Final Exam – Guidelines

Resource Economics - ECON4925
Department of Economics, University of Oslo

December 18, 2023

This exam has 3 questions. The points x of each question are mentioned in brackets as [x p]. The exam sums to a total of 100 points. Provide well-motivated answers that are as concise as possible. If you get stuck in algebra or discover a mistake, explain how you would proceed, what you would expect, or why you think there is a mistake.

1. **Forestry Economics:** In this question, choose only "True" or "False". Do not explain your answer. [15p]

   In a forestry model, assume that the growth function is increasing and concave, forest management is profitable and the optimal Faustmann rotation length is smaller than that of the maximum sustainable yield. Then, under the Faustmann rotation

   (a) when suppliers become more patient, timber supply decreases in the short run but increases in the long run. [3p]

   (b) when the price of timber rises, timber supply increases in the short run but decreases in the long run. [3p]

   (c) timber supply increases in both the short and long run when cost of production falls. [3p]

   (d) land rent increases when costs fall, and thus more land is devoted to forestry. [3p]

   (e) the average annual harvest decreases when suppliers become more patient. [3p]

   Solution: a)T b)F c)F d)T e)F. See slide 36 of the lecture on forestry economics.

2. **Fishery Economics:** Consider a fishery where the biomass of some fish S_t in international waters evolves according to the equation of motion

$$\dot{S}_t = g(S_t) - H_t,$$

   where time is continuous, $H_t \geq 0$ is the size of harvest, and

$$g(S_t) = 2r\left(\sqrt{S_t} - \frac{S_t}{\sqrt{M}}\right),$$

   gives the dynamics of the fish population in the absence of harvesting, where $r > 0$ and $M > 0$ are parameters defined by the biological conditions of the marine ecosystem. Initial fish stock is positive $S_0 > 0$. (Observe that the derivative of the function $f(x) = \sqrt{x} = x^{0.5}$ is $f'(x) = \frac{1}{2\sqrt{x}}$.) [50p]

   (a) Find the maximum sustainable yield S^{MSY}. [3p]

   $g'(S_t) = 0 \Rightarrow r\left(\frac{1}{\sqrt{S_t}} - \frac{2}{\sqrt{M}}\right) = 0 \Rightarrow S^{MSY} = \frac{M}{4}$.

   (b) Find the carrying capacity of the fish population S^{CC}. [3p]

   $g(S_t) = 0 \Rightarrow 2r\left(\sqrt{S_t} - \frac{S_t}{\sqrt{M}}\right) = 0 \Rightarrow \sqrt{S_t} - \frac{S_t}{\sqrt{M}} \Rightarrow S_t = \frac{S_t^2}{M} \Rightarrow S_t^2 - MS_t = 0 \Rightarrow S_t - M = 0 \Rightarrow S^{CC} = M$ (and $g(0) = 0$).

A social planner (a state or government) will choose the optimal sizes of harvest H_t over an infinite time horizon. The population receives some immediate utility $2\sqrt{H_t}$ from the harvest, plus some recreational utility from deep-sea diving $2\alpha_S\sqrt{S_t}$ ($\alpha_S \geq 0$), which decreases the less fish remains in the sea. As a result, the total immediate utility is given by $u(H_t, S_t) = 2\sqrt{H_t} + 2\alpha_S\sqrt{S_t}$. Moreover, the utility stream is discounted at some interest rate $i \geq 0$.

(c) Formulate the dynamic optimization problem that the social planner faces, and identify the control and state variables. [5p]

$$\max_{H_t \geq 0} \int_0^\infty 2\left(\sqrt{H_t} + \alpha_S\sqrt{S_t}\right) e^{-it} dt$$

$$\text{such that } \dot{S}_t = 2r\left(\sqrt{S_t} - \frac{S_t}{\sqrt{M}}\right) - H_t$$

$$\text{and given } S_0 > 0.$$

The state variable is the biomass of the fish (fish stock) S_t and the control variable is the the size of harvest H_t.

(d) Formulate the current-value Hamiltonian and derive the first-order conditions. [5p]

Current-value Hamiltonian $\mathcal{H}^c(S_t, H_t, \mu_t) = 2\left(\sqrt{H_t} + \alpha_S\sqrt{S_t}\right) + \mu_t[g(S_t) - H_t]$. The first-order conditions (FOC):

1. $\dfrac{\partial \mathcal{H}^c}{\partial H_t} = 0 \Rightarrow \dfrac{1}{\sqrt{H_t}} - \mu_t = 0 \Rightarrow \mu_t = \dfrac{1}{\sqrt{H_t}}$ or $H_t = \dfrac{1}{\mu_t^2}$;
2. $\dfrac{\partial \mathcal{H}^c}{\partial S_t} = i\mu_t - \dot{\mu}_t \Rightarrow \dfrac{\alpha_S}{\sqrt{S_t}} + \mu_t g'(S_t) = i\mu_t - \dot{\mu}_t \Rightarrow \dfrac{r + \frac{\alpha_S}{\mu_t}}{\sqrt{S_t}} - \dfrac{2r}{\sqrt{M}} = i - \dfrac{\dot{\mu}_t}{\mu_t}$;
3. $\dfrac{\partial \mathcal{H}^c}{\partial \mu_t} = \dot{S}_t \Rightarrow \dot{S}_t = g(S_t) - H_t$, i.e. the equation of motion.

(e) Show that the Euler equation is given by [4p]

$$\frac{\dot{H}_t}{H_t} = 2\left(\left[\alpha_S\sqrt{\frac{H_t}{S_t}} + \frac{r}{\sqrt{S_t}} - \frac{2r}{\sqrt{M}}\right] - i\right).$$

From the first FOC we know that

$$H_t = \frac{1}{\mu_t^2} \xrightarrow[\text{respect to time}]{\text{Differentiation with}} \dot{H}_t = \frac{-2\dot{\mu}_t\mu_t}{\mu_t^4} = \frac{-2\dot{\mu}_t}{\mu_t} \cdot \underbrace{\frac{1}{\mu_t^2}}_{=H_t} \Rightarrow \frac{\dot{\mu}_t}{\mu_t} = -\frac{1}{2} \cdot \frac{\dot{H}_t}{H_t}.$$

Replacing the expression for $\frac{\dot{\mu}_t}{\mu_t}$ in the second FOC with the expression above and replacing μ_t with $\frac{1}{\sqrt{H_t}}$ from the first FOC, we get

$$\alpha_S\sqrt{\frac{H_t}{S_t}} + \frac{r}{\sqrt{S_t}} - \frac{2r}{\sqrt{M}} = i + \frac{1}{2} \cdot \frac{\dot{H}_t}{H_t},$$

and rearranging we get the Euler equation as in the question.

(f) Provide an economic interpretation for the Euler equation in the steady state. [12p]

In a steady state $\dot{H}_t = 0$, and then the Euler equation implies

$$\underbrace{\alpha_S \sqrt{\frac{H^{stst}}{S^{stst}}}}_{=\frac{u'_S(H^{stst},S^{stst})}{u'_H(H^{stst},S^{stst})}} + \underbrace{\frac{r}{\sqrt{S^{stst}}} - \frac{2r}{\sqrt{M}}}_{=g'(S^{stst})} = i$$

which is the fundamental equation of renewable resources for this model. It states that in the steady state, the annual opportunity cost of postponing sales (represented by the interest rate i) is equal to the Annual biomass growth increase from an additional unit of fish in the sea ($g'(S^{stst})$) plus the annual marginal benefit from leaving an additional unit of fish in the sea relative to its harvesting value $\left(\frac{u'_S(H^{stst},S^{stst})}{u'_H(H^{stst},S^{stst})}\right)$.

(g) Find the equation that provides the optimal steady state level of fish stock S^{stst} resulting from this optimization problem. (Hint: You do not need to find the steady state level itself, just the equation that would produce it in terms of all of the parameters in the model.) [6p]

The steady state level of fish stock S^{stst} is a point S^{stst} where there no longer is change in the dynamic system, i.e. $\dot{H}_t = 0$ and $\dot{S}_t = 0$. The latter, $\dot{S}_t = 0$ yields $H_t = g(S_t)$. On the other hand, $\dot{H}_t = 0$ yields $H_t = 0$ or

$$2\left(\left[\alpha_S\sqrt{\frac{H_t}{S_t}} + \frac{r}{\sqrt{S_t}} - \frac{2r}{\sqrt{M}}\right] - i\right) = 0 \Rightarrow \alpha_S\sqrt{H_t} = i\sqrt{S_t} - r\left(1 - 2\sqrt{\frac{S_t}{M}}\right)$$

$$\Rightarrow H_t = \frac{1}{\alpha_S^2}\left[i\sqrt{S_t} - r\left(1 - \sqrt{\frac{S_t}{S^{MSY}}}\right)\right]^2,$$

where in the last step we have replaced $\frac{M}{4}$ with S^{MSY}. Then combining this expression and $H_t = g(S_t)$ we get

$$g(S_t) = \frac{1}{\alpha_S^2}\left[i\sqrt{S_t} - r\left(1 - \sqrt{\frac{S_t}{S^{MSY}}}\right)\right]^2.$$

Solving this equation for S_t will produce the steady state level of fish stock S^{stst}.
Observe that there are two other stedy states given by where $H_t = 0$. The corresponding fish stock levels at these steady states, are $S = 0$ and $S = S^{CC} = M$.

(h) We can use the equation you found in (g) to show the following result:

1. $S^{stst} > S^{MSY}$ when $\frac{\alpha_S}{i} > Q$,
2. $S^{stst} < S^{MSY}$ when $\frac{\alpha_S}{i} < Q$, and,
3. $S^{stst} = S^{MSY}$ when $\frac{\alpha_S}{i} = Q$,

where $Q > 0$ is determined by parameters in the model. Provide an interpretation of this result in economic terms, and explain why you think it holds. (Hint: Take the result stated as given. You do not need to show that the result holds or what the value of Q is.) [12p]

The result states that the steady state fish stock S^{stst} can be either equal, larger or smaller than maximum sustainable yield S^{MSY} depending on whether the utility weight of non-harvest utility (i.e. deep-sea diving) – represented by α_S – relative to the opportunity cost of postponing capture – represented by the interest rate i – is equal to, less than or larger than some value $Q > 0$. For instance, when non-harvest utility weight relative to the opportunity cost of harvest is lower than this value Q, it is less beneficial to postpone capture, and thus the planner will harvest more frequently, and thus the steady state fish stock is lower than the maximum sustainable yield. However, if the benefits from deep-sea diving are large enough so that the non-harvest utility weight relative to the opportunity cost of harvest exceeds this quota Q, then the planner will postpone capture and thus the steady state fish stock will exceed that of the maximum sustainable yield.

3. **Fossil Fuel Extraction and Climate Economics:** [35p]

(a) Explain the core insights of Hotelling's rule in economic terms for the extraction of a non-renewable resource such as fossil fuels. [5p]

Hotelling's rule is the Euler equation for the optimal extraction problem of a non-renewable resource. If firms are competitive and there are no fixed costs or externalities, the Hotelling rule states that the resource rent grows at the rate of interest, i.e. opportunity cost of operation.

(b) List the three key assumptions for the derivation of the Hotelling rule, and explain in words how the insights of the rule is impacted if these assumptions are violated. [20p]

The main assumptions are that firms are competetive, that there are no fixed costs and that there are no externalities (positive or negative). For a monopolist, the Hotelling rule states that the rental rate grows at a rate higher than the interest rate as the monopolist can sell its product at a mark-up. As a result, it extracts the resource more slowly. If costs grow with deplenishing resource stock (e.g. with fixed costs in production), then the Hotelling rule states that the rental rate of the resource should grow at a rate lower than that of the interest rate. This is due to the fact that we save cost by leaving some resource stock in the ground. Negative externalities such as climate change should make the rental rate grow at a lower rate than the interest (and vice versa for positive ones).

(c) Provide examples of fossil fuels with low and high royalties. The use of which fossil fuel type reacts more strongly to an (unannounced) increase in carbon tax for CO_2 emissions? Explain intuitively. [10p]

An (unannounced) increase in carbon tax for CO_2 emissions reduces emissions more strongly from fuels with low royalty (e.g. coal) compared to those with high royalty (e.g. oil). A Higher social cost of carbon (SSC, i.e. carbon tax) reduces emissions. However, the Hotelling rent is determined by scarcity and terminal conditions that all fossil fuel is used up. Hence, lower fossil fuel demand implies a reduction in Hotelling rent until all fossil fuel is used up again. Therefore, the impact of an increase in carbon tax (SCC) is partly crowded out by a royalty reduction.