Seminar 6

- Sun Gang: Monday, 26.03
- Sun Gang: Wednesday, 28.03

1. (Growth Accounting) The aggregate production function is Cobb-Douglas
$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}.$$
Y_t is output, A_t is total factor productivity (TFP), K_t is the stock of physical capital, N_t is the aggregate labor supply, and $0 < \alpha < 1$.
   (a) Derive the expression for TFP growth by the method of growth accounting.
   (b) How do we get α by aggregate data?

2. (Saving Rate) Consider an individual who lives for two periods. Her life-time utility function is given by
$$U(C_1, C_2) = u(C_1) + \beta u(C_2),$$
where $u' > 0, u'' < 0$,and $0 < \beta < 1$ is the discount factor.She works and consumes for both periods. She has no initial asset but she can save the amount s by buying asset A_2 at interest rate r. Her budget constraint of each period is as follows:
$$C_1 + s = Y_1$$
$$C_2 = (1 + r)A_2 + Y_2$$
$$s = A_2$$
   (a) Rewrite the budget constraints as one life-time budget constraint.
   (b) Write down the individual's maximization problem. Solve for the Euler equation. (An equation linking C_1^* and C_2^*).
   (c) Assume that $\beta(1 + r) = 1$, solve for the C_1^*, C_2^* and the saving rate s^*/Y_1.
   (d) If the individual has to pay a lump sum tax T at period 1 and get the pension b in period 2 and if the pension system is fully funded, i.e. $b = T(1 + r)$, what will happen to the budget constraint, the Euler equation and the saving rate?
   (e) Consider a permanent wage increase, i.e. $Y_i^{'} = (1 + \theta)Y_i, i = 1, 2.$ $\theta > 0$.What is the response of the saving rate?
   (f) Consider a transitory wage increase, i.e. $Y_1^{'} = (1+\theta)Y_1$, $Y_2^{'} = Y_2, \theta > 0$. What is the response of the saving rate?
   (g) The two-period model may seem unrealistic. But will the previous results change if the individual lives for infinite periods and works, consumes and saves in each period?

3. (Current Account) Consider a two-period small open economy. Assume all the individual are identical and the population is 1. This assumption allows us to identify the individual quantity with the national aggregate quantity. We can use the same mathematical model as in Question 2, if we redefine C as the aggregate consumption, Y as the national output, s as the aggregate savings, r as the world interest rate, and A as the value of net foreign asset.

   (a) What is the GDP, GNP, net export, current account and capital account in each period of this country?

   (b) Define the equilibrium of this country.

   (c) Assume $\beta(1+r)=1$, solve for the balance of current account in the equilibrium.

   (d) If there is a goverment spending G_1, G_2 on both periods, what will happen to the budget constraint and Euler equation?

   (e) Consider a permanent change of output, i.e. $Y_i^{'}=(1+\theta)Y_i, i=1,2$. $\theta>0$. What will happen to the equilibrium consumption and current account?

   (f) Consider a transitory change of output, i.e. $Y_1^{'}=(1+\theta)Y_1$, $Y_2^{'}=Y_2, \theta>0$. What will happen to the equilibrium consumption and current account?

   (g) If this country wants to implement the 'Balanced Current Account' policy, how do you formalize it in this model? What will be the welfare implication of this policy?

   (h) If the rest of the world set some limit to the value of foreign asset of this country, i.e. $A_2 \leq X_H$, as well as some borrowing limit, i.e. $A_2 \geq X_L$, where $X_L < 0 < X_H$, what will happen to the equilibrium consumption and current account?