AST 2000 - Part 6
Preparing For The Landing

Welcome to Part 6 of the AST2000 Spacecraft Project. You are now going to examine your destination through a series of calculations and simulations. In addition, you are also going to scout the surface of the planet for potential landing sites. **For this part, you are supposed to deliver a full scientific paper, including introduction, theory (if you wish), method, results, discussion and conclusion (the latter two may be merged to one section). For those working alone: if you manage to get at least 10 points in total on this part (or more) then you will get another 10 points for free here, after rounding up. This means that you get 85 points, if you really get 75 (after rounding up), and 110 points if you really got 100, etc.**

GOALS

- ∇ Enter a low orbit.
- ∇ Use χ^2 minimization to find which molecules the planet's atmosphere contains
- ∇ Build a numerical model of the planet's atmosphere.
- ∇ Scout for potential landing sites and construct a model of their coordinates.

SIMPLIFICATIONS AND ASSUMPTIONS

Once the spacecraft enters a stable orbit around the planet, the gravitational effects from other astronomical bodies become miniscule in comparison. We are therefore ignoring all effects from all other planets and the star.

RELEVANT THEORY

1. A Little Chemistry

1. Subatomic Composition of Relevant Elements

Table I shows the subatomic composition (the number of protons and neutrons in the atomic nucleus) of a number of elements that are relevant in Part 6. You need not consider other isotopes. You can find the mass of the subatomic particles using numerous other sources.

2. Spectral Lines of Common Gases in Planetary Atmospheres

The following table contains the spectral lines of common gases found in planetary atmospheres:
The presence of methane (CH_4) indicates either possible signs of life or geological activity. Nitrous oxide (N_2O), commonly called *laughing gas*, also indicates possible signs of life.

Element	Isotope	A	Z	N
Hydrogen	${}^{1}_{1}\mathrm{H}$	1	1	0
Carbon	${}^{12}_{6}\mathrm{C}$	12	6	6
Nitrogen	${}^{14}_{7}\mathrm{N}$	14	7	7
Oxygen	${}^{16}_{8}\mathrm{O}$	16	8	8

TABLE I: The subatomic composition of four elements. A is the nucleon number, Z is the proton number and N is the neutron number.

Gas	Spectral Lines [nm]		
O_2	632	690	760
H_2O	720	820	940
CO_2	1400	1600	—
CH_4	1660	2200	—
CO	2340	—	—
N_2O	2870	—	—

TABLE II: Spectral lines of common gases found in planetary atmospheres. The blank boxes comes from the fact that each gas does not necessarily have three spectral lines.

2. Relevant Relations from Thermodynamics

The following list includes a couple of relevant relations from thermodynamics:

→ Adiabatic gases:

$$p^{1-\gamma}T^{\gamma} = constant \tag{1}$$

An adiabatic gas in one is which the temperature can change without gaining or losing heat to its environment.

→ Isothermal gases:

$$T = constant \tag{2}$$

You will investigate the origin of these relations in FYS2160.

3. The Method of χ^2 Minimization

Much like the method of least squares, the method of χ^2 minimization is a process of curve fitting. Both methods are excellent when analysing signals with noise. The benefit of χ^2 minimization over least squares is that χ^2 is capable of handling noise that changes, by downweighting noisy data points. It is important not to confuse this *change* with changes in the noise between measurements. Noise is by definition a random fluctuation, meaning the change is in the standard deviation of the noise. You can read more about the difference between the method of least squares and the method of χ^2 minimization in Lecture Notes 1C.

The idea of χ^2 minimization is conceptually identical to that of least squares: find the underlying signal in noisy measurements.

Let f_i denote the measured data points and $f(t_i)$ denote the expected data points based on our mathematical model $f(t)$. The χ^2 function can thus be expressed as

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{f_i - f(t_i)}{\sigma_i} \right]^2 \tag{3}$$

where σ_i is the standard deviation in the noise of the measurements. The model $f(t)$ is generally dependent on a small set of parameters. The method of χ^2 minimization involves finding the set of parameters that either minimize χ^2 or reduce it below a satisfactory error boundary.

CHALLENGES

A. Enter a Low Orbit

1. *Stabilize your Orbit*

This is optional for those working alone: those working alone can get a shortcut to enter stable orbit fro free. In case your orbit is particularly elliptical, you may want to revisit the orbital injection maneuver from Part 5 in order to adjust it. The more elliptical your orbit the more difficult parts 6 and 7 will be.

2. *Descend*

Up until this point, all your spacecraft commands have been given through to the `InterplanetaryTravel` instance generated with the `begin_interplanetary_travel` method. From now on you are instead going to use the `LandingSequence` instance generated with the `begin_landing_sequence` method. While `InterplanetaryTravel` works with AU units relative to the star, `LandingSequence` works with SI units relative to the planet. Moreover, while `InterplanetaryTravel` accepts boost commands with 2 coordinates, `LandingSequence` accepts boost commands with 3 coordinates. For the time being however, just let all z components be 0.

Orbital mechanics tells us that the radius of a circular orbit is determined by the kinetic energy of the orbiting body. Decrease your speed in order to decrease the average distance between your spacecraft and the center of the planet, but be careful! Lower your orbit too much and your spacecraft will enter the atmosphere. We are going to model the atmosphere later in this part, but until then you should stay away.

Use your orientation software to check the distance between your spacecraft and the center of the planet. If the average distance remains constant over time, you can safely conclude that you have not entered the atmosphere.

B. Spectral Analysis of the Atmosphere

You might wonder why we need to study the atmosphere. Well, our interest in the atmosphere lies in its effects on the spacecraft during landing. These effects include air resistance, which is dependent on the atmosphere's *density profile*.

The first step is to know what gases are present in your atmosphere. Let us start with some preparation:

1. Use 10 km/s as an upper boundary for the spacecraft's velocity with respect to the planet during the time when it observes and takes the spectra of the atmosphere. Use this to derive an expression for the maximum Doppler shift that your spacecraft is able to observe when analysing the planet's atmosphere.

2. We are going to model the spectral lines using a Gaussian *line profile*. Use your knowledge of the Gaussian distribution with theory from Lecture Notes 1D in order to derive an expression for the standard deviation of the line profile σ as a function of the gas temperature T, the mass of a particle in the gas m and the central wavelength of the spectral line λ_0.

Downloading the Data:
Your spacecraft has been equipped with a flux meter capable of measuring the flux of light with wavelengths between 600 nm and 3000 nm. The flux meter began collecting data as soon as your spacecraft entered the low orbit. The data has been automagically uploaded to the following website: `https://www.uio.no/studier/emner/matnat/astro/AST2000/h24/undervisningsmateriell/standardlop/data_og_filer_til_oppgaver/filer-til-oppgaver-i-del-1d/`. The website contains data from a range of spacecraft missions, so you will need to choose the correct data. The data files are labeled `seedXX`, so choose the file whose `XX` equals the

last 2 digits of your `SpaceMission` seed like so:

$$\texttt{seed} = 25256 \implies \texttt{XX} = 56$$

In addition to the flux data, you also need to download the file `sigma_noise.txt` for reasons that will become apparent in due time.

The Statistical Analysis:
Your goal is to analyse the flux data from the flux meter using χ^2 minimization in order to examine the presence or absence of common gases found in planetary atmospheres. You can assume that no gas other than the ones listed in section 1 are present.

As mentioned previously, you can use a Gaussian line profile as your model of the spectral line in the χ^2 minimization process. Your χ^2 analysis needs to take the following into account:

→ The parameters in the Gaussian model are:

F_{min} : The flux at the center of the spectral line.

σ : The standard deviation of the spectral line's gaussian line profile.

λ_0 : The central wavelength of the spectral line.

→ The data has been normalized such that $F = 1$ is the expected flux where no spectral lines are present.

→ As opposed to other equipment installed on the spacecraft, measurements taken with the flux meter carry noise. The file `sigma_noise.txt` contains the standard deviation in the noise fluctuations σ_i corresponding to the flux measurements.

→ All spectral lines are expected to obey $F_{\min} > 0.7$. (BUT, note the effect of noise which could make the line lower, also, if you find a line with $F_{\min} < 0.7$, is this most probably a real line or a fluke?? Think twice and ask if you're doubting)

→ The atmospheric temperature T is expected to be somewhere in the range from 150 K to 450 K.

→ Due to your spacecraft's nonzero velocity with respect to atmosphere, spectral lines are subject to the Doppler effect.

Filtering the Real Lines from the Flukes:
If implemented correctly, the method of χ^2 minimization will produce a parameter set for each spectral line regardless of whether or not the data features an actual spectral line. More sophisticated methods include statistical uncertainty calculations, which help researchers filter the real spectral lines from statistical flukes. However, by including these calculations the project's difficulty increases significantly. Instead, you need to filter the spectral lines based on your intuition and the following criteria:

→ You should find that the Doppler shift in the real spectral lines are the same, it is however unrealistic to expect perfectly identical Doppler shifts due to noise.

→ Real spectral lines tend to have lower F_{min} values than the statistical flukes.

→ You will not necessarily find every spectral line of a gas, finding a single spectral line is fine.

→ Theoretically, different gases may have different temperatures, but different spectral lines from the same gas will have the same temperature. But beware of noise, which is sure to give some variation in the temperature.

If you have trouble differentiating between real lines and flukes, you could try to study the σ_i values in the same region as the spectral lines. You should know that the uncertainty margins are expected to be uncomfortably high, this isn't a problem (other than it's more difficult to differentiate real lines from flukes).

The Mean Molecular Weight:
With the spectral lines in hand, you are finally ready to finish this challenge and calculate the mean molecular weight of your atmosphere μ. Let any confirmed spectral line count as whether or not the gas is present, and that all gases come in equal amounts.

For example:
Say you find 2 O_2 lines, 1 CO line and 1 N_2O. Then your atmospheric composition is 33.3% O_2, 33.3% CO and 33.3% N_2O.

It is very difficult to determine which lines are real and which are flukes, you simply do not have sufficient information to get a certain answer (as will often be the case also in a realistic situation). Do not panic, this will be the case for everybody and will not strongly affect your landing. Do your best to make an educated guess. The main point here is that you show clearly which arguments you use to determine the composition of your atmosphere.

C. Model the Atmosphere

By *"modelling the atmosphere"* what is meant is: you need to determine the planet's density profile and its temperature profile. In this context, a "profile" for a quantity, say temperature, is a function that tracks the temperature as a function of the height above the surface.

Base your model of the atmosphere on the following set of simplifications and assumptions:

⊗ Assume the atmosphere is uniform. To exemplify:

Say the atmosphere consists of 40% Nitrogen. Then, not only is the average amount of Nitrogen in the atmosphere 40%, but every point in the atmosphere is comprised of 40% Nitrogen.

⊗ We are also going to assume that the atmosphere is spherically symmetric, i.e. $\rho = \rho(r)$, where r is the radial distance from the center of the planet. The density at the surface of your planet can be obtained from `atmospheric_densities` in `ast2000tools`.

⊗ You can reasonably assume the atmosphere is in hydrostatic equilibrium.

⊗ Approximate the atmosphere as an ideal gas.

⊗ In Part 3, you built a model of the surface temperature of the planets in your solar system. Let T_0 denote your estimate of your destination's surface temperature. Then, assume the atmosphere is

  – adiabatic with $\gamma = 1.4$ for heights up to the point where $T = T_0/2$,

  – and isothermal otherwise.

⊗ The gravitational acceleration from the surface to the top of the atmoshere changes very little as the thickness of the atmosphere is tiny with respect to the radius of the planet (check to which degree this is true for your planet). You may therefore, in particular if you solve this analytically, make a simplifying assumption on the gravitational acceleration (which one?).

You may choose to solve this challenge in whatever way you like, be it analytically or numerically. After completing the atmospheric model you are expected to have found the density profile and the temperature profile, both in the form of a `Pyhton` function that accepts the height above the surface as a parameter.

If you solved this challenge numerically:
The product of a numerical simulation of the atmosphere is a set of discrete arrays. In order to transform these discrete arrays into continuous functions you need to interpolate the arrays. Use the same strategy as you did with the planetary orbits back in Part 3.

D. Scouting for Landing Sites

Now that you have completed a series of detailed calculations on your planet's atmosphere, it is time to start thinking about landing the spacecraft.

Recall that the `LandingSequence` class works in 3 dimensions. You are thus free to explore the entire planetary surface, not just the region of the planet that intersects the x-y plane of the solar system. However, keep in mind that introducing the z-axis will in general lead to more complex commands. *You are not forced to use 3-dimensional commands.*

Figure 1 shows an illustration of how this challenge is imagined to take place. You do not need to follow this scheme however, you are free to do whatever you like.

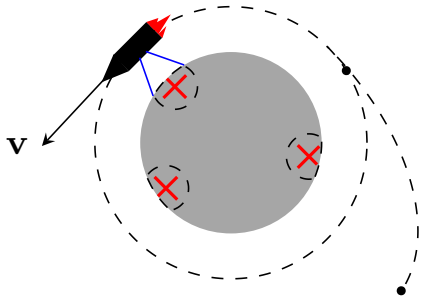


FIG. 1: Scouting for landing sites: Having entered low orbit, the spacecraft takes images of potential landing sites positioned directly below it.

1. *The Rotation of the Planet*

Not only is the planet orbiting the star, it is also rotating about its central axis (the z-axis). As you decide on a potential landing site, you need to know the coordinates of this landing site in order to successfully land after completing an orbit. However, due to the spin of the planet, the coordinates of the landing site change with time. Your task is to write a general function that calculates the new coordinates of a potential landing site using its current coordinates as a function of time. The structure should look something like this:

`input`	`output`
current coordinates time elapsed	new coordinates

To simplify this challenge we strongly encourage you you use a spherical coordinate system. Use the same convention for the angles of this coordinate system as in Part 4. Remember that the planet's spin is constant, i.e., the ϕ coordinate increases linearly.

2. *Scouting*

Complete at least one orbit and take several pictures of the planet's surface. Based on these pictures, choose a reasonable landing site. While taking these pictures, orient yourself in order to calculate the coordinates of the landing site at the time of your picture. **Pro Tip: Land on the day-side of the planet, otherwise you won't be able to see anything in your landing video!**

VERIFICATION/CONFIRMATION

There are no verification steps in Part 6. You need to evaluate your own work based on your experience with numerical analysis from the project and elsewhere. Are you confident that your spectral analysis was correctly

performed? Does your atmospheric model seem plausible? Have you calculated the coordinates of your landing site correctly?

EXTRA CHALLENGE

Are you able to write programs that solve challenges B and C automagically?



FIG. 2: Orbitally oriented, atmospherically motivational, trajectory-planning duck. Launched.

AST 2000 - Part 6
Tips, Hints & Guiding Questions

I. SPECTRAL ANALYSIS

The Gaussian line profile can be written as:

$$F = 1 + (F_{min} - 1) \exp\left[-\frac{1}{2}\left(\frac{\lambda - \lambda_0}{\sigma}\right)^2 \right] \qquad \text{(a)}$$

Your task is to find the set of F_{min}, λ_0 and σ parameters that minimizes χ^2; that is, find values for the parameters such that χ^2 is minimized. The first step is to place minimum and maximum boundaries on the parameters, you should be able to discern reasonable boundaries based on information given to you in the challenge. You will then need to test the parameter ranges and calculate χ^2 for every parameter set.

Note that the flux data comes as one large spectrum. In order to avoid confusion between two or more spectral lines (during the numerical analysis) we encourage you to divide the spectrum into a set of slices and analyse each slice one at a time. Note that the slices do not necessarily need to contain the entire spectrum and that the slices may include unequal numbers of data points. Be careful not to slice the spectrum too narrow such that the spectral line may be partially or completely outside the slice. Can you find a reasonable way to divide the spectrum based on the spectral line's maximum displacement?

II. MODELLING THE ATMOSPHERE

It is important to realize the role of each different assumption. The one underlying our atmospheric model is the assumption of hydrostatic equilibrium; it serves as the base for the differential equation that you will need to solve. The equation of hydrostatic equilibrium relates the atmospheric density $\rho(r)$ to the derivative of atmospheric pressure $\mathrm{d}p(r)/\mathrm{d}r$. The relationship between $\rho(r)$ and $p(r)$ is quantified by the ideal gas law.

Combining hydrostatic equilibrium with the ideal gas law will result in a differential equation for $\rho(r)$ that is dependent on the temperature $T(r)$ and the planetary mass $M(r)$.

You should solve for the isothermal part of the atmosphere first which is the easiest part. If you choose to look for an analytical solution and you are stuck, you might get some hints in the exercise of Lecture Notes 1E. If you manage this analytically, it is also possible to solve the adiabatic part analytically (if you prefer).

III. NEW LANDING SITE COORDINATES

Do not forget that the planet's spin only affects the ϕ component of spherical coordinates. The calculation is pretty straightforward.