FIL2405/FIL4405 – Philosophical logic and the philosophy of mathematics

Instructor
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Requirements
Every student is required to make one oral presentation.
The mark for the course will be based on two 5-page essays due 16 March and 26 May at 2pm.

Readings
The readings are listed below. You *must* have read the readings marked by '*' before class; otherwise you won't be able to follow the discussion. You should expect to have to read the main readings *several times*. I have listed a number of further optional readings, which are *not* part of the official curriculum.

Students should obtain copies of the following two books:

- Øystein Linnebo, *Philosophy of Mathematics* (Princeton University Press, 2017)
- Paul Benacerraf and Hilary Putnam, *Philosophy of Mathematics: Selected Readings* 2nd ed. (Cambridge UP, 1983)

The former is a textbook that provides an introductory to the subject. The latter is a classic anthology containing most of the articles we will study.

If you wish to consult another textbook, a good option (but not part of the curriculum) is

- Stewart Shapiro, *Thinking about Mathematics* (Oxford UP, 2000)

All our main readings will be available in these two books, online, or through Canvas.

Course overview
Pure mathematics appears to be *very* different from the empirical sciences: It appears not to rely on experience but to be completely *a priori*; its truths appear to be *necessary* rather than contingent; and it appears to be concerned with *abstract objects* rather than concrete (spatiotemporal, causally efficacious) ones. These three features of pure mathematics—its apparent apriority, necessity, and concern with abstract objects—give rise to some deep and extremely interesting philosophical questions. Are these features to be taken at face value? If so, how are they to be understood? In particular, how are these features to be reconciled with a scientific world view? Alternatively, if the special features of mathematics are *not* taken at face value, can we give an alternative explanation of mathematics which nevertheless does justice to mathematical practice and mathematical experience?

We will discuss a number of classical and contemporary approaches to these questions and related ones. Topics to be discussed include the following.

- Some traditional philosophical views of mathematics (Plato, Kant)
- Is mathematics reducible to "pure logic"? (Frege, Russell)
- Are mathematical truths just useful conventions? (Hempel)
- Is mathematics a science of mental constructions? (Brouwer, Heyting)
- Is mathematics just a formal game with uninterpreted symbols (Curry, Hilbert)
- Is mathematics empirical after all, just unusually general and abstract? (Quine)
- If there are abstract mathematical objects, how can we know about them? (Benacerraf, Gödel, Maddy)
- Can sense be made of mathematics without postulating mathematical objects? (Field)
- Are mathematical objects just points in mathematical structures? (Benacerraf, Resnik)

Program and readings

<u>1. Mathematics as a philosophical problem</u>
Theme: Mathematics appears to be very different from other sciences in being *a priori*, necessary, and concerned with abstract objects. How might such a science be possible?

Main readings

- *Plato, *Meno*, 80a-86c [8pp]
- *Kant's *Critique of Pure Reason*, B-edition, sections I–V [13pp]
- Linnebo, Introduction and ch. 1 [18pp] [total ca. 41]

Optional further readings

- Shapiro, pp. 49-63, 73-91, though ch.s 1 and 2 are also relevant
- Kant's *Critique of Pure Reason*, B-Edition Introduction, "The Discipline of Pure Reason in Its Dogmatic Use" (Part II, Ch. 1, Section 1; esp. A712/B740-A724/B752)

<u>2. Frege's logicism</u>
Theme: According to Frege, large parts of mathematics are reducible to "pure logic", thus making mathematical truths analytic. But Frege's logicist project was shaken by Russell's paradox.

Main readings

- *Frege's *Foundations of Arithmetic*, sections 1-4 (on Canvas), and sections 55-61 and 106-109 (in B&P) [34pp]
- Linnebo, ch. 2 [17pp] [total ca. 51pp]

Optional further readings: see the end of Linnebo, ch. 2.

3. Formalism and deductivism

Theme: Mathematics is a science, not just a game or an activity of make-believe. And although proof is an essential tool for the discovery of mathematical truths, mathematical truth cannot be reduced to proof.

Main readings

- *Frege, *Basic Laws of Arithmetic* (OUP, 2013), Sections 86-94, 113-4, 118-9, 123-5 (17pp)
- Linnebo, ch. 3 [18pp]
- Putnam, "The thesis that mathematics is logic", in his *Mathematics, matter and method* [30pp] [total ca. 65pp]

Optional further readings

- Frege, Preface to *Begriffsschrift*
- Resnik, *Frege and the Philosophy of Mathematics* (Cornell UP, 1980), pp. 54-65, 65-75, and 119-130

4. Hilbert's formalism

- Hilbert, "On the Infinite", in Benacerraf and Putnam (1983) [20pp]
- Linnebo, ch. 4 [17pp] [total 37pp]

Optional further readings

- Shapiro, *Thinking about Mathematics*, ch. 6
- Curry, "Remarks on the Definition and Nature of Mathematics", *Dialectica* 8 (1954) and in Benacerraf and Putnam (1983) [5pp]
- Detlefsen, *Hilbert's Program* (Reidel, 1986)
- Resnik, *Frege and the Philosophy of Mathematics* (Cornell UP, 1980), pp. 76-104
- Tait, "Finitism", *Journal of Philosophy*, 1981
- Zach, "Hilbert's Program", *Stanford Encyclopedia of Philosophy*

5. Intuitionism

- Heyting, "The Intuitionist Foundations of Mathematics" and "Disputation," both in B&P [19pp]
- Brouwer, "Intuitionism and Formalism", in B&P [13pp]
- Linnebo, ch. 5 [15pp] [total ca. 47pp]

Optional further readings

- Shapiro, ch. 7
- The remaining articles from B&P on intuitionism

6. Quine's empiricist platonism

Theme: Is Quine right that there is no difference of kind between the truths of mathematics and other scientific truths?

Main readings

- *Quine, "Two Dogmas of Empiricism" (esp. the final two sections), in his *From a Logical Point of View* (Harvard UP, 1953) [23pp]
- Quine, *Pursuit of Truth* (Harvard UP, 1990), Section 40 [1p]
- Quine, *From Stimulus to Science* (Harvard UP, 1995), ch. 5 [7pp]
- Colyvan, "Indispensability Arguments in the Philosophy of Mathematics," *Stanford Encyclopedia of Philosophy* [7pp]
- Linnebo, ch. 6 [12pp] [total ca. 50pp]

Optional further readings

- Shapiro, pp. 212-20
- A.J. Ayer, "The *A Priori*," in B&P
- Shapiro, pp. 124-133
- Rudolf Carnap, "Empiricism, Semantics, and Ontology," in B&P
- Quine, "Carnap on Logical Truth," in B&P
- Parsons, "Quine and the Philosophy of Mathematics"

7. Nominalism

Theme: Is Field right that science can and should be rewritten in a way that eliminates all reference to mathematical objects? Or can nominalism be established in some easier way?

Main readings

- *Field, "Realism and Anti-Realism about Mathematics," in his *Realism, Mathematics, and Modality* (Blackwell, 1989) [25pp]
- Melia, "On What There's Not", *Analysis* 55 (1995): 223-229 [7pp]
- Linnebo, ch. 7 [15pp] [total ca. 47pp]

Optional further readings

- Paul Benacerraf, "Mathematical Truth," *Journal of Philosophy* 70 (1973) and in B&P (18pp)
- Shapiro, pp. 226-237, 243-249
- Eklund, "Fictionalism", *Stanford Encyclopedia of Philosophy*
- Yablo, "The Myth of the Seven" (available from his home page)
- Yablo, "Abstract Objects: A Case Study", *Philosophical Issues* 12 (2002)
- Yablo, "Go Figure", *Midwest Studies in Philosophy*, 25 (2001), Appendix pp. 93-102

8. Mathematical intuition

Theme: Do we have some form of intuition of abstract mathematical objects?

Main readings

- Parsons, "Mathematical intuition" [23pp]

- Føllesdal, "Gödel and Husserl", available on Canvas [16pp]
- Linnebo, ch. 8 [10pp] [total 49pp]

Optional further reading

- Penelope Maddy, *Realism in Mathematics* (Oxford University Press, 1990), pp. 50-75
- Parsons, "Platonism and mathematical intuition in Kurt Gödel's thought", *Bulletin of Symbolic Logic*, 1 (1995): 44–74

9. Abstraction reconsidered

Theme: Might an account of abstraction, perhaps inspired by Frege, explain the nature of mathematics and of mathematical knowledge?

Main readings

- Heck, "An Introduction to Frege's Theorem", *Harvard Review of Philosophy* 7 (1999), pp. 56-73 [18pp]
- Wright, "On the Philosophical Significance of Frege's Theorem", in *Reason's Proper Study* (OUP 2001), Sections I and II [10pp]
- Linnebo, ch. 9 [13pp] [total ca. 41pp]

Optional further readings

- Shapiro, pp. 107-115, 133-138
- Dummett, *Frege: Philosophy of Mathematics*, pp. 111-119 and ch. 11 [23 pp]
- Bob Hale and Crispin Wright, *Reason's Proper Study* (OUP, 2001), esp. the Introduction [27pp]
- G. Boolos, "Gottlob Frege and the Foundations of Arithmetic," in his *Logic, Logic, and Logic* (Harvard UP, 1998)

10. The iterative conception of sets

Theme: The iterative conception of sets. Does this conception provide a justification for the axioms of standard ZFC set theory?

Main readings

- *Boolos, "The Iterative Conception of Set", esp. Sections I and II, in B&P [10pp]
- Parsons, "What is the Iterative Conception of Set?", esp. pp. 503-515, in B&P [13pp]
- Linnebo, ch. 10 [15pp] [total 33pp, though try to read *all* of the two assigned articles]

Optional further readings

- Boolos, "Iteration Again"
- Bernays, "On Platonism in Mathematics," in B&P

11. Structuralism

Theme: It is often asserted that mathematics is the science of abstract structures. What might this mean, and might it help us explain the nature of mathematics and mathematical knowledge?

Main readings

- Benacerraf, "What Numbers Could Not Be," in B&P [23pp]
- *Resnik, "Mathematics as a Science of Patterns: Ontology and Reference," *Noûs* 15 (1981), pp. 529-550 [22pp]
- Linnebo, ch. 11 [15pp] [total 60pp]

Optional further readings

- Shapiro, *Thinking about Mathematics*, ch. 10
- Parsons, "The Structuralist View of Mathematical Objects", *Synthese* 84 (1990), pp. 303-46.
- Shapiro, *Philosophy of Mathematics: Structure and Ontology* (OUP, 1997), pp. 71-106
- Resnik, *Mathematics as a Science of Patterns* (OUP, 1997), chapters 10-11
- Shapiro, *Philosophy of Mathematics: Structure and Ontology* (OUP, 1997), ch. 4
- MacBride, "Structuralism Reconsidered", *Oxford Handbook of Philosophy of Mathematics and Logic* (OUP, 2005), Section 3-4
- Linnebo, "Structuralism and the Notion of Dependence", *Philosophical Quarterly* 58 (2008), pp. 59-79
- Hellman, "Three Varieties of Mathematical Structuralism", *Philosophia Mathematica* 9 (2001), pp. 184-211
- MacBride, "Can Structuralism Solve the 'Access' Problem?", *Analysis* 64 (2004), pp. 309-17

12. The quest for new mathematical axioms

Theme: Cantor's continuum hypothesis and other mathematical questions are left open by our current axioms. Do such questions have objective answers? If so, can we find new axioms that enable us to *prove* these answers?

Main readings

- Russell, "The Regressive Method in Philosophy," repr. in Lackey ed. *Essays in Analysis by Bertrand Russell* (George, Allen & Unwin, 1973), pp 272-83 [12pp]
- *Gödel, "What is Cantor's Continuum Problem", esp. the Supplement (pp. 482-85), in B&P [4pp]
- Gödel, "Russell's Mathematical Logic", p. 449, in B&P [1p]
- Linnebo, ch. 12 [13pp] [total 30pp, but try to read all of the two assigned Gödel articles]

Optional further readings

- Field, “Which Undecidable Mathematical Sentences have Determinate Truth-Values?”, in his *Truth and the Absence of Fact* (2001)
- Koellner, “The Question of Absolute Undecidability”