EXERCISE 1

A) Why the evolution of a Fourier mode of the density perturbations $\delta_k(t)$ on scales >> Jeans length λ_J can be described by

$$\ddot{\delta}_k + 2H(t)\dot{\delta}_k = 0 \qquad (1)$$

if the Universe is dominated by a scalar field (w/negligible clustering)

(i.e. describe how you get this from eq 4.26 in the notes)

start from $\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \delta_k(4\pi G\rho_m - k^2 c_s^2)$ (1a)

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B) say that i) $H(t) \sim \frac{1}{3t}$; ii) $H(t) = \text{constant}$

$a \sim t^{1/3}$ $\qquad$ $a \sim$?

solve eq (1)

two ways i) a) try with t^n solve for n i.e. $\delta = A t^n$

$$n(n-1) + \frac{2}{3}n = 0 \rightarrow n = 0 \quad n = 1/3$$

$$\delta = \text{const} + c t^{1/3}$$

b) say $y = \dot{\delta}$ $\rightarrow \dot{y} = -\frac{2}{3t}y \rightarrow y = A' t^{-2/3}$

integrate $\delta_k = c t^{1/3} + b$ $\qquad$ P.S $c = 3A'$

ii) $\dot{y} = -2y \rightarrow y = e^{-2t} \rightarrow y = a' + b' e^{-2t}$

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what does it tell us about the evolution of the structure formation in such Universe ?

perturbations do not grow or grow slowly -

NEUTRINOS!

in standard ΛCDM ν' are considered massless (& 3 ν families are assumed)
oscillation experiments however have shown that ν's have non zero masses.
but have not established a ν's mass scale.
(if neutrinos have a mass below 1 eV) they are hot, & have a high velocity dispersion. they are very (very!) weakly interacting.

Assume that the fraction of (Dark) matter in ν's can be written as

$$f_\nu = \frac{\Omega_\nu}{\Omega_m} \quad \text{where } (\Omega_m = \Omega_c + \Omega_\nu)$$

write down how the differential eq. for the growth of perturbations

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \bar{\rho}\,\delta - c_s^2 k^2 \delta$$

can be written for the (cold) dark matter overdensity field δ_c and expressed in terms of f_ν; motivate it.
consider scales ~~[illegible]~~ above the Jeans length but below ν's free streaming scale.
compute growth of perturbations assuming that f_ν is small.

matter domination is when it matters

$$\ddot{\delta}_c + 2H\dot{\delta}_c = 4\pi G \frac{\rho_m}{\rho_{cr}} \rho_{cr} (1 - f_\nu)$$

$$= \frac{3}{2} H^2 \Omega_m (1 - f_\nu)\,\delta_c$$

in matter domination $H \sim \frac{2}{3}\frac{1}{t}$ $a \propto t^{2/3}$

$$\ddot{\delta}_c + \frac{4}{3}\frac{1}{t}\dot{\delta}_c = \frac{2}{3}\frac{3}{2}\frac{2}{3}\frac{1}{t^2}\Omega_m(1-f_\nu)\,\delta_c$$

$\cancel{\delta = \delta_i t^{n}}$; $\cancel{\dot\delta = \delta_i n t^{n-1}}$; $\cancel{\ddot\delta = \delta_i n(n-1) t^{n-2}}$

$\cancel{-\frac{4}{9} t^{-4/3} \ldots \frac{8}{9} t^{-4/3} = \frac{3}{2}\Omega_m(1-f_\nu) t^{-4/3}}$

$\cancel{\ldots = \frac{2}{3}\Omega_m(1-f_\nu)}$

$\delta = \delta_i t^n$

$\dot{\delta} = \delta_i \, n \, t^{n-1}$

$\ddot{\delta} = \delta_i \, n(n-1) \, t^{n-2}$

$$\delta_i \, n(n-1) \cancel{t^{n-2}} + \frac{4}{3} \delta_i \, n \cancel{t^{n-2}} = \frac{2}{3} \Omega_m (1-f_\nu) \delta_i \cancel{t^{n-2}}$$

$$n(n-1) + \frac{4}{3} n = \frac{2}{3} \Omega_m (1-f_\nu)$$

$$3n^2 - 3n + 4n = 2 \Omega_m (1-f_\nu)$$

$$3n^2 + n - 2 \Omega_m (1-f_\nu) = 0$$

$$n = \frac{-1 \pm \sqrt{1 + 4 \cdot 3 \cdot 2 \Omega_m (1-f_\nu)}}{6}$$

$$= \frac{-1 \pm \sqrt{1 + 24 \Omega_m (1-f_\nu)}}{6}$$

$$= \frac{-1 \pm \sqrt{1 + 24 - 24 f_\nu}}{6}$$

$$= \frac{-1 \pm 5 \sqrt{1 - \frac{24}{25} f_\nu}}{6}$$

$$= -\frac{1}{6} \pm \frac{5}{6} \left(1 - \frac{1}{2} \frac{24}{25} f_\nu\right)$$

$$\rightarrow \frac{2}{3} - \frac{2}{5} f_\nu$$

$$\frac{\delta_\nu}{\delta_{\nu=0}} \sim t^{-\frac{2}{5} f_\nu}$$

$\delta \sim t^{2/3}$ if no f_ν

$\sim t^{2/3 - \frac{2}{5} f_\nu}$ if $f_\nu \neq 0$

slow growth.

c) how would this affect LSS ....