initial state where the decision maker is indifferent to converge to either of the steady states. If the initial state is not right at the Skiba point, then it is optimal to control the dynamic system in such away that one converges to the steady state that is on the same side of the Skiba point as the initial state. E.g., if we are modeling shallow lake pollution, it is optimal to clean up the lake only if the current pollution level is lower than the Skiba point. If it is higher, we give up on the lake and converge to a steady state whith high pollution and lots of algea and little life in the lake.

3. Extraction of Gold, 50%.
A social planner wants to build an optimal extraction model for gold. You are asked to help with this task. Extraction costs $C(q_t, R_t)$ depend on the extraction flow (q_t) of the resource as well as the remaining stock in the ground (R_t).

i) Set up the continuous time infinite horizon optimization model including objective and equation of motion. Think carefully about the objective and discuss your modeling choice. How does it differ from the case of oil?

Solution: The value of oil derives mostly from fossil fuel burning, a process that uses the resource up in the process. The value of gold is mostly proportional to the stock extracted and wearing or storing gold does not use the resource up. Thus, we assume a value of the stock of the extracted resource, say S_t. So we have a utlity function $U(S_t)$ where $S_t = S_0 + \int_0^t q_t$, or simply, $S_t = S_0 + R_0 - R_t$, i.e., the gold in circulation in some future period is the gold currently in circulation plus the amount dug out between now and period t.[2] Given S_0 and R_0 are given constants, it's convenient to define

$$u(R_t) \equiv U(S_0 + R_0 - R_t) \text{ implying}$$
$$u'(R_t) = -U'(S_0 + R_0 - R_t) < 0 \text{ and}$$
$$u''(R_t) = U''(S_0 + R_0 - R_t) < 0$$

if we assume positive but falling marginal utility from gold $U'(R_0)$.

Then the intertemporal optimization problem is

$$\max \int_0^\infty (u(R_t) - C(q_t, R_t))e^{-rt}\mathrm{d}t \quad \text{s.t. } \dot{R} = -q_t \;,\; R_t \geq 0.$$

with R_0 given.

ii) Set up the current value Hamiltonian and derive the necessary conditions for an optimal extraction path. Do not worry about the transversality conditions.

Solution: The Hamiltonian is

$$H^c(q_t, R_t, \mu_t) = u(R_t) - C(q_t, R_t) - \mu_t q_t$$

and we obtain the necessary conditions

$$\frac{\partial H^c(q_t, R_t, \mu_t)}{\partial q_t} = -\frac{\partial C(q_t, R_t)}{\partial q_t} - \mu_t \overset{!}{=} 0$$

$$\Rightarrow \mu_t = -C_q(q_t, R_t) \tag{1}$$

$$\frac{\partial H^c(q_t, R_t, \mu_t)}{\partial R_t} \overset{!}{=} r\mu_t - \dot{\mu}_t$$

$$\Rightarrow u'(R_t) - \frac{\partial C(q_t, R_t)}{\partial R_t} = r\mu_t - \dot{\mu}_t$$

$$\Rightarrow u'(R_t) - C_R(q_t, R_t) = r\mu_t - \dot{\mu}_t \tag{2}$$

where we abbreviated $C_q(q_t, R_t) \equiv \frac{\partial C(q_t,R_t)}{\partial q_t}$ and $C_R(q_t, R_t) \equiv \frac{\partial C(q_t,R_t)}{\partial R_t}$.

iii) Derive the Euler equation and discuss the sign of the different terms.

Solution: Using equation (1) for the shadow value of gold in the ground and its time derivative in equation equation (2) we obtain

$$u'(R_t) - C_R(q_t, R_t) = -rC_q(q_t, R_t) + \frac{d}{dt}C_q(q_t, R_t). \tag{3}$$

The lower the remaining stock of the resource, the more expensive extraction will usually be, so that $C_R(q_t, R_t) < 0$ (less of the stock left implies higher costs).

Extracting gold faster will usually be more expensive and, thus, $C_q(q_t, R_t) > 0$. The sign of u' is negative as we have argued above (recall that R_t is the part of the gold *not* extracted). We can analyze the time derivative of $C_q(q_t, R_t)$ further by carrying out the time derivative

$$\frac{d}{dt} C_q(q_t, R_t) = \frac{\partial C_q(q_t, R_t)}{\partial q_t} \dot{q}_t + \frac{\partial C_q(q_t, R_t)}{\partial R_t} \dot{R}_t \tag{4}$$

where the only obvious sign is $\dot{R}_t < 0$, i.e., the resource left in the ground will decrease over time. If we have convex extraction costs in the resource flow we also have $\frac{\partial C_q(q_t, R_t)}{\partial q_t} > 0$. If the extraction flow is decreasing over time we have $\dot{q}_t < 0$, which is the likely prediction of the present model as it does not incorporate technological progress or a possible increase in demand. Finally, one could argue either sign for the cross-derivative of the cost function $\frac{\partial C_q(q_t, R_t)}{\partial R_t}$, but it stands to reason that the marginal costs maintaining a given resource flow become more expensive as the amount of the resource left in the ground a decreasing so that $\frac{\partial C_q(q_t, R_t)}{\partial R_t} < 0$.

Under these assumptions, the first term is negative and the second term is positive, making it hard to sign the overall contribution of $\frac{d}{dt} C_q(q_t, R_t)$ without further more restrictive assumptions.

iv) Interpret the Euler equation.

Solution: Using $U'(S_t) = -u'(R_t) > 0$ let's rearrange the Euler equation (5) to the form

$$-C_R(q_t, R_t) + rC_q(q_t, R_t) - \frac{d}{dt} C_q(q_t, R_t) = U'(S_t). \tag{5}$$

On the left, we have summarized the benefits of leaving a unit of gold in the ground for one more (marginal) period. On the right, we have summarized the benefits of taking out the unit of gold at the beginning of the (marginal) period.

- $-C_R(q_t, R_t) > 0$: The value of leaving another unit of gold in the ground over the course of an extraction period because it lowers extraction costs.
- $rC_q(q_t, R_t) > 0$: The savings of postponing extraction for one period (interest on not spending the marginal extraction costs today but postponing them for a period).

- $\frac{d}{dt}C_q(q_t, R_t)$: If $\frac{d}{dt}C_q(q_t, R_t) < 0$, benefit from postponing extraction for one period because marginal costs will fall in the next period (e.g. because extraction flow falls, see discussion of 4 for details). If $\frac{d}{dt}C_q(q_t, R_t) > 0$, cost of postponing extraction for one period because marginal costs will be higher in the next period (e.g. because dominating effect is that resource grows more scarce and is harder to extract as level or remaining resource falls).
- $U'(S_t) > 0$: Benefit of extracing the unit of gold already now and enjoying it over the course of the period rather than leaving it in the ground.

The Euler equation states that the sum of the benefits of leaving a unit of gold in the ground has to equal the marginal benefit of already extracting that additional unit now.

v) How and why does the Euler equation differ from a similar model describing oil extraction?

Solution: The main difference is that, here, the benefits derive from the stock of the resource already extracted rather than from the flow of resource extraction. In our usual fossil fuel extraction problem, we expect to find the rate of return on the other side of the Euler equation, i.e., the term containing the interest rate "has the opposite sign". That's because in the fossil fuel extraction problem, the marginal extraction value contains an immediate benefit from extraction $u(q_t)$ and postponing the net benefit of extraction $u(q_t) - c_q(q_t, R_t) > 0$ for one period would be a cost rather than a benefit. Here, we only have the immediate costs deriving from extraction, i.e., the part $c_q(q_t, R_t) < 0$, which we therefore rearranged to the opposite side. Here, the benefits derive directly from the stock of gold already extracted, S_t. These benefits remain on the other side and characterize the value of extracting the resource now rather than tomorrow. They appear without the interest rate (multiplying the marginal benefit) because we derive one additional period of marginal utility from the unit of gold extracted as opposed to merely gaining the benefit from burning a given ton of fossil fuel one period earlier (but only deriving the benefit from burning the fuel once).