8 Part 8: Making experiments with special relativity

You have now landed on your destination planet and you start making experiments on the surface of the planet. For part 8 and part 9 you need to have a partner student. Even if you work in a group, you need a partner group or partner student. You may do everything alone, but it requires more work. You will create a set of videos of the experiments you are performing on the surface of your planet. You will need to decide between you and your partner (or your group and your partner group) on which planet you will do the experiments: the planet where you have landed or the planet where your partner has landed. You may also do some experiments on your planet and others on your partners planet, but it is very important that for the same experiment, you use the same planet for all frames of reference.

Your task is to do a selection (listed below) of the weekly exercises of part 2A and 2B (with the addition of two extensions to the exercises at the end of this document), but you will not use the standard xml published on the web-site, you will rather generate your own xml of the experiments taking place on your (or your partners) planet. Before you continue reading, you now need to read section 7 of part 2A remembering that you are supposed to generate the xml yourself.

8.1 Creating xml for relativity exercises

Creating the xml files is easy: Make sure you have the latest version of AST2000SolarSystem.pyc. Then you use exercise-specific methods to generate xml. The format is the same for all exercises except 2A.8 which does not take input parameters. For instance to generate xml for both frames of exercise 2A.4:

```
mySS.part2A_4(chosen_planet)
```

assuming that your instance of AST2000SolarSystem is called `mySS`. This will create the xml files on your destination planet (specified in `chosen_planet`). If instead you have chosed to use your partners destination planet for this exercise:

```
mySS.part2A_4(chosen_planet, friend_seed = 114)
```

if the seed of your partner is 114, her/his destination planet is then specified in `chosen_planet`.

In some cases the videos generated will take place too close to the surface of the planet. If your video does not make sense, or does not show what you expect from the description of the exercises then you should try to regenerate you xml files using:

```
mySS.part2A_4(chosen_planet, increase_height = True)
```

which will move the events high above the surface where the landscape should not cause problems, or

```
mySS.part2A_4(chosen_planet, increase_height = number)
```

where `number` is a number between 0.5 and 5: below 1 means that you will be moved even closer to the surface, this may make the video look even nicer, but only if the landscape does not hide the events which you need. A number above 1 means that you are moved further away from the surface, but not as high as when using `increase_height = True`. In this way you may still get a nice video and see the events.

All exercises are created in the same way, except 2A.8 which is created by:

```
mySS.part2A_8()
```

since fixed planets are used for this exercise.

8.2 Reporting the relativity experiments

You need to do all off the following exercises: 2A.4, 2A.5, 2A.8, 2B.1, 2B.3, 2B.4 and 2B.5 as well as the extensions in this document. However, you only need to write an extensive report on parts of these exercises as detailed below.

In each experiment you are supposed to take the role of one of the observers and you should therefore report on the results from your frame of reference only, but tell us who the other observer was, and in some cases when the exercises ask for it, you may need to report some numbers/results which you obtain from your partner. In this part of the project, you should, even more than before, focus your reporting on **showing understanding**. For this reason, here is a list of parts of the parts of the exercises which you should focus your reporting on. You will need to report partially on the other parts of these exercises in order to make your report complete and understandable, but you only need to go into detail in the following parts. You will mainly be evaluated on how you explain the meaning of the results and show physical understanding:

1. **2A.4 space ship frame:** point 2c, **2A.4 planet frame:** point 3e and 3f, **2A.4 both observers:** all point from 4a to 4j.

2. **2A.5** point 9 to 12

3. **2A.8** point 11 to 15 including all the additional exercises for 2A.8 given below in this document.

4. **2B.1** the full exercise

5. **2B.3** points 2, 3, 6 and 7 as well as the additional exercises for 2B.3 given below (note 2B.5 should be done before the additional exercises to 2B.3)

6. **2B.4** points 9 to 11.

7. **2B.5** all points from 4 to 14

In your reporting, you should therefore give emphasis to these exercises, and in particular show that you understand the physics behind and also that you explain what you have learned about relativity from the given exercise.

8.3 Addition to exercise 2A.8

After having finished all of exercise 2A.8, make sure you understand very well all your results and calculations. We will now try to go even deeper in understanding the twin paradox and look in detail to what happens with time and space during the accelerated phase. At the end we will connect the accelerated phase to gravity and general relativity.

1. Return to point 11 to 15 of 2A.8. This is where the solution to the paradox appears: It took the space ship 202 years to arrive at P2 measured on clocks in the planet system, it took only 28.5 years to arrive, measured on the space ships clocks, **BUT**, for observers in the outgoing space ship frame, only 4 years had passed on P1 when the space ship arrived at P2. We now want to repeat this calculation using invariance of the space time interval: Write the time and positions of event B and B' in terms of L_0 and v and show again that $t_{B'} = L_0/v - L_0 v = L_0/(v\gamma^2)$.

2. Further, in point 15 you show that, just after the astronaut arrived in the returning elevator, 396 years had now passed on P1. Time on P1, as measured by an elevator satellite at the position of P1, suddenly made a huge jump when shifting from one system of reference to the other. Clearly the change of reference system is not instantaneous as we assumed in 2A.8. We will now assume that the space ship, starting at P2 starts accelerating with a constant negative acceleration (decceleration) g measured in the planet frame. Some time ΔT after reaching P2, measured on planet system clocks, the space ship has reached velocity 0 and will start returning to P2. Show that $\Delta T = -v_0/g$ where v_0 is the velocity of the space ship just before starting to accelerate.

3. What is the velocity of the space ship when it returns to P2? (assuming the space ship continues accelerating with the same constant acceleration (in the planet frame) also after reaching $v = 0$)

4. We will now calculate how time on P1 runs for the space ship system during the accelerated phase. Assume that during the accelerated phase, there are constantly events similar to event B and B' happening. We will call these series of events Y and Y'. We will use T_Y to denote planet system time after reaching P2, $T_Y = 0$ when starting to accelerate and $T_Y = \Delta T$ when the velocity has reached 0. At some time T_Y, event Y refers to an event happening at the current position of the space ship and Y' is an event happening simultaneously with event Y in the space ship frame. Event Y' happens at the position of P1: an astronaut in an outgoing space ship elevator having the same velocity as our space ship at time T_Y is just passing P1 checking the time on P1 clocks. This time is called $T_{Y'}$ and measures time passed on P1 clocks after the space ship arrived at P2 in the space ship frame (time $T_{Y'}$ can also be seen as the time which has passed since event B' happened measured on P1 clocks). Show that the

times and positions of an event Y and Y' can be written as

$$x_Y = L_0 + v_0 T_Y + \frac{1}{2} g T_Y^2 \qquad x'_Y = 0$$

$$t_Y = \frac{L_0}{v_0} + T_Y \qquad t'_Y = t'_Y$$

$$x_{Y'} = 0 \qquad x'_{Y'} = \frac{L_0 + v_0 T_Y + \frac{1}{2} g T_Y^2}{\gamma(T_Y)}$$

$$t_{Y'} = \frac{L_0}{v_0 \gamma_0^2} + T_{Y'} \qquad t'_{Y'} = t'_Y$$

Note that $\gamma(T_Y)$ here refers to γ taken with the velocity of the space ship at time T_Y. Make sure you understand and can deduce all these expressions.

5. Use again invariance of the space time interval to obtain an expression for $t_{Y'}$. Note: if you make this right, the equations will not be ugly. Make sure to have the square containing $t_{Y'}$ on the left handside and everything else on the right. Note that the full expression on the right can be written as a square. Taking the square root on both sides and making sure to use the correct sign, you will easily get a result.

6. Now assume a constant acceleration (decceleration) of $g = -0.1\text{m/s}^2$ in the planet frame . What is g written in units where distance and time are both measured in seconds?

7. Find numbers for t_Y and $t_{Y'}$ (expressed in years) when $T_Y = \Delta T$.

8. You should find that they are equal, why?

9. Plot the time $t_{Y'}$ elapsed on P1 measured in the space ship frame as a function of time t_Y elapsed on P1 in the planet frame in the full range $t_Y = 0$ to $t_Y = \frac{L_0}{v_0} + \Delta T$, i.e. during both the constant velocity as well as the decceleration phase. Does the P1 clock suddenly start to run quicker (as observed from the space ship frame) during the decceleration phase?

10. Use numbers derived above together with symmetry arguments to show that the P1 clock shows 588 years (seen from the returning elevator frame) as the space ship again reaches P2. We have now seen how P1 time suddenly could go from 4 years to 588 years during the acceleration phase.

11. We will finally calculate how much the astronaut aged during the acceleration phase. We will need to add this time to the 57 years he aged during the two constant velocity phases. In order to make the calculation easier, we will do this for the returning phase, starting at the moment when the velocity reached 0 (called event E) and the space ship starts returning to P2 accelerating. We will assume an infinite number of space ship elevators going towards P2 (and beyond), all with different velocities from 0 to v_0. The astronaut stays in one elevator during a short time interval $\Delta t'$ and then accelerates step by step by jumping to the neighbouring elevator having a slightly larger velocity than the previous. Show that at a time t, after event E measured on planet clocks, the relation between a time

interval in the space ship frame and a time interval in the planet frame can be written as

$$\Delta t' = \sqrt{1-g^2t^2}\Delta t \tag{1}$$

Thus, when a time t has passed after event E and the astronaut is in the elevator corresponding to current velocity, the time $\Delta t'$ which he remains in the elevator corresponds to a time interval Δt on planet clocks. **hint:** the usual formula for time dilation, but make sure you use it properly.

12. Show that the time the astronaut ages from event E to the astronaut reaches P2 again can be written as

$$t' = \int_0^{v_0/g} \sqrt{1-g^2t^2}dt$$

13. Using i.e. The Integrator, or tables of integrals, you can show that this integral can be written as

$$t' = \frac{v_0\sqrt{1-v_0^2} + \arcsin(v_0)}{2g}$$

Insert numbers and find the total time the astronaut has aged when returning to P1.

14. Finally we will link this to gravitation: We will call the distance from event E for r such that event E happens at $r = 0$ and the space ship accelerates in the positive r direction towards P2. Show that after time t, the position of the space ship can be written as $r = \frac{1}{2}gt^2$.

15. Show that the time elapsed when the space ship has reached distance r from event E can be written as $t = \sqrt{\frac{2r}{g}}$.

16. Show that eq.1 above can thus be written as

$$\Delta t' = \sqrt{1-2gr}\Delta t$$

17. The equivalence principle states that you cannot judge whether you are in an accelerated frame or in a gravitational field. We will now see one way to show this, by observing that clocks in an acclerated frame ticks with the same rate as in a gravitational field. We know that the gravitational acceleration is given by $g = \frac{GM}{r^2}$. Insert this expression for acceleration to show that

$$\Delta t' = \sqrt{1-\frac{2GM}{r}}\Delta t$$

In 2 weeks we will show that this is identical to the formula for time dilation in the gravitational field. We see that the form of the expressions for time dilation in an accelerated frame, or in a gravitational field take the same form. We would have got the same results if the astronaut had been in a gravitational field for some time instead of being accelerated. In both cases, time runs slower for the astronaut being either accelerated or in a gravitational field compared to observers in the frame not being accelerated or being outside of a gravitational field. This was one of Einstein's starting points when deducing the general theory of relativity.

Note that in our example, the acceleration is constant in the planet frame which means that the acceleration is not constant for the space ship observer. In order to make a direct comparison to a person in a constant gravitational field which we will consider in the lectures on general relativity (the so-called shell-observer), we should have used constant acceleration for the space ship which would have given a time dependent acceleration in the planet frame. We have avoided this due to the much uglyer calculations for the case of constant space ship acceleration.

8.4 Addition to exercise 2B.3

Go back to exercise 2B.3 and repeat what you did in that exercise. Look at both videos for the exercise. Note: it is important that you also completed exercise 2B.5 before starting this exercise.

1. We will again imagine a stick with the two laser beams as end points. Your task is now to find the length of the stick L' measured in the space ship frame, expresse only in terms of the length L in the planet frame and the relative velocity v between the frames. You should use invariance of the space time interval, but you need to define the events yourself. You get no help here.

2. Having obtained L', find an expression for the ratio L/L' in terms of v, insert numbers and compare to what you measured i the last questions of exercise 2B.3.

3. Imagine now that the laser beams are wave tops of an electromagnetic wave. Use your results for L and L' to derive the formula for relativistic Doppler shift obtained in exercise 2B.5? You have now seen that we can find the relativistic formula for Doppler shift, both by transforming energy as well as by considering the space time interval for events happening along the world lines of light beams.