AST2000 Lecture Notes

Part 3A
The cosmic distance ladder

Questions to ponder before the lecture

1. How do we know that the distance to our closest star is 4 light years?
2. How do we know that our galaxy is 100.000 light years across?
3. How do we know that the distance to our neighbour galaxy, Andromeda, is 2.5 million light years?
4. How do we know that the most distant objects we observe are more than 10 billion light years away?
5. The answer to the above questions are all different. Can you imagine at least one of the ways to measure distances?

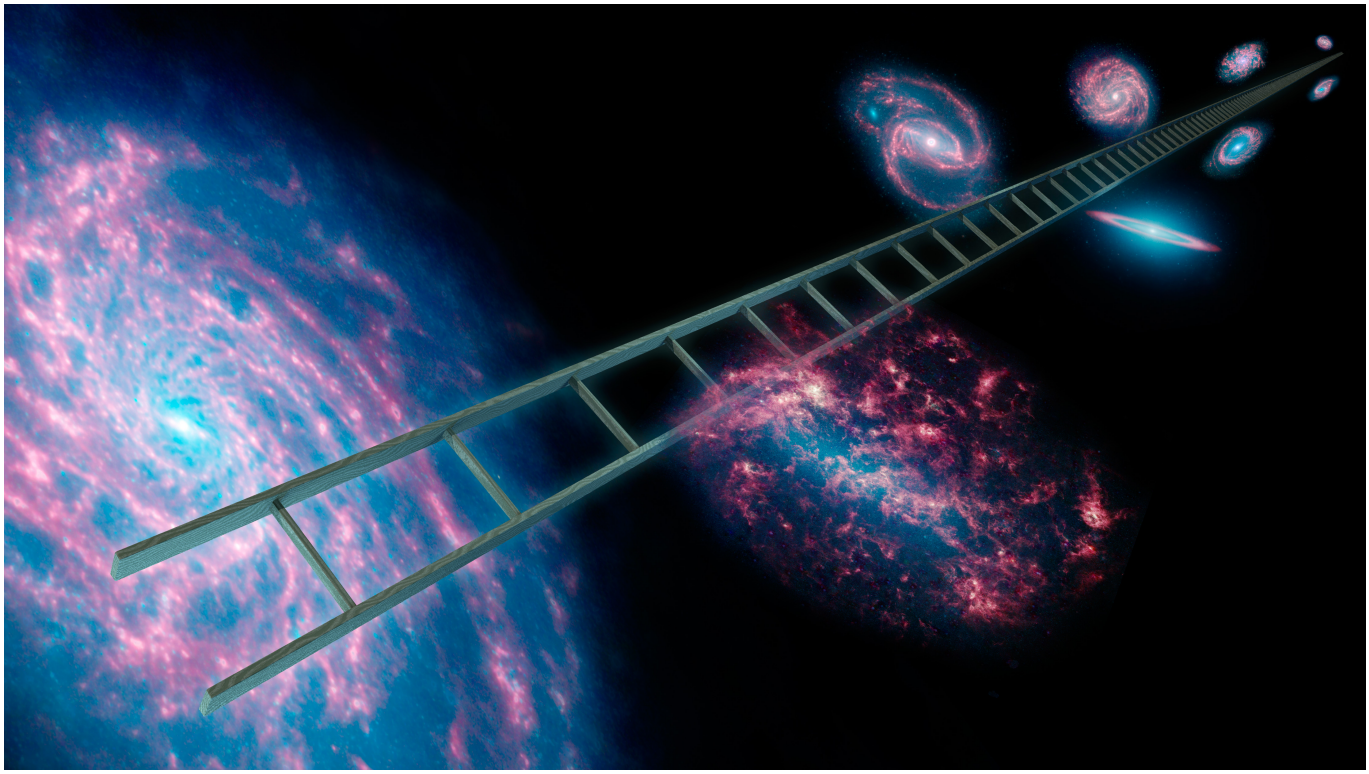


Image: NASA/JPL-Caltech.

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Part 3A
The cosmic distance ladder

How do we measure the distance to distant objects in the universe? There are several methods available, most of which suffer from large uncertainties. Particularly the methods to measure the largest distances are often based on assumptions which have not been properly verified. Fortunately, we do have several methods available which are based on different and independent assumptions. Using cross-checks between these different methods we can often obtain more exact distance measurements.

Why do we want to measure distances to distant objects in the universe? In order to understand the physics of these distant objects, it is often necessary to be able to measure how large they are (their physical extension) or how much energy that they emit. When looking through a telescope, what we observe is not the physical extension or the real energy that the object emits, what we observe is the *appaerent magnitude* and the *angular extension* of an object. We have seen several times during this course that in order to convert these to *absolute magnitudes* (and thereby luminosity/energy) and *physical sizes* we need to know the distance (look back to the formula for converting appaerent magnitude to absolute magnitude as well as the small angle formula for angular extension of distant objects). In cosmology it is important to make 3D maps of the structure in the universe in order to understand how these structures originated in the Big Bang. To make such 3D maps, again knowledge of distances are indispensable.

There are 4 main classes of methods to measure distances:

1. Triognometric parallax (or simply parallax)
2. Methods based on the Hertzsprung-Russel diagram: main sequence fitting
3. Distance indicators: Cepheid stars, supernovae and the Tully-Fisher relation
4. The Hubble law for the expansion of the universe.

We will now look at each of these in turn.

1 Parallax

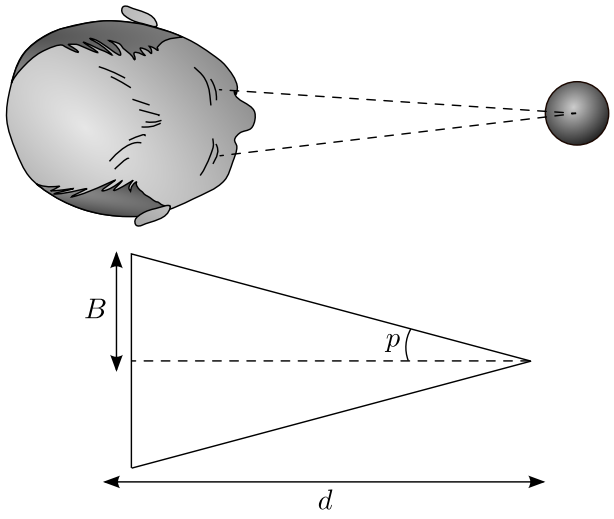


Figure 1: Definition of parallax: above is a face seen from above looking at an object at distance d. Below is the enlarged triangle showing the geometry.

Shut your left eye. Look at an object which is close to you and another object which is far away. Note the position of the close object with respect to the distant object. Now, open you left eye and shut you right. Look again at the position of the

close object with respect to the distant. Has it changed? If the close object was close enough and the distant object was distant enough, then the answer should be yes. You have just experienced *parallax*. The apparent angular shift of the position of the close object with respect to the distant is called the *parallax angle* (actually the parallax angle is defined as half this angle). The further away the close object is, the smaller is the parallax angle. We can thus use the parallax angle to measure distance.

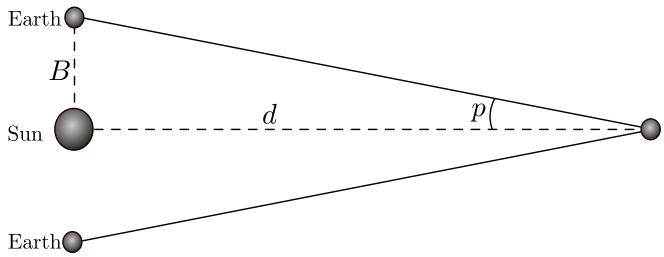


Figure 2: The Earth shown at two different positions half a year apart. The parallax angle p for a distant object at distance d is defined with respect to the Earth-Sun distance as baseline B.

In figure (1) we show the situation: It is the fact that your two eyes are located at different positions with respect to the close object that causes the effect. The larger distance between two observations (between the 'eyes'), the larger the parallax angle. The closer the object is to the two points of observation, the larger the parallax angle. From the figure we see that the relation between parallax angle p, baseline B (B is defined as half the distance between your eyes or between two observations) and distance d to the object is

$$\tan p = \frac{B}{d}.$$

For small angles, $\tan p \approx p$ (when the angle p is measured in radians) giving,

$$B = dp, \qquad (1)$$

which is just the small angle formula that we encountered in the lectures on extrasolar planets. For distant objects we can use the Sun-Earth distance as the baseline by making two observations half a year apart as depicted in figure 2. In this case the distance measured in AU can be written (using equation (1) with $B = 1$ AU)

$$d = \frac{1}{p} \text{ AU} \approx \frac{206265}{p''} \text{ AU}.$$

Here p'' is the angle p measured in arcseconds instead of radians (I just converted from radians to arcseconds, check that you get the same result!). For a **par**allax of one arc**sec**ond (par-sec), the distance is thus 206265 AU which equals 3.26 ly. This is the definition of one *parsec* (pc). We can thus also write

$$d = \frac{1}{p''} \text{ pc}.$$

The Hipparcos satellite measured the parallax of 120 000 stars with a precision of $0.001''$. This is far better than the precision which can be achieved by a normal telescope. A large number of observations of each star combined with advanced optical techniques allowed for such high precision even with a relatively small telescope. With such a precision, distances of stars out to about 1000 pc (= 1 kpc) could be measured (do you see this from the formula?). The diameter of the Milky Way is about 30 kpc so only the distance to stars in our vicinity can be measured using parallax.

2 The Hertzsprung-Russell diagram and distance measurements

You will encounter the Hertzsprung-Russell (HR) diagram on several occations during this course. Here you will only get a short introduction and just enough information in order to be able to use it for the estimation of distances. In the lectures on stellar evolution, you will get more details.

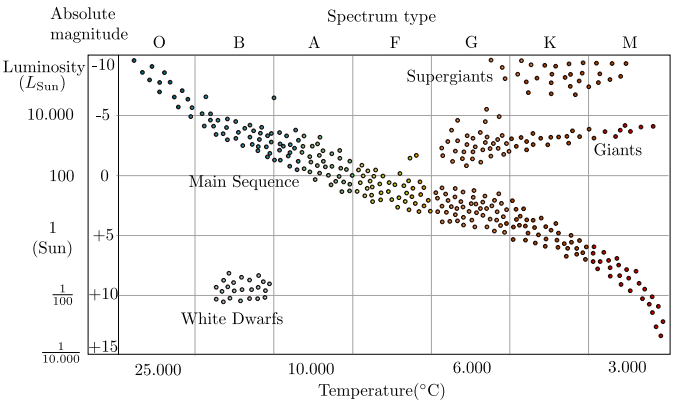


Figure 3: The Hertzsprung-Russell diagram.

There are many different versions of the HR-diagram. In this lecture we will study the HR-diagram as a plot with surface temperature of stars on the x-axis and absolute magnitude on the y-axis. In figure 3 you see a typical HR-diagram: Stars plotted according to their surface temperature (or color) and absolute magnitude. The y-axis shows both the luminosity and the absolute magnitude M of the stars (remember: these are just two different measures of the same property, check that you understand this). Note that the temperature increases towards the left: The red and cold stars are plotted on the right hand side and the warm and blue stars on the left.

We clearly see that the stars are not randomly distributed in this diagram: There is a diagonal line going from the left to right. This line is called the *main sequence* and the stars on this line are called *main sequence stars*. The Sun is a typical main sequence star. In the upper right part of the diagram we find the so-called giants and super-giants, cold stars with very large radii up to hundreds of times larger than the Sun. Among these are the *red giants*, stars which are in the final phase of their lifetime. Finally, there are also some stars found in the lower part of the diagram. Stars with relatively high temperatures, but extremely low luminosities. These are *white dwarfs*, stars with radii similar to the Earth. These are dead and compact stars which have stopped energy production by nuclear fusion and are slowly becoming colder and colder.

In the lectures on stellar evolution we will come back to why stars are not randomly distributed in a HR-diagram and why they follow certain lines in this diagram. Here we will use this fact to measure distances. The HR-diagram in figure 3 has been made from stars with known distances (the stars were so close that their distance could be measured with parallax). For these stars, the absolute magnitude M (thus the luminosity, total energy emitted per time interval) could be calculated using the apparent magnitude m and distance r,

$$M = m - 5\log_{10}\left(\frac{r}{10\text{ pc}}\right). \qquad (2)$$

HR-diagrams are often made from stellar clusters, a collection of stars which have been born from the same cloud of gas and which are still gravitationally bound to each other. The advantage with this is that all stars have very similar age. This makes it easier to predict the distribution of the stars in the HR-diagram based on the theory of stellar evolution. Another advantage with clusters is that all stars in the cluster have roughly the same distance to us. For studies of the main sequence, so-called *open clusters* are used. These are clusters containing a few thousand stars and are usually located in the galactic disc of the Milky Way and other spiral galaxies.

Now, consider that we have observed a few hundred stars in an open cluster which is located so far away that parallax measurements are impossible. We have measured the surface temperature (how?) and the apparent magnitude of all stars. We now make an HR-diagram where we, as usual, plot the surface temperature on the x-axis. However, we do not know the distance to the cluster and therefore the absolute magnitudes are unknown. We will have to use the apparent magnitudes on the y-axis. It turns out that this is not so bad at all: Since the cluster is far away, the distance to all the stars in the cluster is more or less the same. Looking at equation (2), we thus find,

$$M - m = -5\log_{10}\left(\frac{r}{10\text{ pc}}\right) = \text{constant},$$

for all stars in the cluster (assuming r is constant). The HR-diagram with apparent magnitude instead of absolute magnitude will thus show the same pattern of stars as the HR-diagram with absolute magnitudes on the y-axis. The only difference is a constant shift $m-M$ in the magnitude of all stars given by the distance of the cluster. Thus, by finding the shift in magnitude between the observed HR-diagram with apparent magnitudes and the HR-diagram in figure (3) based on absolute magnitudes, the distance to the cluster can be found. This method is called *main sequence fitting*.

Example

We observe a distant star cluster with unknown distance, measure the temperature and apparent magnitude of each of the stars in the cluster and plot these results in a diagram shown in figure 4 (lower plot) (note: spectral class is just a different measure of temperature, we will come to this in later lectures). In the same figure (upper plot) you see the HR-diagram taken from a cluster with a known distance (measured by parallax). Since the distance is known, the apparent magnitudes could be converted to absolute magnitudes and for this reason we plot absolute magnitude on the y-axis for this diagram. We know that the main sequence is similar in all clusters since stars evolve similarly. For this reason, we know that the two diagrams should be almost identical. We find that by shifting all the observed stars in the lower diagram upwards by 2 magnitudes (to higher luminosities but lower magnitudes), the two diagrams will look almost identical (look at the figure and check that you agree!). Thus, there is a difference between the apparent magnitude and the absolute magnitude of $M - m = -2$ and the distance is found by

$$-2 = -5 \log_{10} \left(\frac{r}{10 \text{ pc}} \right),$$

giving $r = 25$ pc.

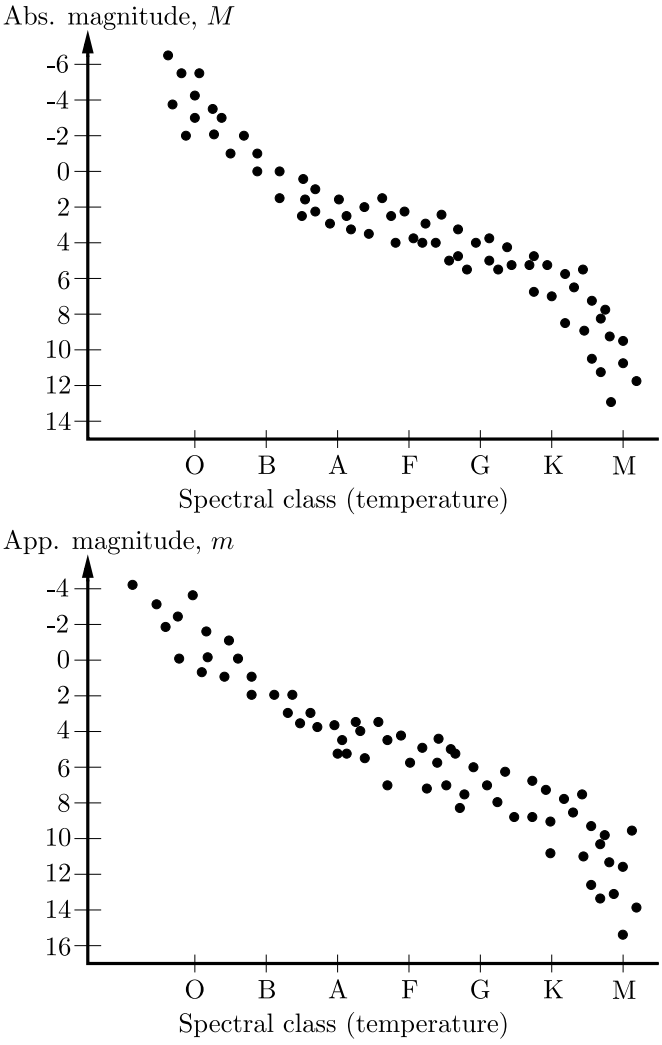


Figure 4: The HR-diagrams for the example exercise (note: spectral class is just a different measure of temperature). The upper plot shows the HR-diagram of a cluster with a known distance. Since the distance is known, we have been able to convert the apparent magnitudes to absolute magnitudes and we therefore plot absolute magnitudes on the y-axis. The lower plot is the HR-diagram of a cluster with unknown distance. Because of the unknown distance, we only have information about the apparent magnitude of the stars and therefore we now have apparent magnitude on the y-axis.

Main sequence fitting can be used out to distances of about 7 kpc, still not reaching out of our galaxy. We now see why we use the phrase 'cosmic distance ladder'. The parallax method reaches out to about 1000 pc. After that, main sequence fitting needs to be used. But in order to use main sequence fitting, we needed a calibratet HR-diagram like figure 3. But in order to obtain such a diagram, the parallax method needed to be used on nearby clusters. So we need to go step

by step, first the parallax method which we use to calibrate the HR-diagram to be used for the main sequence fitting at larger distances. Now we will continue one more step up the ladder. We use stars in clusters which distance is calibrated with main sequence fitting in order to calibrate the *distance indicators* to be used for larger distances.

3 Distance indicators

Again the method is based on equation (2). We can always measure the apparent magnitude m of a distant object. From the equation, we see that all we need in order to obtain the distance is the absolute magnitude. If we know the absolute magnitude (luminosity) for an object, we can find its distance. But how do we know the absolute magnitude? There are a few classes of objects, called *standard candles*, which reveal their absolute magnitude in different ways. Examples of these 'standard candles' can be Cepheid stars or supernova explosions.

Another class of distance indicators are the so-called 'standard rulers'. The basis for the distance determination with standard rulers is the small-angle formula,

$$d = \theta r,$$

where d is the physical length of an object, r is the distance to the object and θ is the apparent angular extension (length) of the object. We can often measure the angular extension of an observed object. All that we need in order to find the distance is the physical length d. There are some objects for which we know the physical length. These objects are called *standard rulers*. For instance a special kind of galaxy which has been shown to always have the same dimensions could be used as a standard ruler.

3.1 Cepheid stars as distance indicators

Several stars show periodic changes in their apparent magnitudes. This was first thought to be caused by dark spots on a rotating star's surface: When the dark spots were turned towards us, the star appeared fainter, when the spots were turned away from us, the star appeared brighter. Today we know that these periodic variations in the star's magnitude is due to pulsations. The star is pulsating and therefore periodically changing its radius and surface temperature.

The Milky Way has two small satellite galaxies orbiting it, the Large and the Small Magellanic Cloud (LMC and SMC). They contain $10^9 - 10^{10}$ stars, less than one tenth of the number of stars in the Milky Way and are located at a distance of about 160 000 ly (LMC) and 200 000 ly (SMC) from the Sun. In 1908, Henrietta Leavitt at Harvard University discovered about 2400 of these pulsating stars in the SMC. The pulsation period of these stars were found to be in the range between 1 and 50 days. These stars were called *Cepheids* named after one of the first pulsating stars to be discovered, δ Cephei. She found a relationship between the stars' apparent magnitude and pulsation period. The shorter/longer the pulsation period, the fainter/brighter the star. Since all these stars were in the SMC they were all at roughly the same distance to us. We have seen above that for stars at the same distance, there is a constant difference $M - m$ in apparent and absolute magnitude. So the stars with a larger/smaller apparent magnitude also had a larger/smaller absolute magnitude. Since absolute magnitude is a measure of luminosity, what she had found was a *period-luminosity relation*.

Pulsating stars with higher luminosity were thus found to be pulsating with longer periods, pulsating stars with low luminosity were found to be pulsating with short periods. We can now reverse the argument: By measuring the period one can obtain the luminosity. There was one problem however: The method could not be calibrated as the distance to the SMC was unknown and therefore also the constant in $m - M =$ constant was unknown. Without this constant one cannot find M. One had to find Cepheids in our vicinity for which the distance was known. Only in that way could this constant and thus the relation between period and absolute magnitude be established.

Today the distance to several Cepheids in our galaxy are known by other methods. One of the most recent measurements of the constants in the period-luminosity relation came from the parallax

measurements of several Cepheids by the Hipparcos satellite. The relation was found to be

$$M_V = -2.81 \log_{10} P_d - 1.43,$$

where P_d is the period in days. Here M_V is the absolute magnitude in the *Visual* part of the electromagnetic spectrum instead of the normal magnutide M which is based on the flux integrated over all wavelengths λ. Before describing in detail the difference between M and M_V, we will end our discussion on the Cepheid stars.

When pulsating stars were first used to measure distances one did not know that there are three different types of pulsating stars with different period-luminosity relations:

1. The classical Cepheids which belong to a class of giants, are very luminous stars. These are the most useful distance indicators for large distances because of their high luminosity.
2. W Virginis stars, or type II Cepheids are pulsating stars which on average have lower luminosity than the classical Cepheids.
3. RR Lyrae stars are small stars which usually have less mass than the Sun. Their luminosity is much lower than the luminosity of classical Cepheids and RR Lyrae stars are therfore less useful for distance determination at large distances. The advantage with RR Lyrae stars however, is that they are much more numerous than classical Cepheids.

When Edwin Hubble tried to estimate the distance to our neighbour galaxy Andromeda, he obtained a distance of about one million light years whereas the real distance is about twice as large. The reason for this error was that he observed W Virginis stars in Andromeda and applied the period-luminosity relation for classical Cepheids, thinking that they were the same. In this course we will mainly discuss the classical Cepheids.

Since Cepheids are very lumious (about 10^3 to 10^4 times higher luminosity than the Sun) they can be observed in distant galaxies. In order to determine the distance of a whole galaxy it suffices to find Cepheid stars in that galaxy and determine their distance. In this manner, the distance to several galaxies out to about 30 Mpc has been measured. Beyond 30 Mpc other methods need to be applied.

At the moment we will use the period-luminosity relation for Cepheids to determine distances without questioning why it works. When we come to the lectures on stellar structure we will study the physics behind these pulsations and see if we can deduce the period-luminosity relation theoretically by doing physics in the interior of stars.

We have now learned about our first distance indicator: We can find the absolute magnitude M_V at visual wavelength of Cepheids by observing their plusation period. Having the absolute magnitude M_V we can find the distance. We will now look at a different approach to find M_V for a distant object, but first we will discuss some extended definitions of magnitudes.

3.2 Visual magnitudes and other filters

Looking back at the definition of absolute magnitude, we see that we can write the absolute magnitude M as

$$\begin{aligned} M &= M^{\text{ref}} - 2.5 \log_{10} \left(\frac{F(10 \text{ pc})}{F^{\text{ref}}(10 \text{ pc})} \right) \\ &= M^{\text{ref}} - 2.5 \log_{10} \left(\frac{L}{L^{\text{ref}}} \right), \end{aligned}$$

where M_{ref} and F_{ref} are the absolute magnitude and flux (observed flux if the distance had been 10 pc) of a reference star used for calibration (as we have seen before, the star Vega with its magnitude defined to be 0, has often been used for this purpose). The flux is here the total flux of the star integrated over all wavelengths

$$F = \int_0^\infty F(\lambda) d\lambda. \tag{3}$$

The magnitude M which is based on flux integrated over all wavelengths is called *the bolometric magnitude*.

The *visual magnitude* M_V on the other hand, is based on the flux over a wavelength region defined by a *filter function* $S_V(\lambda)$. The filter function is a function which is centered at $\lambda = 550$ nm with an effective bandwidth of 89 nm. The flux F_V which

is used instead of F to define visual magnitude can be written as

$$F_V = \int_0^\infty F(\lambda) S_V(\lambda) d\lambda.$$

Compare with expression (3): The main difference is that a limited wavelength range is selected by $S_V(\lambda)$. The magnitude is then defined as

$$M_V = M_V^{\text{ref}} - 2.5 \log_{10} \left(\frac{F_V}{F_V^{\text{ref}}} \right).$$

As for the bolometric magnitude, the relation between absolute and apparent visual magnitude is also given by

$$M_V - m_V = -5 \log_{10} \left(\frac{r}{10 \text{ pc}} \right).$$

The concept of the visual magnitude originates from the fact that detectors normally do not observe the flux over all wavelengths. Instead detectors are centered on a given wavelength and integrate over wavelengths around this center wavelength in a given *bandwidth*. There are three of these filters which are in common use:

- U-filter (ultraviolet), $\lambda_0 = 365$ nm, $\Delta\lambda_{FWHM} = 68$ nm
- B-filter (blue), $\lambda_0 = 440$ nm, $\Delta\lambda_{FWHM} = 98$ nm
- V-filter (visual), $\lambda_0 = 550$ nm, $\Delta\lambda_{FWHM} = 89$ nm

3.3 OPTIONAL: colour indices

The absolute magnitudes M_V, M_B and M_U are used to define *colour indices*. These colour indices $(U-B)$ and $(B-V)$ are defined as

$$U - B = M_U - M_B = m_U - m_B,$$
$$B - V = M_B - M_V = m_B - m_V.$$

Note that these indices are written as a difference in apparent or absolute magnitudes: The colour indices are independent of distance and will therefore give the same results if they are obtained using apparent magnitudes or absolute magnitudes (check that you can show this mathematically!). These indices are used to measure several properties of a star related to its colour. The period-luminosity relation for a Cepheid can be improved using information about its colour in terms of the $(B-V)$ colour index as

$$M_V = -3.53 \log_{10} P_d - 2.13 + 2.13(B-V).$$

For Cepheids, the $B-V$ colour index is usually in the range 0.4 to 1.1. Thus, a more exact M_V and thereby a more exact distance (using relation (2)) can be obtained using the additional distance independent information contained in the color of the star. It suffices to observe the star with three color filters instead of one to obtain this additional information.

3.4 Supernovae as distance indicators

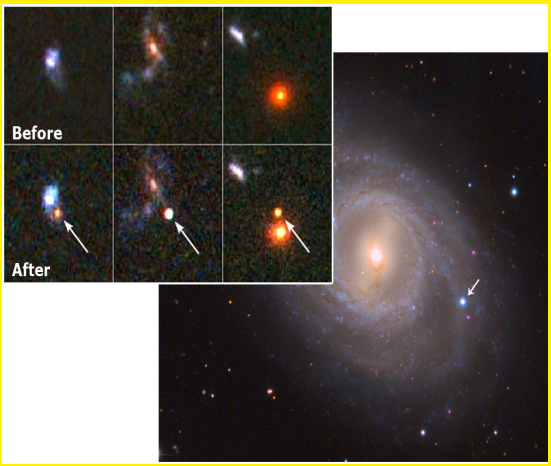
One of the most energetic events in the Universe are the Supernova explosions. In such an explosion, one star might emit more energy than the total energy emitted by all the stars in a galaxy. For this reason, supernova explosions can be seen at very large distances. The last confirmed supernova in the Milky way was seen in 1604 and was studied by Kepler. It reached an appaerent magnitude of about -2.5, similar to Jupiter at its brightest. There have been other reports of supernovae in the Milky way during the last 2000–3000 years, both in Europe and Asia. Some of these were so bright that they were seen clearly in the sky during daylight. Written material from Europe, Asia and the middle East all report about a supernova in 1006 which was so bright that one could use it to read at night time. The nearest supernova in modern times, called SN1987A, was observed in 1987 in the Large Magellanic Cloud at a distance of 51 kpc. It was visible by the naked eye from the southern hemisphere.

Supernovae can be classified as type I or type II,

1. Type I supernovae: These explosions show no hydrogen lines. There are three sub types, defined according to their spectra: Type Ia, Ib and Ic.
2. Type II supernovae: These are explosions with strong hydrogen lines. Type II supernova have several properties in common with type Ib and Ic.

It is now clear that supernovae of type Ib, Ic and II are *core collapse supernovae*. This is a star ending its life in a huge explosion, leaving behind a neutron star or a black hole. In the lectures on stellar evolution we will come back to the details of a core collapse supernova. Type Ia supernovae are usually brighter. These have the

Fact sheet: Upper panel: Three very distant type Ia supernovae observed with the Hubble Space Telescope. The stars exploded back when the universe was approximately half its current age. Since supernovae are so bright and their absolute magnitude can be obtained from their light-curves, astronomers can trace the expansion rate of the universe by observing these standard candles at various distances. Lower panel: A relatively close example of another type of supernova, the type II supernova SN 2012aw discovered in March 2012 in M95, a barred spiral galaxy around 10 Mpc away in the constellation Leo. (Figure: A. Block, Univ. of Arizona, NASA/ESA and A. Riess/STScI)



property which is desirable for a standard candle: Their luminosity is relatively constant and there is a receipe for finding their exact luminosity. The origin of type Ia supernovae are still under discussion, but according to the most popular hypothesis, the explosion occurs in a white dwarf star which has a binary companion. A white dwarf star is the result of one of the possible ways that a star can end its life: in the form of a very compact star consisting mainly of carbon and oxygen which are the end products from the nuclear fusion processes taking place in the final phase of a star's life. If a white dwarf is part of a binary star system (two stars orbiting a common center of mass), the white dwarf may start accreating material from the other star. At a certain point, the increased pressure and temperature from the accreted material may reignite fusion processes in the core of the white dwarf. This is the cause of the explosion. We will again defer details about the process to later lectures.

It can be shown that this explosion occurs when the mass of the white dwarf is close to the so-called *Chandrasekhar limit* which is about $1.4M_{\odot}$. Since the mass of the exploding star is always very similar, the luminosity of the explosions will also be very similar. The absolute magnitude of a type Ia supernova is $M_V \approx M_B \approx= -19.3$ with a spread of about 0.3 magnitudes. A more exact estimate of the absolute magnitude of a supernova may be obtained by the light curve. After reaching maximum magnitude, the supernova fades off during days, weeks or months. By observing the rate at which the supernova fades, one can determine the absolute magnitude of the supernova at its brightest.

Again, here we will only use the fact that the absolute magnitude of type Ia supernovae can be obtained from its light curve in order to determine distances. More details about the physical processes giving rise to the explosion and to the fact that the light curve can be used to obtain the luminosity will be presented in later lectures. Due to their strong luminosity, supernovae can be used to determine distances to galaxies beyond 1000 Mpc.

3.5 The Tully-Fisher relation

The Tully-Fisher relation is a relation between the width of the 21 cm line of a galaxy and its absolute magnitude. As we remember, the 21 cm radiation is radiation from neutral hydrogen (look back at the lecture on electromagnetic radiation). Spiral galaxies have large quantities of neutral hydrogen and therefore emit 21 cm radiation from the whole disc. The 21 cm line is wide because of Doppler shifts: Hydrogen gas at different distances from the center of the galaxy orbits the center at different speeds giving rise to several different Doppler shifts. We also remember that the rotation curve for galaxies towards the edge of the galaxy was flat. So, gas clouds orbiting the galactic center at large distances all have the same orbital velocity $v_{\max}$ and thus the same Doppler shift. There are therefore many more gas clouds with velocity $v_{\max}$ than with any other velocity. The flux at the wavelength corresponding to the

Doppler shift

$$\frac{\Delta\lambda_{\max}}{\lambda_0} = \frac{v_{\max}}{c},$$

is therefore larger than for instance at a wavelength of 21 cm itself. The result is a peak in the flux of the spectral line at either side of 21 cm at the wavelength $21 \pm \Delta\lambda_{\max}$ cm. The wavelength of this peak is a measure of the maximal velocity in the rotation curve:

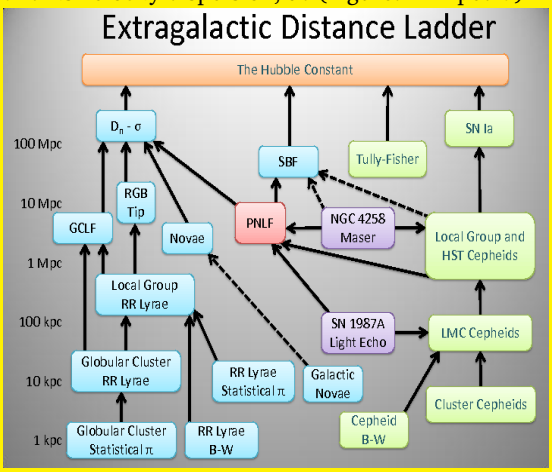
$$v_{\max} = c\frac{\Delta\lambda_{\max}}{\lambda_0}.$$

We have seen in a previous lecture that the maximum velocity is related to the total mass of the galaxy. The higher the maximum velocity, the higher the mass (why?). If we assume that a higher total mass also means a higher content of lumious matter and therefore a higher luminosity, it is not difficult to imagine that a relation can be found between the maximal speed measured from the 21 cm line and the luminosity, or absolute magnitude of the galaxy. The relation can be written as

$$M_B = C_1 \log_{10} v_{\max} + C_2,$$

where M_B is the absolute magnitude at blue wavelenghts and C_1 and C_2 are constants depending on the type of spiral galaxy. The constant C_1 is normally in the range -9 to -10 and C_2 in the range 2.7 to 3.3. The Tully-Fisher relation can be used as a distance indicator out to distances beyond 100 Mpc.

Fact sheet: There are many more steps on the cosmic distance ladder than discussed in this course. Light green boxes: technique applicable to star-forming galaxies. Light blue boxes: technique applicable to Population II galaxies. Light purple boxes: geometric distance technique. Light red box: the planetary nebula luminosity function (PNLF) technique is applicable to all populations of the Virgo Supercluster. Solid black lines denote well-calibrated ladder steps, while dashed black lines denote uncertain calibration steps. Symbols and acronyms: π = parallax, GCLF = globular cluster luminosity function, SBF = surface brightness fluctuation, B-W = Baade-Wesselink method, RGB = red giant branch, LMC = Large Magellanic Cloud, Dn – σ = relation between the angular diameter, D, of the galaxy and its velocity dispersion, σ. (Figure: Wikipedia)



3.6 Other distance indicators

Some other distance indicators:

- The globular cluster luminosity function: Globular clusters are clusters of stars containing a few 100 000 stars. These clusters are usually orbiting a galaxy. A galaxy has typically a few hundred globular clusters orbiting. It has been found that the luminosity function, i.e. the percentage of globular clusters with a given luminosity, is similar for all galaxies. By finding this luminosity function for galaxies with a known distance, the globular clusters can be used as distance indicators for other galaxies.
- The planetary nebula luminosity function: Planetary nebulae (which have nothing to do with planets) are clouds composed of gas which dying stars ejected at the end of their lifetime. The planetary nebulae have

a known luminosity function which can be used as distance indicators for distant galaxies.

- The brightest galaxies in clusters: It has been found that the brightest galaxies in clusters of galaxies have a very similar absolute magnitude in all clusters. They can therefore be used as distance indicators to clusters of galaxies.

4 The Hubble law

At the top of the distance ladder, we find the Hubble law. Edwin Hubble discovered in 1926 that all remote galaxies are moving away from us. The further away the galaxy, the faster it was moving away from us. This has later been found to be due to the expansion of the Universe: The galaxies are not moving away from us, the space between us and distant galaxies is expanding inducing a Doppler shift similar to that induced by a moving galaxy. Waves emitted by an object moving away from us have larger wavelengths than in the rest frame of the emitter. Thus, light from distant galaxies are red shifted. By measuring the red shift of distant galaxies, we can measure their velocities, or in reality the speed with which the distance is increasing due to the expanstion of space. From this velocity we can find their distance using the Hubble law

$$v = H_0 r,$$

where r is the distance to the galaxy, $H_0 \approx 71$ km/s/Mpc is the Hubble constant and v is the velocity measured by the redshift.

$$v = c\frac{\Delta\lambda}{\lambda}.$$

The Hubble law is only valid for large distances.

5 OPTIONAL: Uncertainties in distance measurements

There are several uncertainties connected with distance measurements. One of the main problems is caused by *interstellar extinction*. Our galaxy contains huge clouds of dust between the stars. Light which passes through these dust clouds loose flux as

$$F(\lambda) = F_0(\lambda)e^{-\tau(\lambda)}, \quad (4)$$

where $F(\lambda)$ is the observed flux and $F_0(\lambda)$ is the flux we would have observed had there not been any dust clouds between us and the emitting object. Finally, the quantity $\tau(\lambda)$ is called the *optical depth* and is given by

$$\tau(\lambda) = \int_0^r dr' n(r')\sigma(\lambda, r').$$

Here $n(r)$ is the number density of dust grains at distance r from us and $\sigma(\lambda, r)$ is a measure of the probability for a photon to be scattered on a dust grain. The optical depth is simply an integral along the line of sight from us to the emitting object of the density of dust grains times the probability of scattering. The larger the density of dust grains or the larger the probability of scattering, the larger the optical depth. The optical depth is a measure of how many photons which are scattered away during the trip from the radiation source to us. If the scattering probability is constant along the line of sight (this depends on properties of the dust grains), we can write the optical depths as

$$\tau(\lambda) = \sigma(\lambda)\int_0^r dr' n(r') = N(r)\sigma(\lambda),$$

where $N(r)$ is the total number of dust grains that the photons encounter during the trip from the emitter at distance r.

Interstellar extinction increases the apparent magnitude (decreases the flux) of an object. Photons are scattered away from the line of sight and the objects appear dimmer. Taking this into account we need to correct our formula for the relation between the apparent and the absolute magnitude

$$m(\lambda) = M(\lambda) + 5\log_{10}\left(\frac{r}{10\text{ pc}}\right) + A(\lambda),$$

where $A(\lambda)$ is the total extinction at wavelength λ and $m(\lambda)$ and $M(\lambda)$ are the apparent and absolute magnitudes based on the flux at wavelength λ only. Using the formula for the difference between two apparent magnitudes in lecture 6, we can write the change in apparent magnitude due to extinction as

$$m(\lambda) - m_0(\lambda) = -2.5\log_{10}\left(\frac{F(\lambda)}{F_0(\lambda)}\right) = -2.5\log_{10}(e^{-\tau(\lambda)}) = 1.086\tau(\lambda),$$

where also equation (4) was used (check that you can deduce this formula!). Here $m_0(\lambda)$ and $F_0(\lambda)$ is the apparent magnitude and flux we would have had if there hadn't been any extinction. Thus, we see that

$$m(\lambda) = M(\lambda) + 5\log_{10}\left(\frac{r}{10\text{ pc}}\right) + 1.086\tau(\lambda).$$

Clearly, if we use a distance indicator and do not take into account interstellar extinction, we obtain the wrong distance. It is often difficult to know the exact optical depth from scattering on dust grains. This is an important source of error in distance measurements. Note that the extinction does not only increase the apparent magnitude of an object, it also changes the color. We have seen that the optical depth $\tau(\lambda)$ depends on wavelength λ. The scattering on dust grains is larger on smaller wavelength. Thus, it affects red light less than blue light with the result that the light from the object appears redder. This is called *interstellar reddening*.

Another source of error in the measurement of large distances in the Universe is the fact that objects observed at a large distance are also observed at an earlier phase in the history of the universe. The light from an object at a distance of 1000 Mpc or 3260 million light years has travelled for 3260 million years or roughly one fourth of the lifetime of the Universe. Thus, we observe this object as it was 3260 millions years ago. The universe has been evolving all the time since the Big Bang until today. We do not know if the galaxies and stars at this early epoch had the same properties as they have today. Actually, we have good reasons to believe that they did not. We will come to this later. This could imply that for instance the relation between light curve and absolute magnitudes of supernovae were different at that time than today. Using relations obtained from obervations of the present day universe to observations in the younger universe may lead to errors in measurements of the distance.

6 Exercises

Exercise 3A.1

1. A star is observed to change its angular position with respect to very distant stars by $1''$ in half a year. Assuming that the star does not have any peculiar velocity with respect to us, what is the parallax angle for the star? And its distance?
2. What is the parallax angle for our nearest star Proxima Centauri at a distance of 4.22 ly? (Assume again that the observations are made with a distance of half a year).
3. An open star cluster is observed to have red stars (surface temperature $3000 - 4000$ K) with apparent magnitudes in the range $m = [10, 12]$, yellow stars (about 6000 K) in the apparent magnitude range $m = [6, 9]$ and a few hotter white stars (10000 K) in the apparent magnitude range $m = [1, 5]$. What is the distance to the cluster? Use the diagram in figure 3.
4. A supernova explosion of type Ia is detected today in a distant galaxy. Its apparent visual magnitude at maximum was $m_V = 20$. You still need to wait a few days to obtain the light curve and thereby the exact absolute magnitude. But you can already find an approximate distance. In which distance range do you expect to find the supernova?
5. A distant galaxy is measured to have the center of its 21 cm line ($\lambda_0 = 21.2$ cm) shifted to $\lambda = 29.7$ cm. What is the distance of the galaxy?