Final Exam

Resource Economics - ECON4925
Department of Economics, University of Oslo

December 14^{th}, 2022

This exam has 3 questions. The points of each question x are mentioned in brackets as [x p]. The exam sums to a total of 100 points. Provide well-motivated answers that are as concise as possible. If you get stuck in algebra or discover a mistake, explain how you would proceed, what you would expect, or why you think there is a mistake.

1. In this question, choose the right answer – "True" or "False" for part i, and "True", "False" or "It Depends" for part ii. [30p]

   i. True or false? The Faustmann rotation [10p]

   (a) considers both logging and non-logging benefits of harvests in forestry.
   (b) gives the optimal rotation length of a model with one hundred rotations.
   (c) assumes that operation costs within a rotation increase the longer said rotation is.
   (d) is shorter than the rotation that would maximize the mean annual increment.
   (e) is longer than the optimal single rotation.

   ii. True or false or it depends? If you choose "it depends" then you need to provide an explanation on what factor(s) and how it depends. Keep the answers concise.

   Assume a finite supply of a non-renewable resource. Consider the following scenarios regarding the optimal extraction plan. The optimal time horizon could be either finite or infinite. True, false or it depends? [20p]

   (a) If firms are competitive, it is not possible to decentralize resource extraction such that the outcome is socially optimal.
   (b) A social planner with constant absolute risk aversion ($\frac{1-e^{-at}}{a} + b, a > 0, b \in \mathbb{R}$) will optimally choose to extract over a finite time horizon, and will exhaust the resource stock fully.

2. **Fishery Economics:** Consider a fishery where the biomass of some fish S_t in international waters evolves according to the equation of motion

$$\dot{S}_t = g(S_t) - H_t,$$

where time is continuous, $H_t \geq 0$ is the size of harvest, and

$$g(S_t) = r\left(2\sqrt{S_t} - \frac{S_t}{\sqrt{M}}\right),$$

gives the dynamics of the fish population in the absence of harvesting, where $r > 0$ and $M > 0$ are parameters defined by the biological conditions of the marine ecosystem. Initial fish stock is positive $S_0 > 0$. (Observe that the derivative of the function $f(x) = \sqrt{x} = x^{0.5}$ is $f'(x) = \frac{1}{2\sqrt{x}}$.) [40p]

(a) Find the maximum sustainable yield S^{MSY}. [3p]

(b) Find the carrying capacity of the fish population S^{CC}. [3p]

A social planner (a state or government) will choose the optimal sizes of harvest H_t over an infinite time horizon, with immediate utility $u(H_t) = 2\sqrt{H_t}$, and discounts the utility stream at an interest rate $i \geq 0$.

(c) Formulate the dynamic optimization problem that the social planner faces, and identify the control and state variables. [5p]

(d) Formulate the current-value Hamiltonian and derive the first-order conditions. [5p]

(e) Show that the Euler equation is given by [4p]

$$\frac{\dot{H}_t}{H_t} = 2\left(r\left[\frac{1}{\sqrt{S_t}} - \frac{1}{\sqrt{M}}\right] - i\right).$$

(f) Find the optimal steady state level of fish stock S^{stst} resulting from this optimization problem. [4p]

(g) Under which circumstances is the steady state level of fish stock equal to the maximum sustainable yield ($S^{stst} = S^{MSY}$)? Is one larger than the other in general? If yes, which? Explain. [6p]

(h) Use the equation of motion for the fish stock and the Euler equation to perform phase plane analysis on the solutions to the problem. Put S_t and H_t on the horizontal and vertical axes respectively. Illustrate the separatrix, and explain what it represents economically. [10p]

3. **Lake Pollution:** A social planner wants to feed their population. Agricultural activity at each point in continuous time t, however, will entail polluting a nearby lake by a phosphorous loading c_t, which will increase the stock of phosphorous x_t. The planner, hence, faces the following welfare maximization problem:

$$\max_{c_t \geq 0} \int_0^\infty [u(c_t) - \beta x_t^2] e^{-\rho t} dt,$$
$$\text{such that } \dot{x}_t = c_t - \alpha x_t + f(x_t),$$

where $u'(c) > 0$ and $u''(c) < 0$, and initial stock of pollution $x_0 > 0$ is given. Moreover, $f(x) = \frac{x^2}{1+x^2}$. (Observe that $f'(x) = \frac{2x}{(1+x^2)^2}$, $f''(x) = \frac{2-6x^2}{(1+x^2)^3}$.) Moreover, $0 < \alpha < 1$, $\rho > 0$ and $0 < \beta < 1$. [30p]

(a) Explain the concept of a Skiba point in a resource management problem. What differs in the underlying dynamic system as opposed to typical dynamic systems that do not contain Skiba points? What are the implications for the solution set? Explain why it may emerge in the context of the optimization problem above. (No equations or graphs are required, but provide any that would help in explaining the concept.) [20p]

The function $u(c_t)$ represents the immediate utility stream from agricultural activity. The planner, however, is unsure of the exact benefits of the farming products for the population. It is therefore considering two utility functions: $u_1(c_t) = \log c_t$ and $u_2(c_t) = \frac{1-e^{-ac_t}}{a}$, $a > 0$.

(b) Show that if $u(c_t) = u_2(c_t)$, then $c_t = 0$ when $\mu_t \leq -1$, where μ_t is the current-value shadow value of pollution stock. What is the corresponding condition for $u(c_t) = u_1(c_t)$? [6p]

(c) Which of these two utility structures is more likely to induce a Skiba point for a given parameter space? Why? Motivate your answer concisely. [4p]