1 Course Objectives

This is a course in the basic tools of numerical analysis that can be used to address analytically intractable problems in economics. A large class of problems cannot be analyzed with analytical tools, and numerical methods are increasingly expanding the questions we can address.

Numerical methods are vital to all types of applied economic research. The generality with which the techniques will be presented in this course will make them applicable to a wide range of fields, including macroeconomics, finance, econometrics, game theory, public finance, contract theory and others.

In order to learn how to use computational tools in an informed and intelligent way, this course endeavors to explain not only when and how to use various numerical algorithms but also how and why they work; in other words, the course opens up the “black boxes” and provide the students with a versatile toolbox for their own research.

At the end of the course special attention will be paid to dynamic economic problems, including methods for solving models with heterogenous agents.

2 Overview

We will use the stochastic neoclassical model as an organizing framework throughout the course. In the beginning we will use this model to motivate the study of elementary numerical methods for differentiation, optimization, root-finding, approximation and integration. We will then to combine these elementary methods to analyze functional problems, in particular value function iteration on a discrete grid, finite elements methods, and Chebyshev collocation, pertubation methods, linear quadratic methods, Euler equation methods incl. projection methods and weighted residual
methods. Finally, we will see that the unconditional time-series properties of the neoclassical growth model can be interpreted as the unconditional cross-sectional properties of different model and we will use this to solve a model with heterogeneous individuals and idiosyncratic risk.

3 Administrative

3.1 Lectures

Thursday afternoons.

3.2 Problem sets and seminar

One problem set per week.

3.3 Evaluation

t.b.a.

4 Suggested preparations

Suggested preparations for the class.

Programming  Elementary structures in Matlab at the level of pp. 1-10 of the Matlab Primer. Available for free on the internet.

Dynamic macroeconomics:

- The recursive formulation of the stochastic neoclassical growth model.
  Recommended readings: Chapter 3 of Per Krusell’s textbook manuscript; Krusell [2006].

Math preliminaries

- Taylor’s theorem, Orders of convergence, Difference equations, Computer arithmetic
  Recommended readings: Chapters 1 and 2 (Math preliminaries and Computer Arithmetic) of Kincaid and Cheney [2001]

- Metric spaces and normed vector spaces, the contraction mapping theorem
  Recommended readings: Chapter 3 of Stokey and Lucas [1989]
5 Software

This course will not teach programming per se, but it will teach and emphasize general principles of programming, such as simplicity, clarity, structure, replicability, and testing. Since one of the objectives of this course is to teach students what is going on inside the “black boxes” of numerical algorithms, students should avoid the use of pre-programmed numerical tool-boxes.

Students may use any programming language to complete the problem sets, including Fortran, C, C++, Matlab, and Gauss. If you intend to solve large-scale problems in the future, it is well worth the effort to learn Fortran or C++.

6 Textbooks


Students are also encouraged to buy the following book:


Other (optional) books students might find useful include:

- Recursive Macroeconomic Theory by Tom Sargent and Lars Ljungqvist (MIT Press, 2005).
- Dynamic General Equilibrium Modelling by Heer and Maßner (Springer Verlag, 2005)
7 Lecture Plan

1. **Thursday January 17**
   Introduction and overview
   Numerical Differentiation
   - Numerical Recipes: Chapter 5.7
   - Judd: Chapters 1, 2, and 7.7

2. **Thursday January 24**
   Root-finding (bisection, secant method, Newton’s method, fixed-point iteration, Gauss-Jacobi, Gauss-Seidel, Brent’s method)
   - Numerical Recipes: Chapter 9
   - Judd: Chapter 5
   Minimization in one or more dimension
   - Numerical Recipes: Chapter 10
   - Judd: Chapter 4

3. **Thursday January 31**
   Interpolation and approximation of functions, finite elements methods
   - Numerical Recipes: Chapters 3 and 6
   - Judd: Chapter 6

4. **Thursday February 7**
   Interpolation and approximation of functions, (orthogonal) polynomials
   - Numerical Recipes: Chapters 3 and 6
   - Judd: Chapter 6

5. **Thursday February 14**
   Numerical Integration (Newton-Coates quadrature, Gaussian quadrature, Monte Carlo integration)
6. Thursday February 21
Solving Recursive Problems: Value function iteration, Discretization

- Ljungqvist and Sargent [2004, Ch. 1, 3 and 4.1-4.7]
- Krusell [2006, Ch. 3]
- Burnside [1999]
- Judd [1998, Ch. 12]
- Stokey and Lucas [1989, Ch. 2]
- Hansen and Prescott [1995]

7. Thursday February 28

8. Thursday March 6

9. Thursday March 13
Value function approximation: Finite Element Methods and Chebyshev Collocation

- Ljungqvist and Sargent [2004, Ch. 4.7-4.8]
- Judd [1998, Ch. 6 and 11]
- McGrattan [1996]
- Trick and Zin [1997]

10. Thursday March 20
Solving Recursive Problems: Value function iteration, Linear Quadratic Methods and Log-Linearization

- Ljungqvist and Sargent [2004, Ch. 5]
- Díaz-Giménez [1999]
- Backus [2004]
- Hansen and Prescott [1995]
- Tallarini [2000]

11. Thursday April 3rd
General perturbation methods

- Judd [1998, Ch. 13-14]
12. **Thursday April 10th**
   Euler equation methods: Weighted residual method
   - Judd [1998, Ch. 11]
   - McGrattan [1999]

Euler equation methods: Parameterized Expectations
- Christiano and Fisher [2000]

13. **Thursday April 17st**
   Stationary incomplete market models, heterogenous agents – Bewley-Aiyagari-Hugget-models
   - Aiyagari [1994]
   - Ljungqvist and Sargent [2004, Ch. 16-17]
   - Huggett [1993], Huggett [1997]
   - Krusell and Smith [1998]

8 **Seminars**

  t.b.a.

**References**


