UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Mid-term exam for AST3220 — Cosmology I

Day of exam: Friday 22nd of March 2013

Time for exam: 15.00 – 18.00

This problem set consists of 3 pages.

Attachments: None

Allowed aids: All non-communicative aids.

Make sure that the problem set is complete before you start answering the questions.

Throughout the equation set you can assume that today's value of the Hubble constant is:

$$\begin{aligned} H_0 &= 70 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1} \\ &= (14 \times 10^9 yr)^{-1} \end{aligned}$$

You may also set the scale factor today to unity

$$a_0 = 1$$

Problem 1

Cosmological Distances

Typical peculiar velocities of galaxies are about 300 km/s. The mean distance between large galaxies is about 1 Mpc. How distant must a galaxy be from us for its peculiar velocity to be small compared to its comoving (Hubble) velocity?

Problem 2

The Einstein-de Sitter model

Consider the spatially flat model with only matter, $\Omega_m = 1$.

a) Calculate the scale factor $a(t)$, the age-redshift relationship $t(z)$ and the angular diameter distance $d_A(z)$. Express the latter two in terms of inverse Hubble time (H_0^{-1}).

b) What is the angular diameter distance to the particle horizon in units of inverse Hubble time?

c) What is the age of the universe (in years) today and at $z = 1090$?

d) What is the angular diameter distance (in Mpc) to redshift $z = 1090$? This is in principle the furthest redshift we can observe. Compare with your answer in problem 1. Can you find the comoving velocity by studying single galaxies?

e) The function $d_A(z)$ has a maximum. At which redshift is the maximum?

Problem 3

The concordance model

Consider a universe filled with dust and a comsological constant with $\Omega_{m0} = 0.3$ and $\Omega_\Lambda = 0.7$, so that the universe is spatially flat.

a) Find the age of the universe today and at redshift $z = 1090$. Compare this to your answers to problem 2c).

   Hint: Use the substitution $x^{3/2} = b \sinh \psi$ for the integral

$$\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$$

   . You may also need that

$$\sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right)$$

b) When was the matter density equal to the vacuum energy density? (Give both t (in years) and z.)

c) In this model the expansion rate has an inflection point, where $\ddot{a} = 0$, at which the expansion starts to accelerate. When did this happen, in t (in years) and in z?