

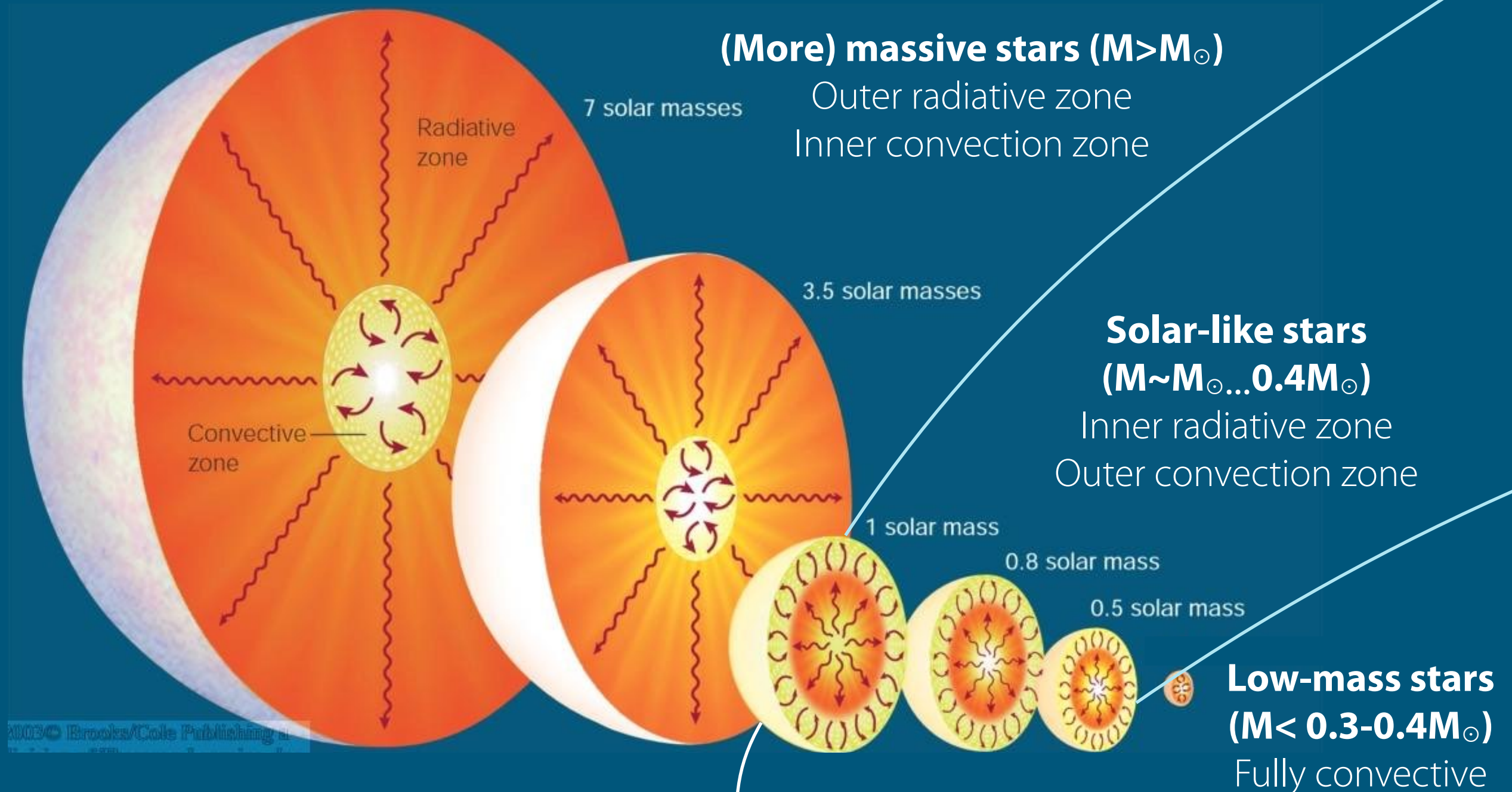
AST5770

Solar and stellar physics

Sven Wedemeyer, University of Oslo, 2025

Recap — Energy transport

Differences along the main sequence



Dominant fusion process

CNO cycle

pp chain

Recap

Differences along the main sequence

(More) massive stars ($M > M_{\odot}$)

Outer radiative zone
Inner convection zone

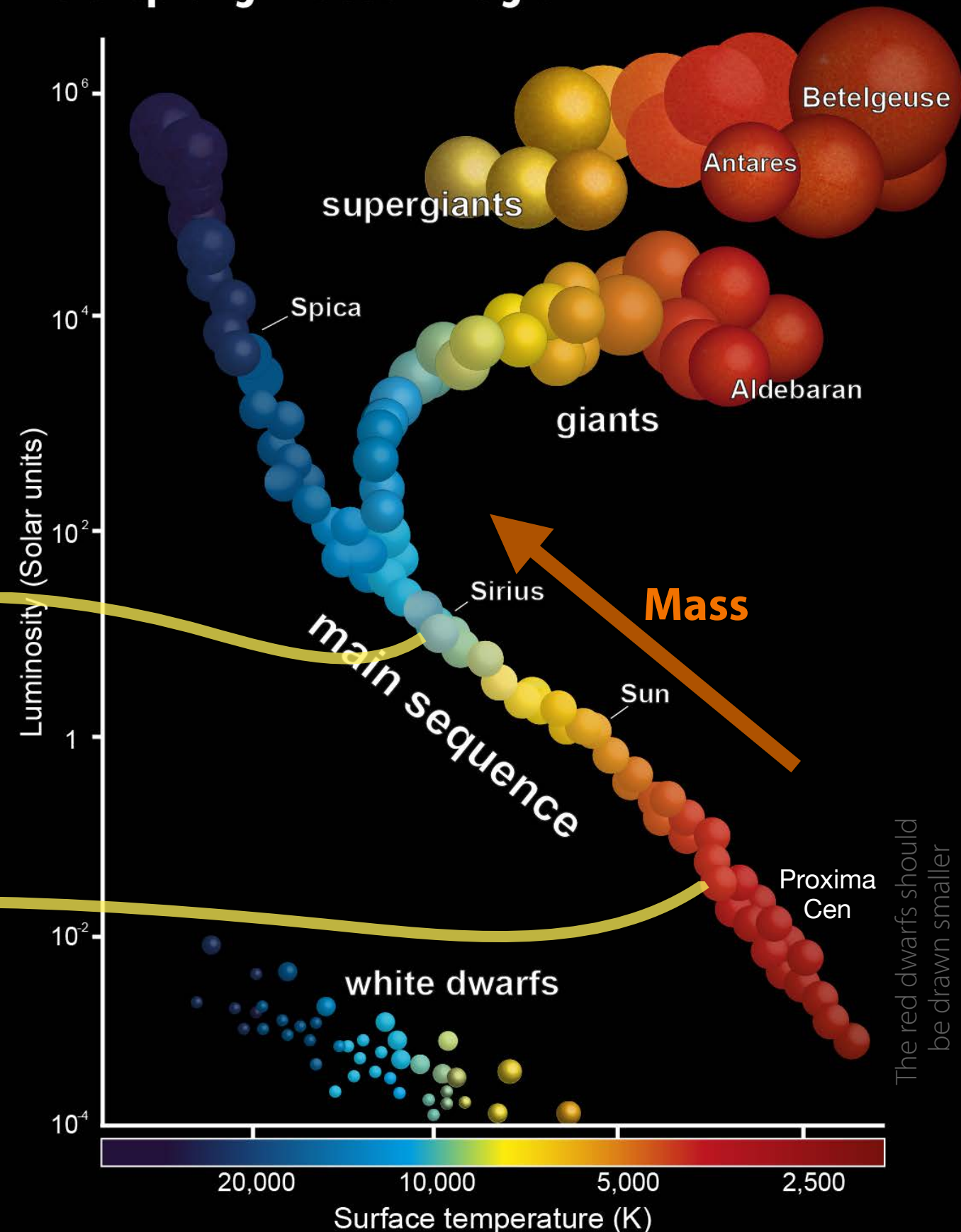
**Solar-like stars
($M \sim M_{\odot} \dots 0.4 M_{\odot}$)**

Inner radiative zone
Outer convection zone

**Low-mass stars
($M < 0.3 - 0.4 M_{\odot}$)**

Fully convective

Hertzprung–Russell Diagram



Recap — Energy transport

Radiation	Photons carry energy, travels a distance between emission and being absorbed again	Occurs whenever a temperature gradient is maintained.
Conduction	Particles carry energy, travels a distance between collisions with other particles (during which energy is exchanged)	Occurs whenever a temperature gradient is maintained. In Sun/solar-like stars only relevant in the corona.
Convection	(Macroscopic) mass motion of gas elements	Occurs only if the temperature gradient exceeds a critical value.

- Radiation and conduction: diffusion process driven by a temperature gradient ∇T

➔ Net energy flux $\mathbf{F} = -K \nabla T$

(K: diffusion constant)

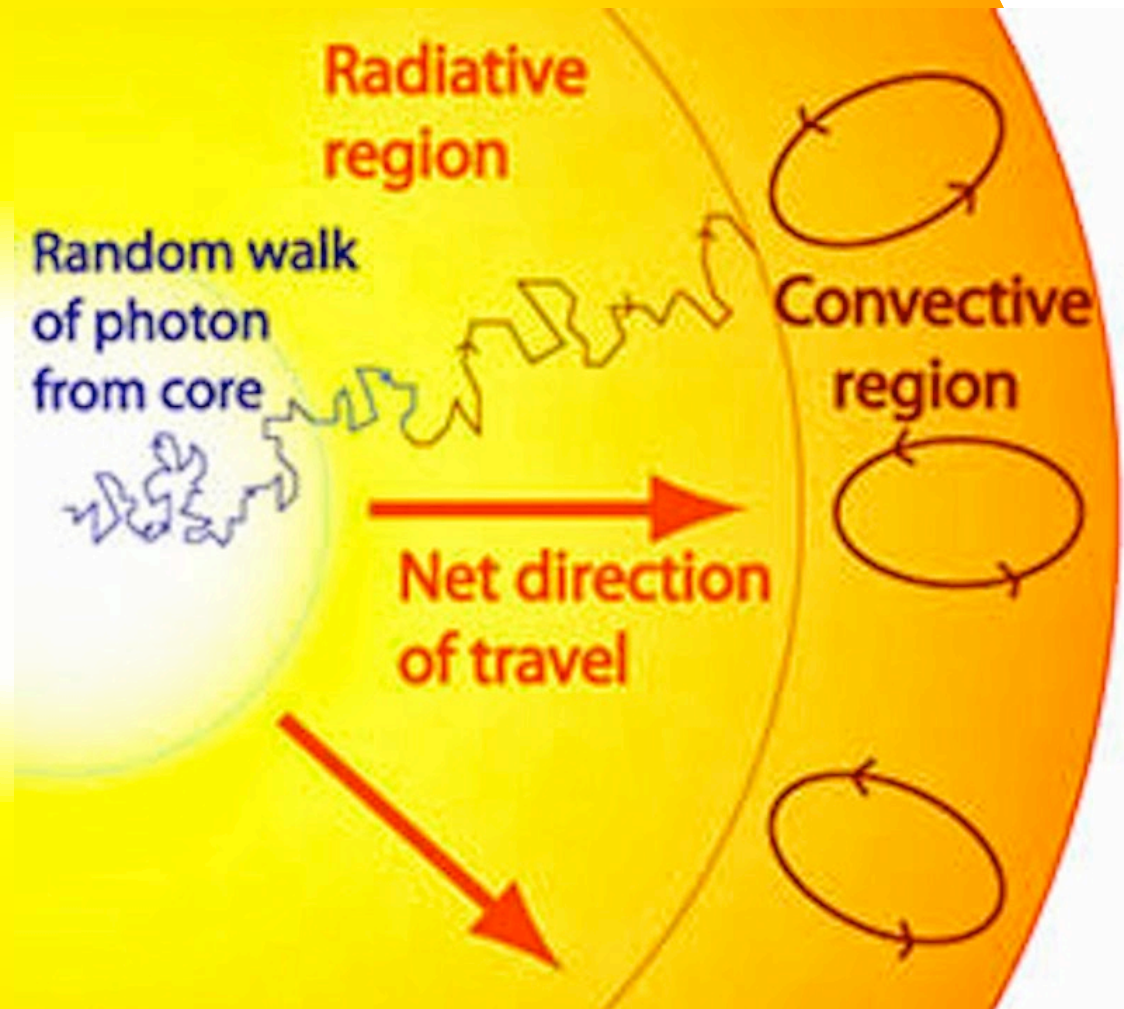
➔ Radiative temperature gradient $\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{r^4 T^3}$

- ➔ Rosseland mean opacity (to incorporate freq. dependence)

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

Energy transport

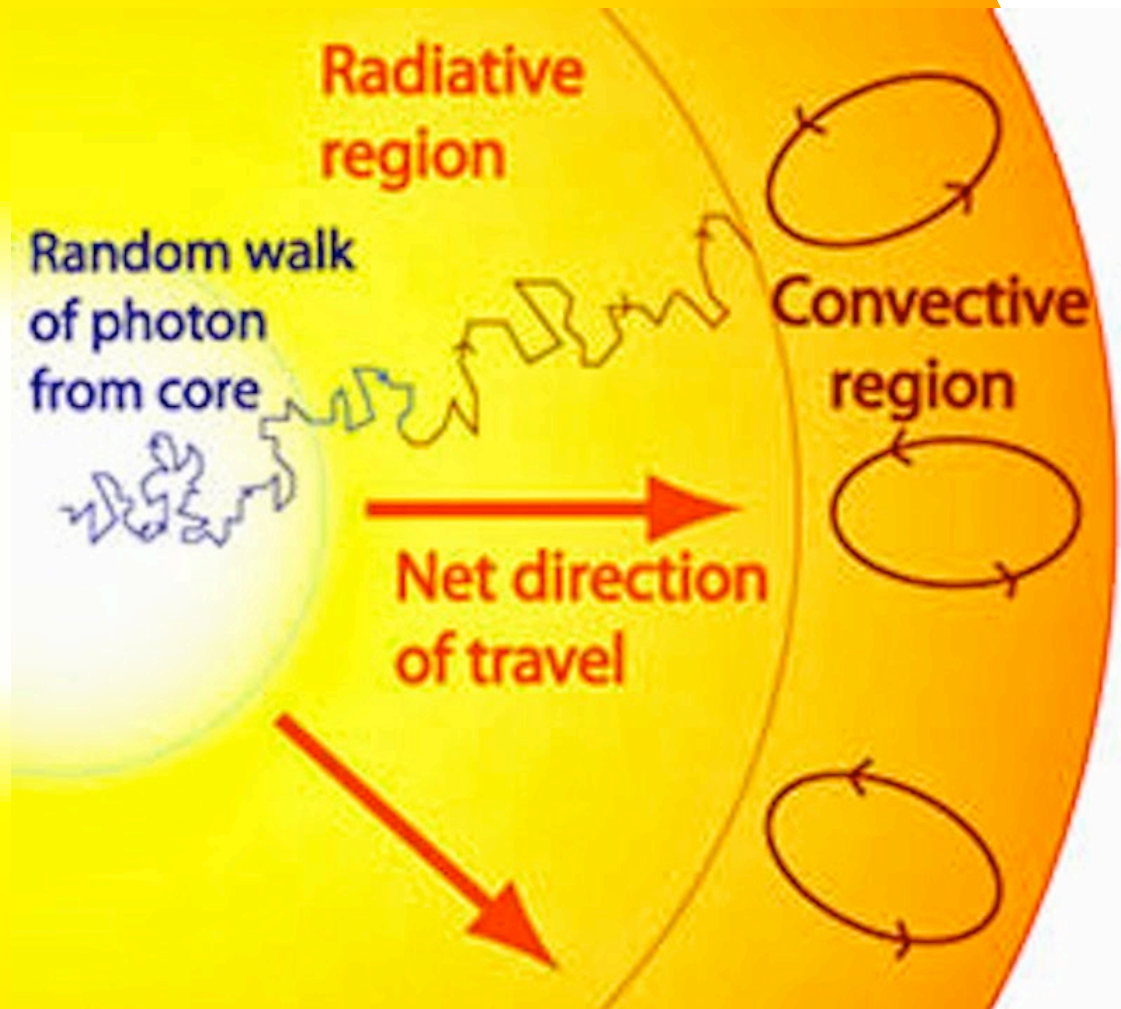
Radiative energy transport



- Time for a photon to travel from centre to surface without interaction:
 $\sim 2s$
- **But: mean free path of photons very small!**
- In the dense solar interior: Mean free path of a photon only 10^{-2} m
- Random walk,
photon absorbed and re-emitted
 $\sim 10^{22}$ times before reaching surface
- ➡ Time $\sim$ thermal timescale of the Sun
 $\sim 2 \cdot 10^7 \text{ yr}$
- ➡ Observed radiation due to fusion reactions
(on average) tens of millions of years ago.
- Net direction towards surface due to
gradients (opacity)

Energy transport

Radiative energy transport



- **Radiation pressure** $P_{\text{rad}} = \frac{1}{3} a T^4$
- Outward force that must be smaller than gravitational force in order to maintain **hydrostatic equilibrium** (HE)!

$$\left| \frac{dP_{\text{rad}}}{dr} \right| < \left| \left(\frac{dP}{dr} \right)_{\text{HE}} \right| \Rightarrow \frac{\kappa \rho}{4\pi c} \frac{l}{r^2} < \frac{Gm\rho}{r^2}$$

- ➡ Upper limit to the local luminosity: (local) **Eddington luminosity**:
Maximum energy flux that can be carried by radiation

$$l < \frac{4\pi c G m}{\kappa} = l_{\text{Edd}}$$

- ➡ Can get violated by intense nuclear burning
- ➡ In these situations, radiative energy transport insufficient for maintaining hydrostatic equilibrium

- At the surface ($m=M$): $L < L_{\text{Edd}} = \frac{4\pi c G M}{\kappa}$

$\uparrow$
 In photosphere

Energy transport

Radiative energy transport

- **Radiation pressure** $P_{\text{rad}} = 1/3 a T^4$
- Outward force that must be smaller than gravitational force in order to maintain **hydrostatic equilibrium** (HE)!

$$\left| \frac{dP_{\text{rad}}}{dr} \right| < \left| \left(\frac{dP}{dr} \right)_{\text{HE}} \right| \Rightarrow \frac{\kappa \rho}{4\pi c} \frac{l}{r^2} < \frac{Gm\rho}{r^2}$$

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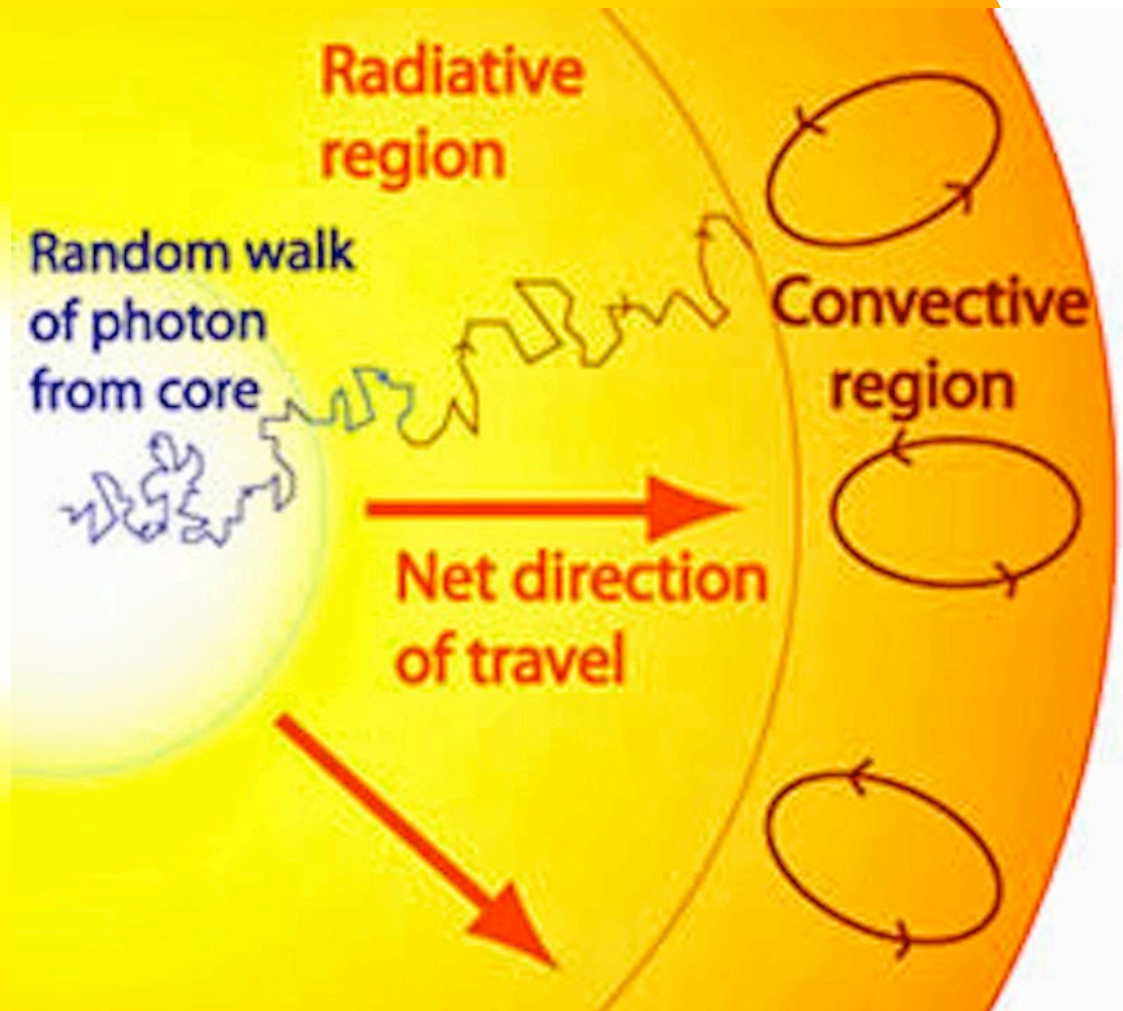
↑
In photosphere

Condition violated?

- ➡ No hydrostatic equilibrium!
- ➡ Gas accelerated outwards due to radiative pressure.
- ➡ Can potentially lead to mass loss!

Energy transport

Radiative energy transport



- Note: At high temperature all atom are completely ionised
 - ➡ Photons move through a plasma consisting of free electrons and atomic nuclei (incl. protons)
- Towards surface (in solar-like stars): Not all atoms are completely ionised anymore
 - ➡ Changes in mean molecular weight and gradients
 - ➡ Convection becomes the dominant mode of energy transport subject to stability criterion
- Note: Radiative energy transport the “default” in convectively stable regions

Energy transport

Convection

$$F = F_{\text{rad}} + F_{\text{c}} + F_{\text{cd}} = L / 4\pi r^2$$

$$F_{\text{rad}} \propto \nabla T$$

- Transport for larger energy flux (luminosity) requires larger temperature gradient required.
- But: Upper limit to ∇T in stellar interior
 - ➔ Stratification becomes unstable if limit exceeded
 - ➔ (Additional) dominating energy transport via convection

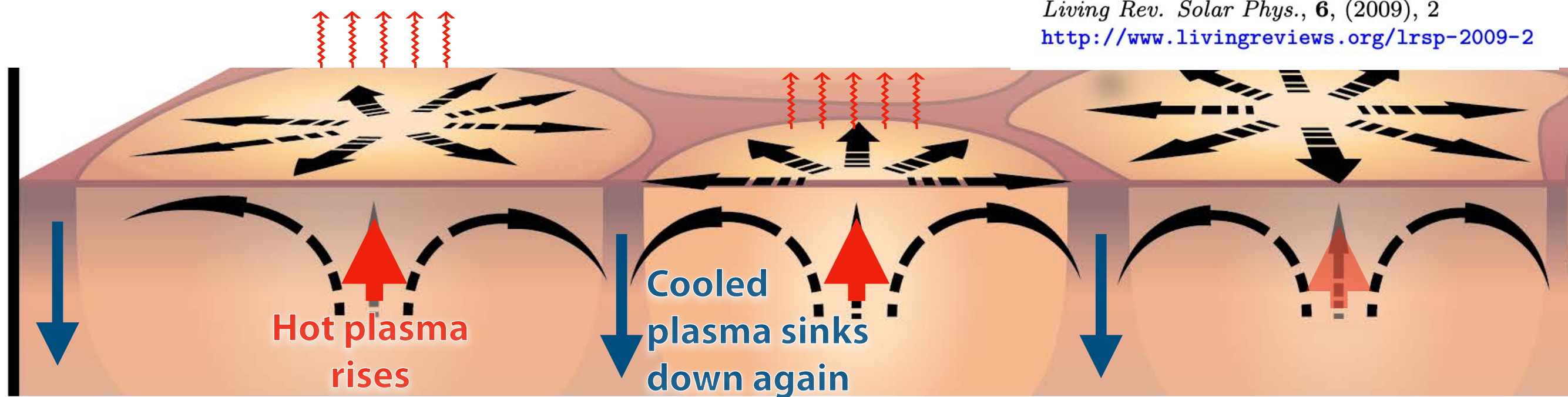
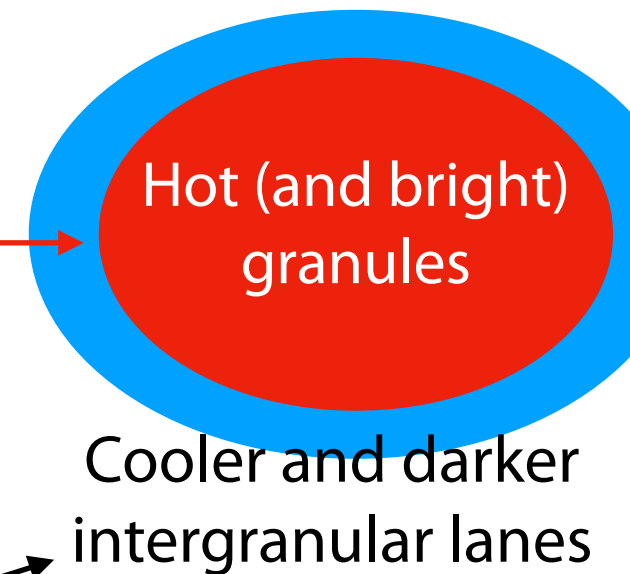


Energy transport

Surface convection — granulation

- Density, temperature decreases with radius
- Eventually plasma transparent enough (longer mean free path)
- ➡ **Radiation effectively removes heat from rising convective cells** at surface
- ➡ Plasma cools
- ➡ Advected sideways (pushed away from more upwelling gas below)
- ➡ Cooled and dense **plasma sinks down again**

Literature: Nordlund, Stein, Asplund
Living Rev. Solar Phys., 6, (2009), 2
<http://www.livingreviews.org/lrsp-2009-2>

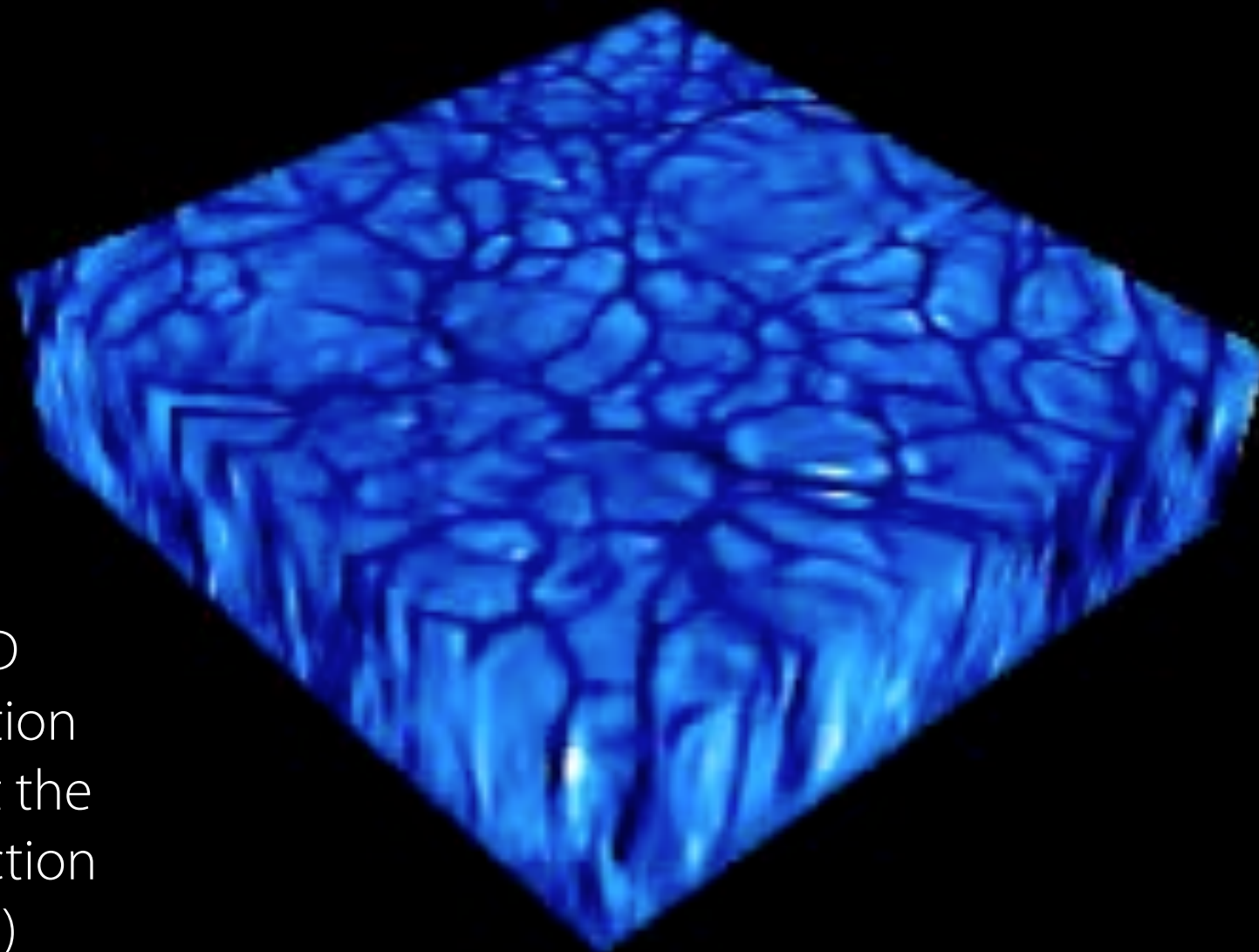


Living Rev. Solar Phys., 6, (2009), 2
<http://www.livingreviews.org/lrsp-2009-2>

Top of convection zone — energy transport via convection (bulk motion)

Energy transport

Convection — Solar surface convection

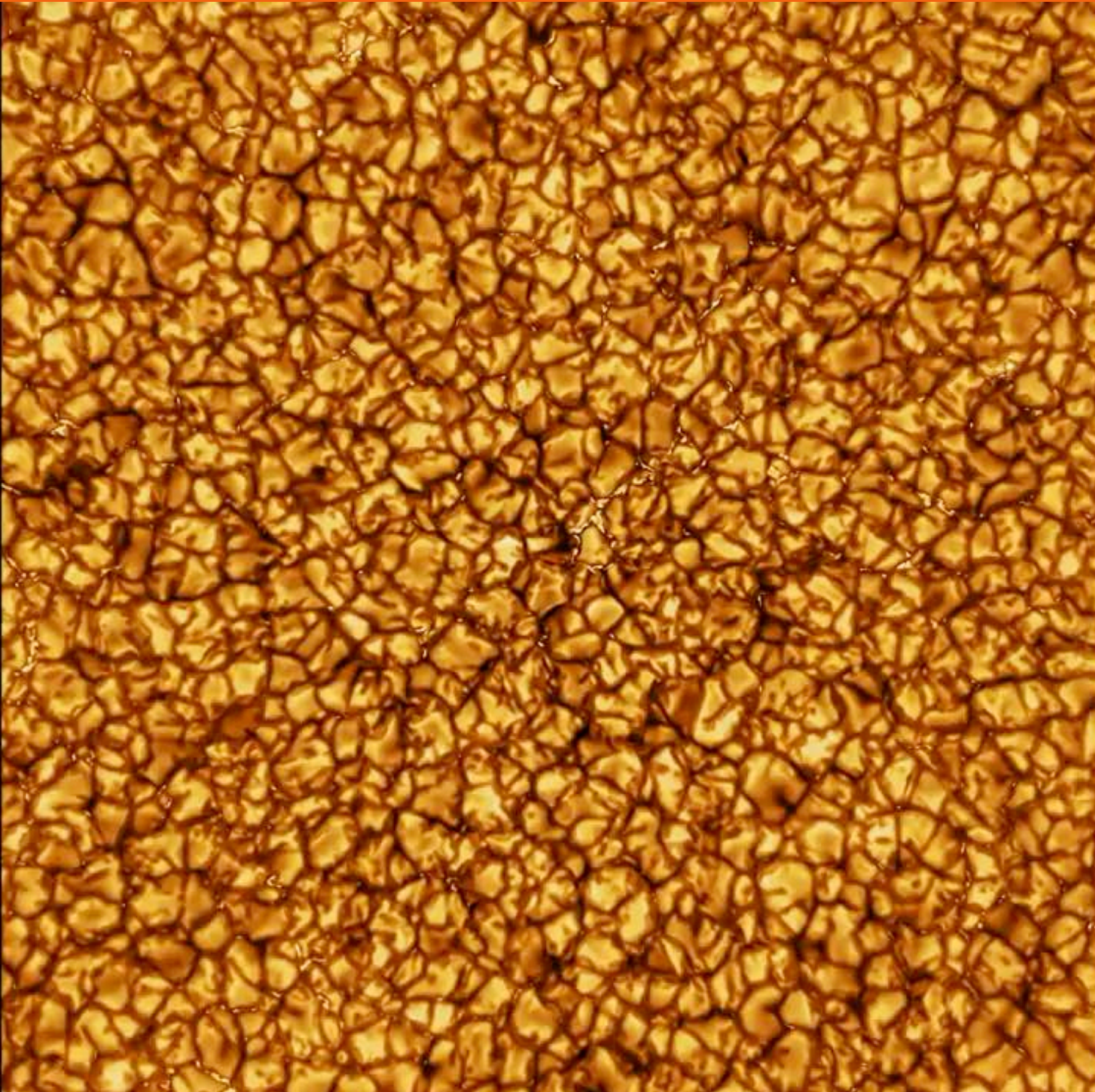


Time-dependent 3D hydrodynamic simulation showing convection at the top of the Sun's convection zone (at the surface)

Energy transport

Highest-resolution
observations of the Sun's
granulation ever taken.

DKIST (4m)
(NSO/AURA/NSF)



KRajala ©

10 Dec. 2019 19:24:31 UT

Energy transport

Surface convection — solar granulation

Highest-resolution observations of the Sun's granulation ever taken (DKIST, NSO/AURA/NSF)

Granules

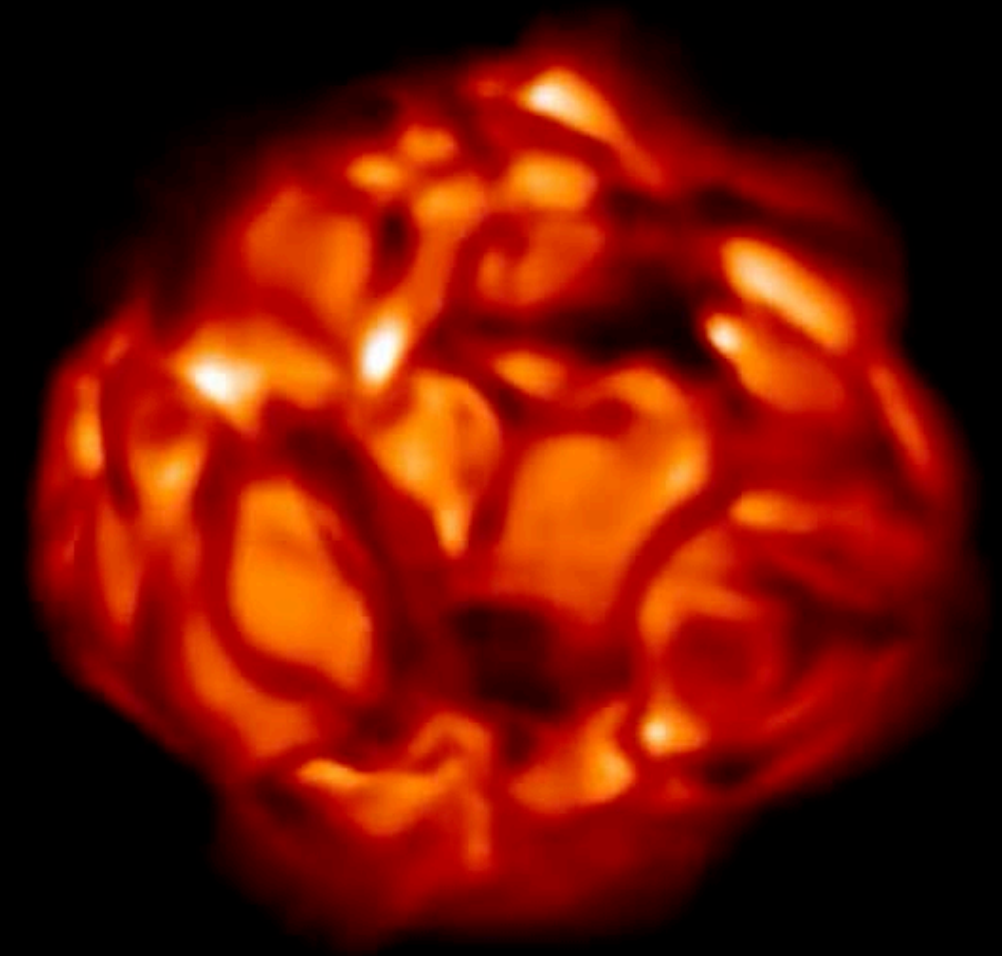
- Diameters $\sim 1000\text{-}2000\text{ km}$
- Lifetime $\sim 8\text{ min}$

Bright features in inter-granular lanes

- (Mostly) due to magnetic field concentrations

Energy transport

Convection



Betelgeuse's gravity:
0.0023 m/s

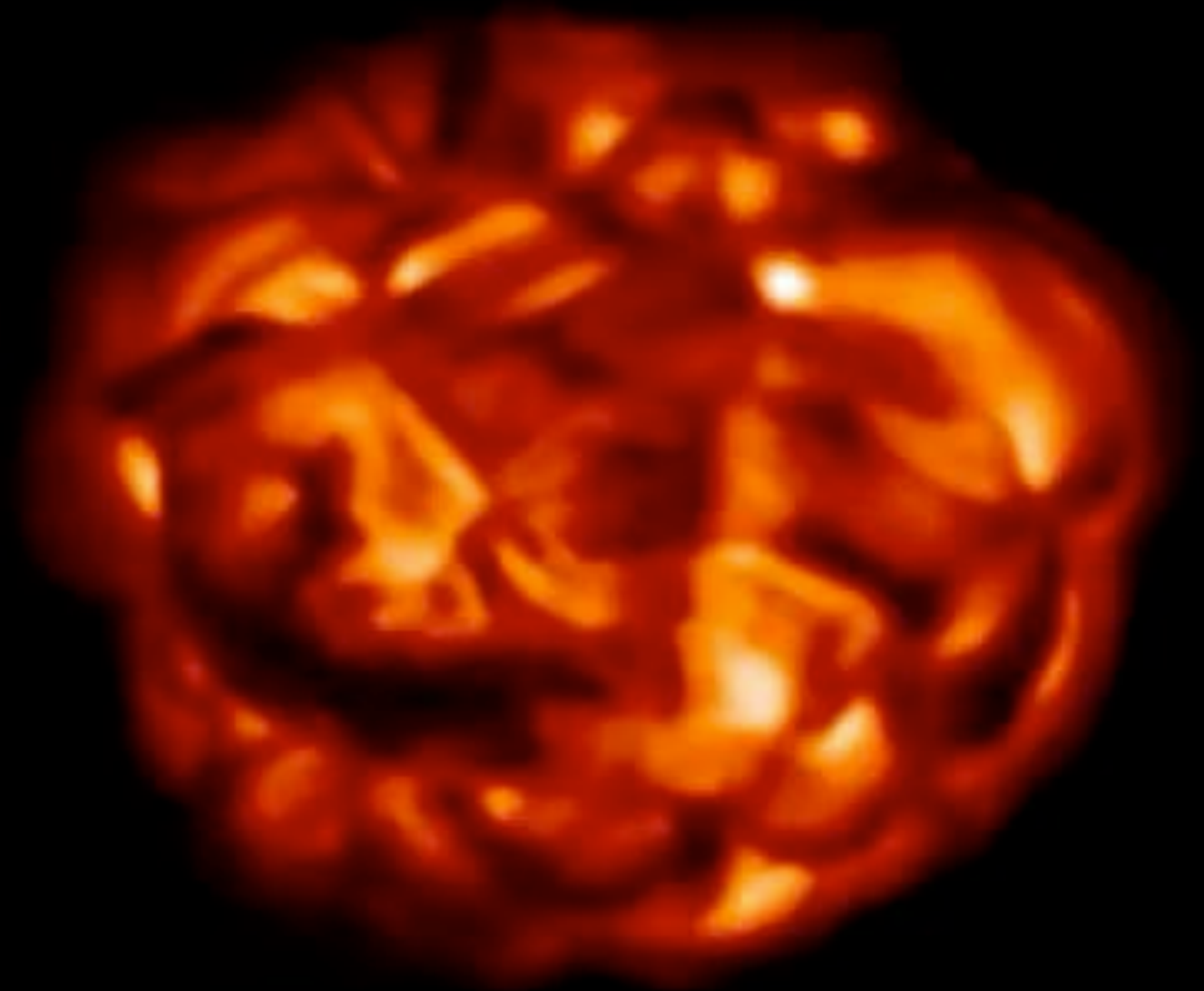
Energy transport

Convection

st35gm04n26: Surface Intensity(3I), time(0.0)=30.263 yrs

Time-dependent 3D
hydrodynamic simulation of
Betelgeuse,
here **intensity**

- Large convection cells
- Slower temporal evolution
(Note time at top right)



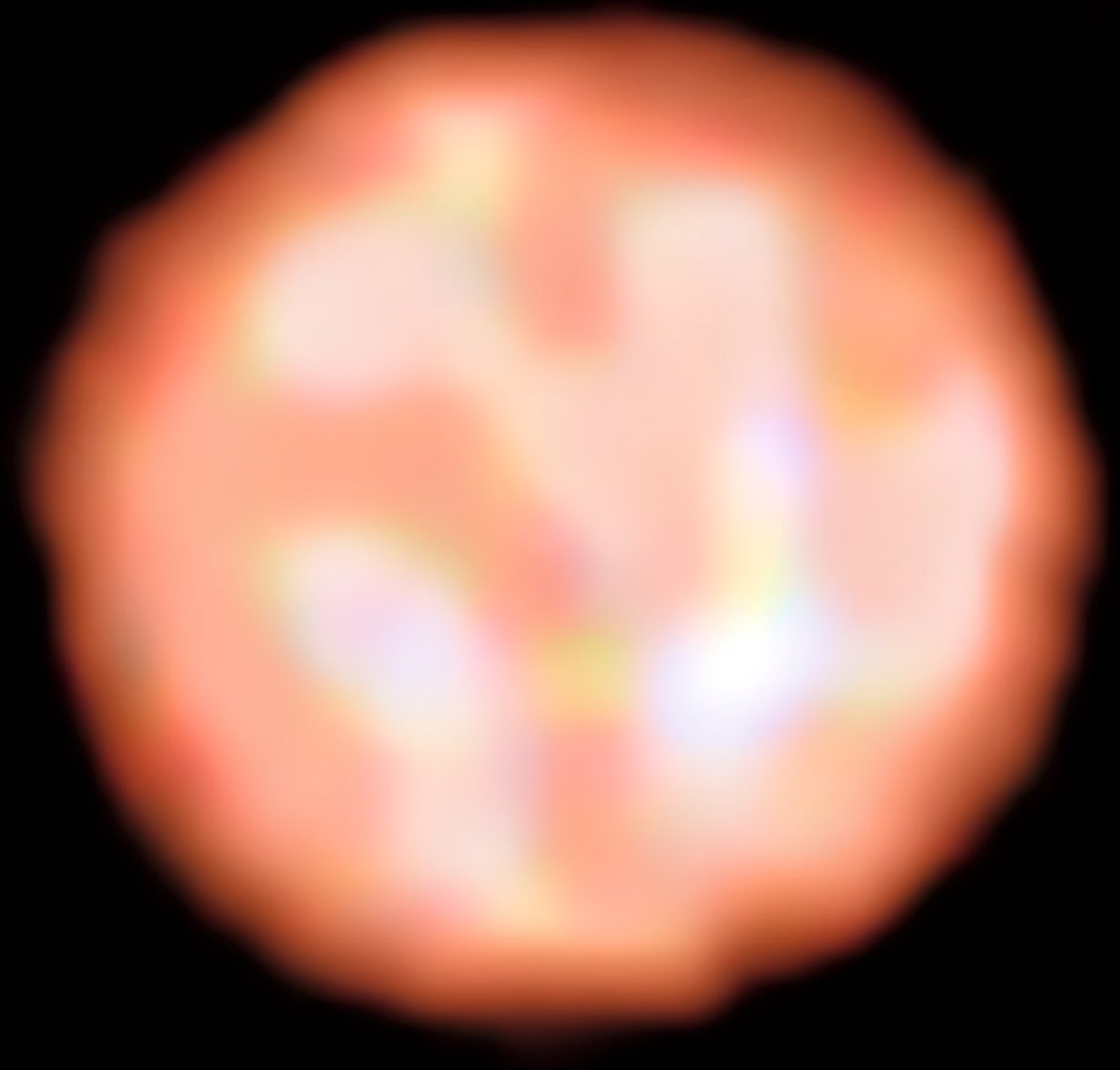
*Simulation produced with the same
code as the models for the project
assignment (Freytag et al. 2012)*

Freytag

Energy transport

Surface convection on other stars

- Observations of the cool red giant π^1 Gruis (with PIONIER/VLT)
- Spectral type M1I
- 1.5 solar masses
- 350 times the diameter of the Sun
- Few giant convection cells



Energy transport

Surface convection on other stars

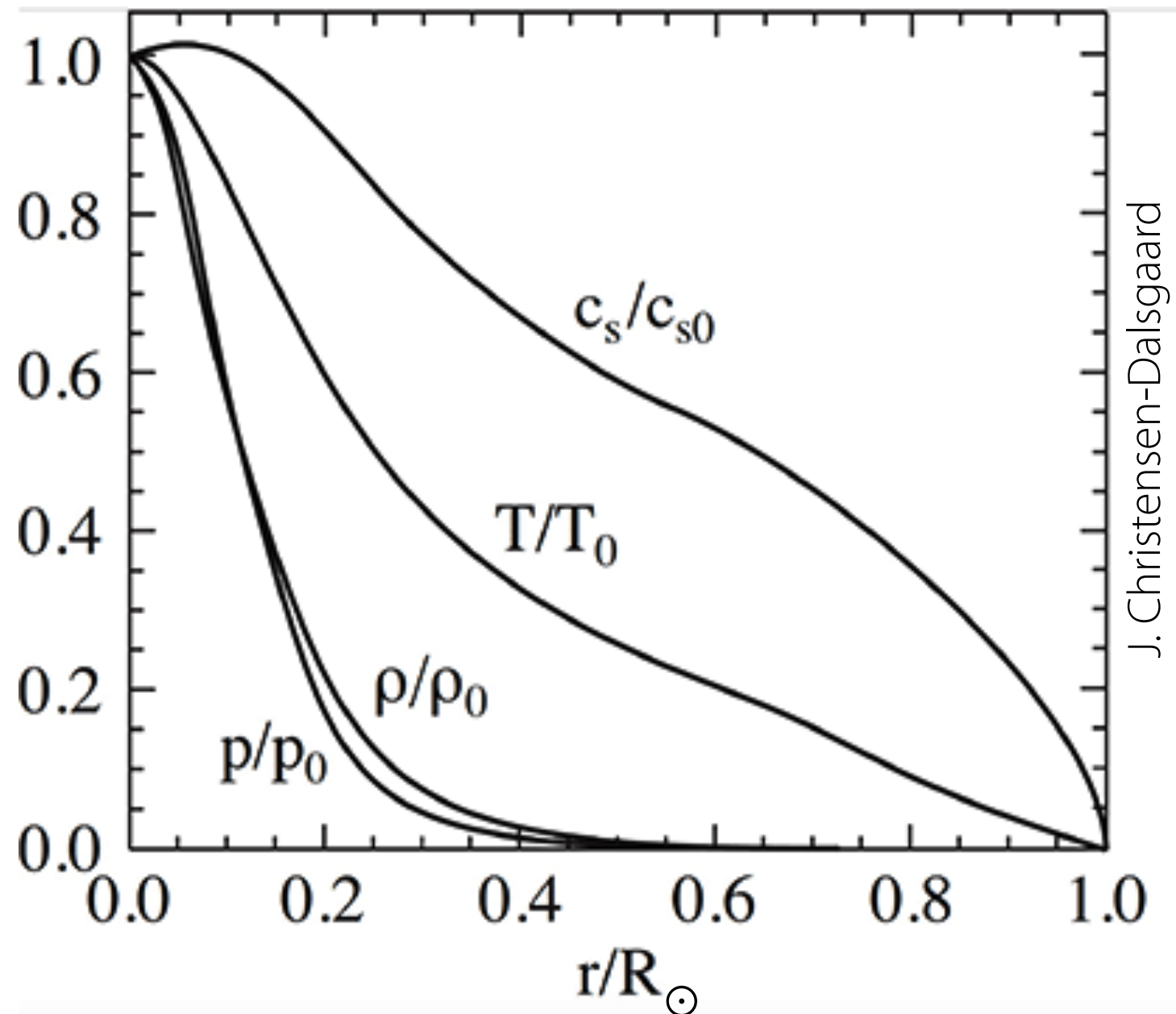
- Observations of the cool red supergiant Antares (with AMBER/VLT)
- Spectral type M1.5I
- ~ 11 -14 solar masses
- ~ 700 times the diameter of the Sun



Stellar interior

Standard model of the solar interior

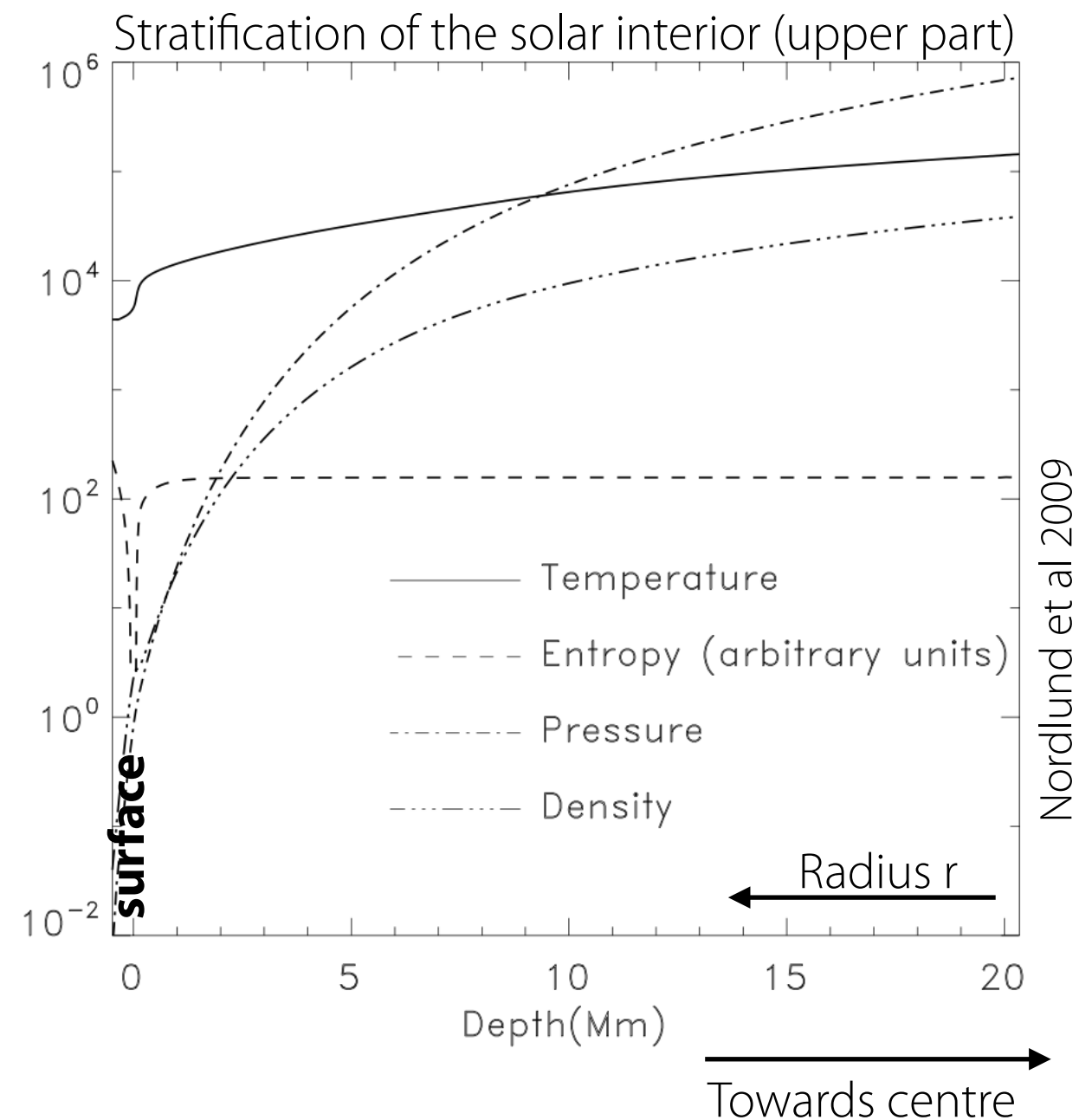
- Variation of (average) quantities as function of radius in the solar interior ($r/R_{\odot}$)
- Scaled to value at solar centre
- Temperature $T_0 = 1.57 \cdot 10^7 \text{ K}$
- Mass density $\rho_0 = 1.54 \cdot 10^5 \text{ kg m}^{-3}$
- Pressure $p_0 = 2.35 \cdot 10^{16} \text{ Nm}^{-2}$
- Sound speed $c_{s,0} = 5.05 \cdot 10^5 \text{ ms}^{-1}$



Energy transport

Convection — Stability criterion

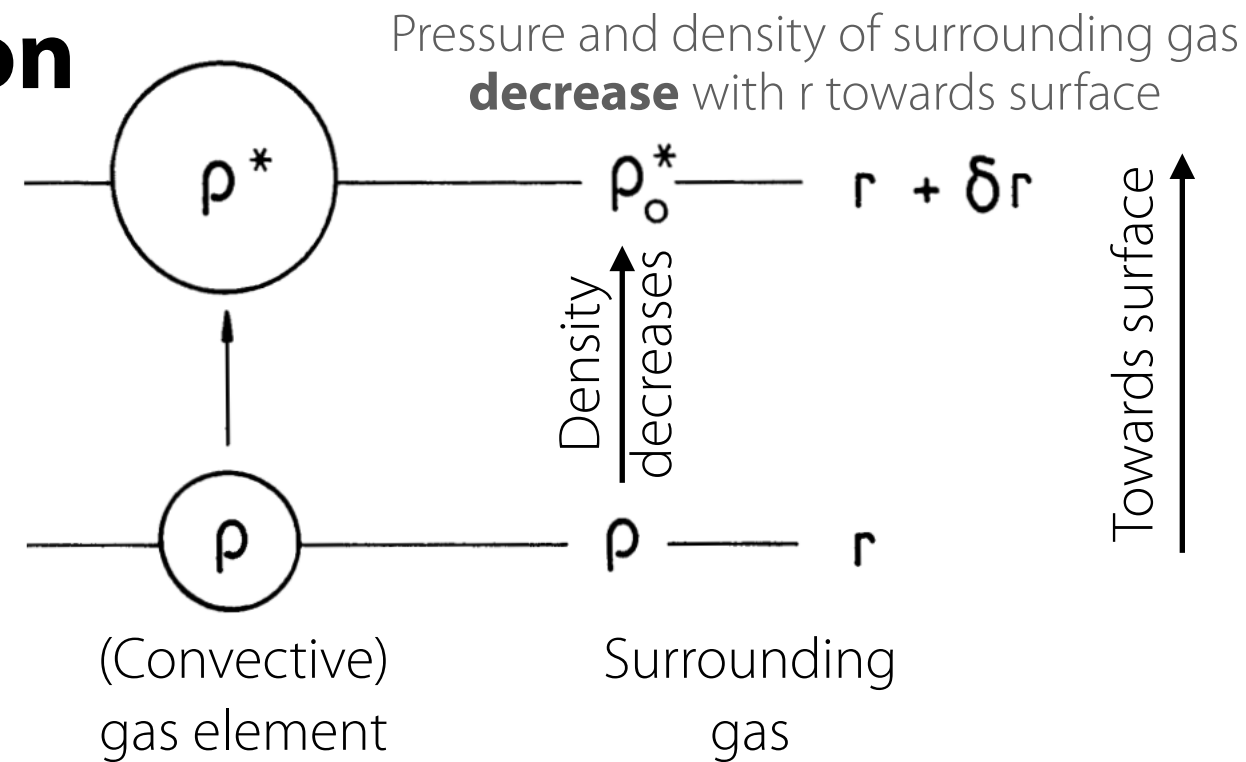
- **Plasma/gas inside a star not a perfectly stratified but small perturbations occur**
 - ➡ Is a layer stable against small perturbations?
 - ➡ Or can initially small perturbations grow and result in significant deviations?
- **In the stratified stellar interior:**
Pressure and density decreases with radius r towards surface



Energy transport

Convection — Stability criterion

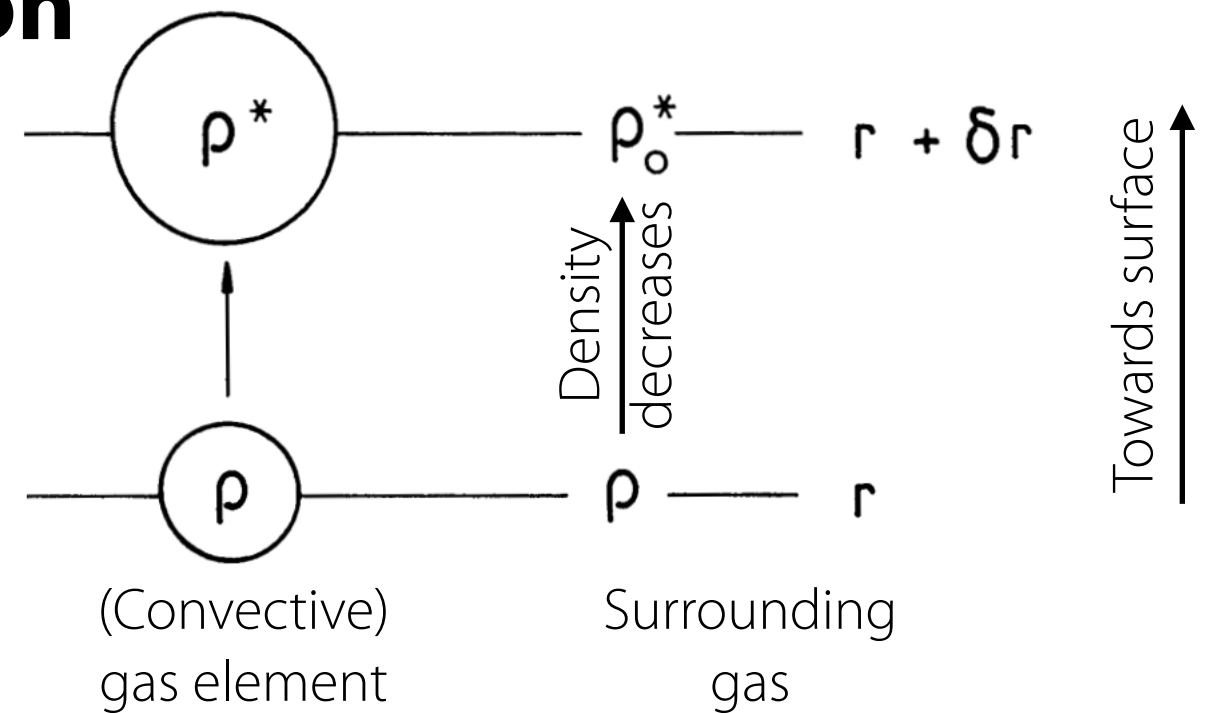
- Gas element at distance r from the centre of the star
- Initially: Element in equilibrium with its surroundings at r :
 - ➡ Pressure P and density ρ are the same as in its surroundings.
- Now — perturbation: **element** displaced (**rises**) a vertical distance δr **adiabatically** (no heat exchanged with environment) but slow enough that pressure is adjusted to new balance with outside pressure
 - Occurs when time scale of heat exchange is long compared to **time scale of expansion** of the element (the latter = local dynamical time scale, set by local sound speed); happens in the optically thick solar interior
- ➡ Element expands to restore pressure equilibrium with surrounding
- ➡ Pressure in the element reduce as it rises
- ➡ Now at $r + \delta r$: Compare density of the element ρ^* with density of (new) surrounding ρ_0^*



Energy transport

Convection — Stability criterion

- Gas element at distance r from the centre of the star rises ...
- Initially in equilibrium with its surroundings at r



- $\rho^* > \rho_0^*$: Element will fall back to initial height — stratification is **convectively stable**
- $\rho^* < \rho_0^*$: Element will keep rising up (net buoyancy!) — stratification is **convectively unstable**

- At $r + \delta r$: Density difference $\rho^* - \rho_0^*$ — gradients!

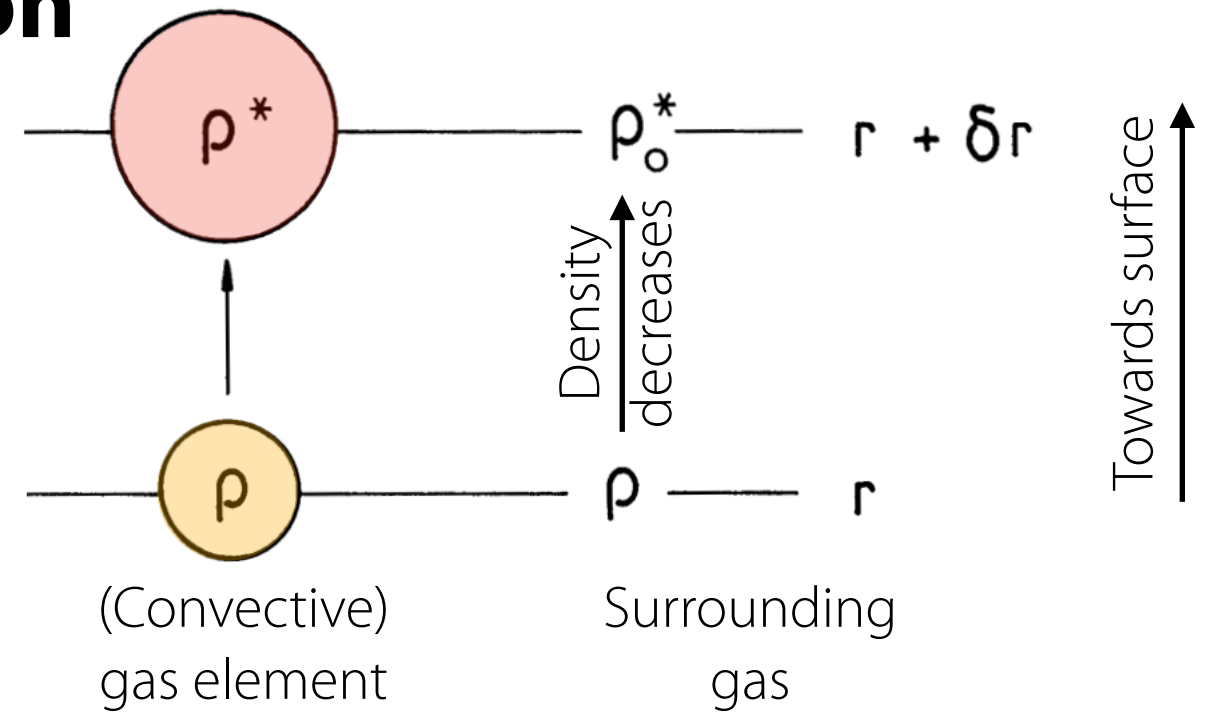
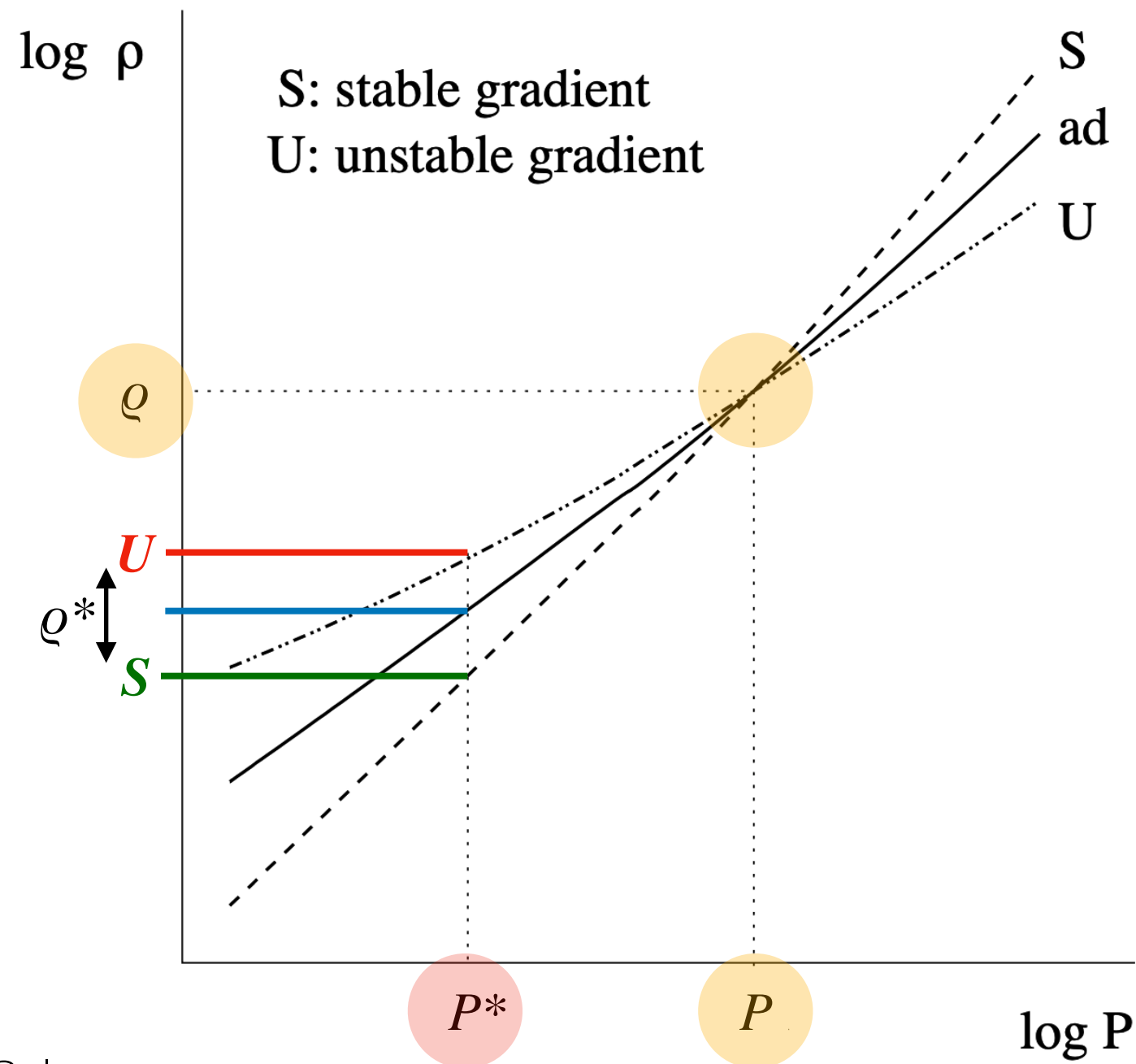
- For adiabatic expansion:
$$\frac{\delta P^*}{P^*} = \gamma_{\text{ad}} \frac{\delta \rho^*}{\rho^*}$$

- ➔ For the situation above, we derive the following criterion for the gas remaining convectively stable:

$$\frac{d \log \rho}{d \log P} > \frac{1}{\gamma_{\text{ad}}}$$

Energy transport

Convection — Stability criterion



$$\frac{d \log \rho}{d \log P} > \frac{1}{\gamma_{\text{ad}}}$$

Energy transport

Convection — Stability criterion

- Compare gradient ∇_{rad} for convectively stable stratification with adiabatic temperature gradient $\nabla_{\text{ad}} \equiv \left(\frac{\partial \ln T}{\partial \ln P} \right)$

- Ledoux criterion** of stability against convection

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} - \frac{\chi_{\mu}}{\chi_T} \nabla_{\mu}$$

∇_{rad} : spatial gradient of temperature

∇_{μ} : spatial gradient of mean molecular weight

∇_{ad} : adiabatic temperature variation in a gas element undergoing a change in pressure.

$$\chi_T = \left(\frac{\partial \log P}{\partial \log T} \right)_{\rho, X_i} \quad \chi_{\rho} = \left(\frac{\partial \log P}{\partial \log \rho} \right)_{T, X_i} : \text{Indices: quantities held constant}$$

- For chemically homogeneous gas: $\nabla_{\mu} = 0$:

➔ **Schwarzschild criterion** of stability against convection

$$\nabla_{\text{rad}} < \nabla_{\text{ad}}$$

- Note: In presence of fusion reactions: $\nabla_{\mu} \geq 0$
- Stabilising effect! (An upwards displaced element is heavier due to higher μ)

Energy transport

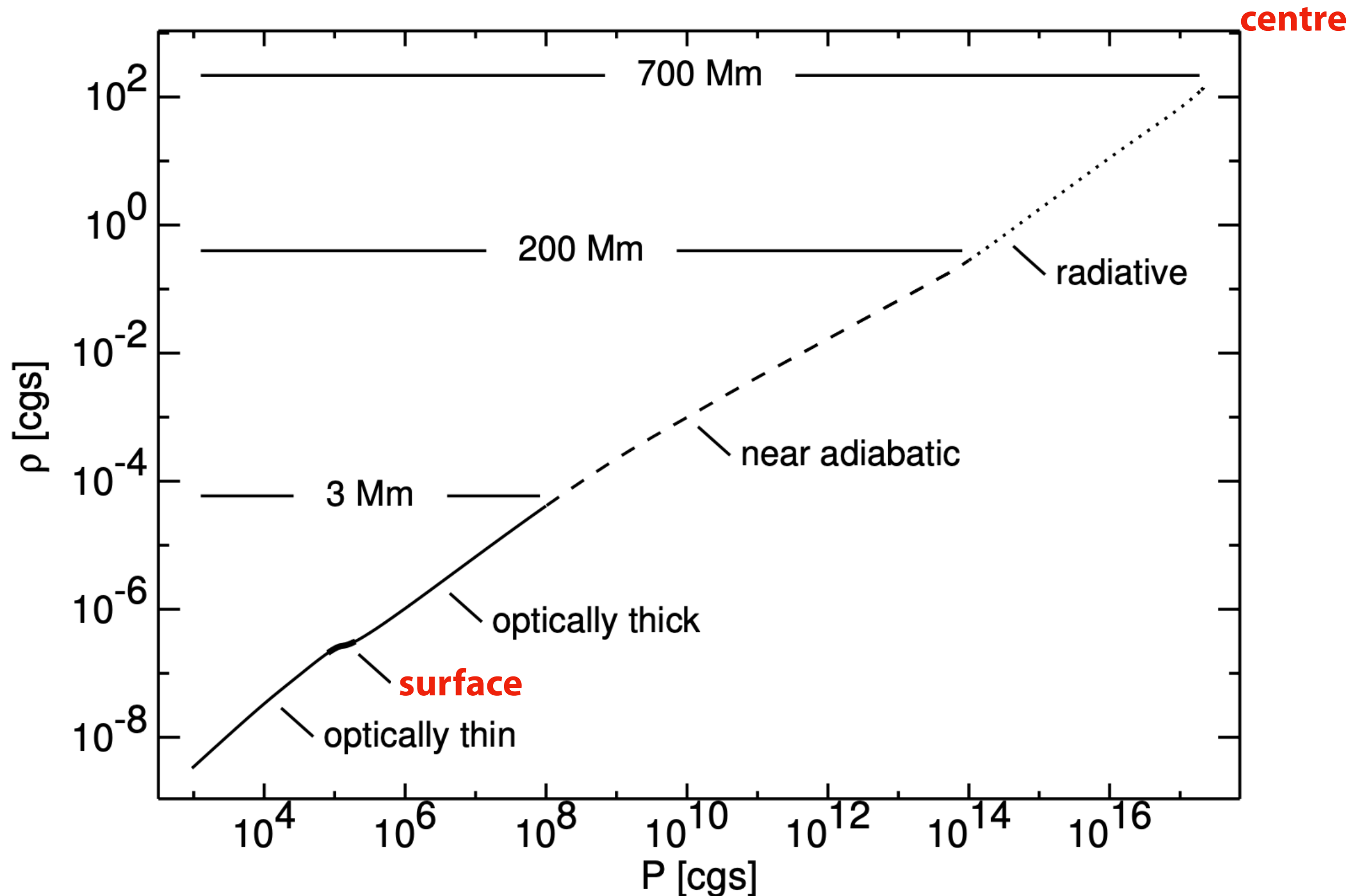
Convection — Stability criterion

- Compare gradient ∇_{rad} for convectively stable stratification with adiabatic temperature gradient $\nabla_{\text{ad}} \equiv \left(\frac{\partial \ln T}{\partial \ln P} \right)$
- **Ledoux criterion** of stability against convection

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} - \frac{\chi_{\mu}}{\chi_T} \nabla_{\mu}$$
- Stratification **convectively stable: Energy transport by radiation**
- Stratification **convectively unstable: Energy transport by convection**

Energy transport

Convection — Stability criterion in the solar interior

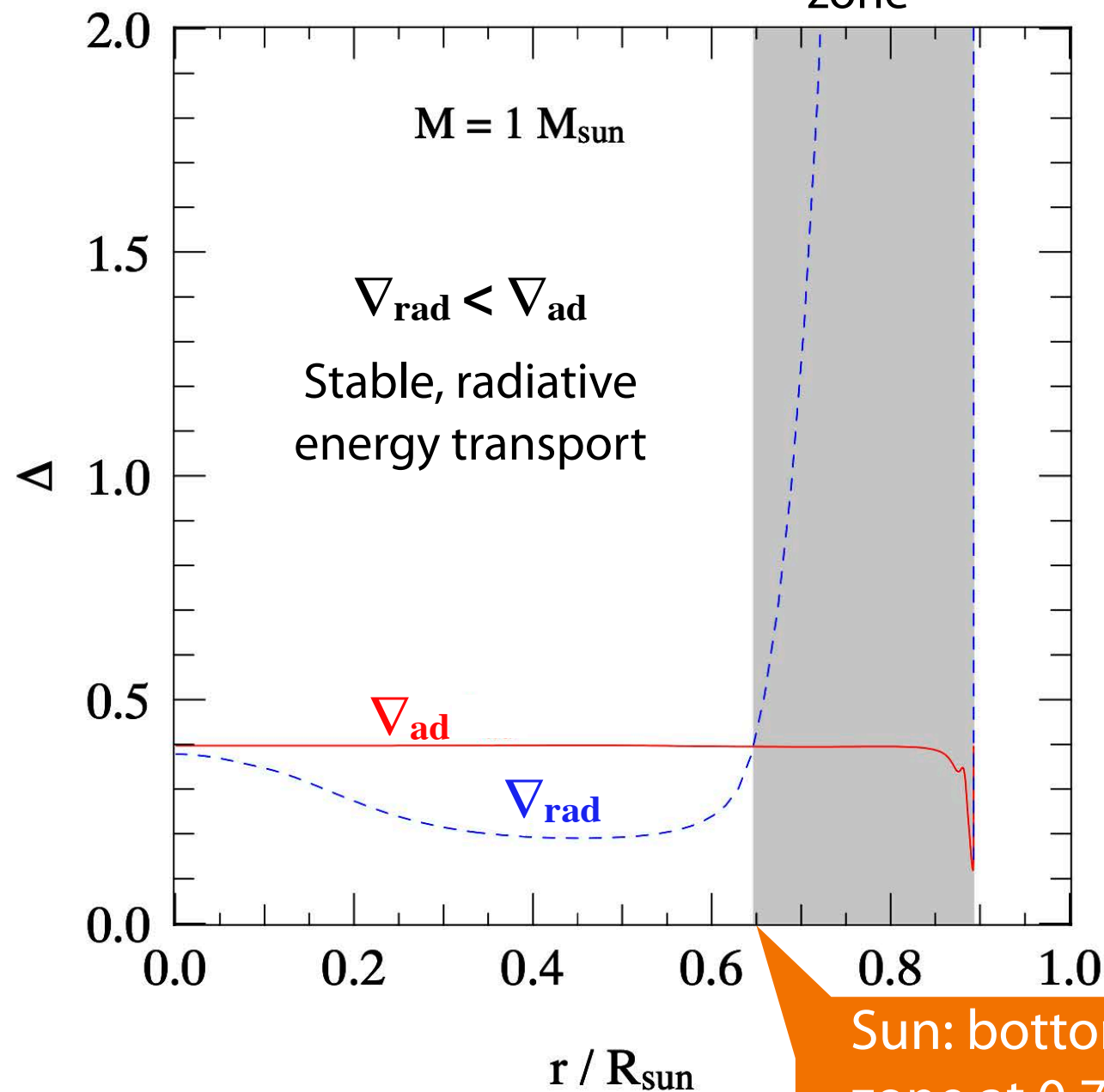


Energy transport

Convection — Stability criterion

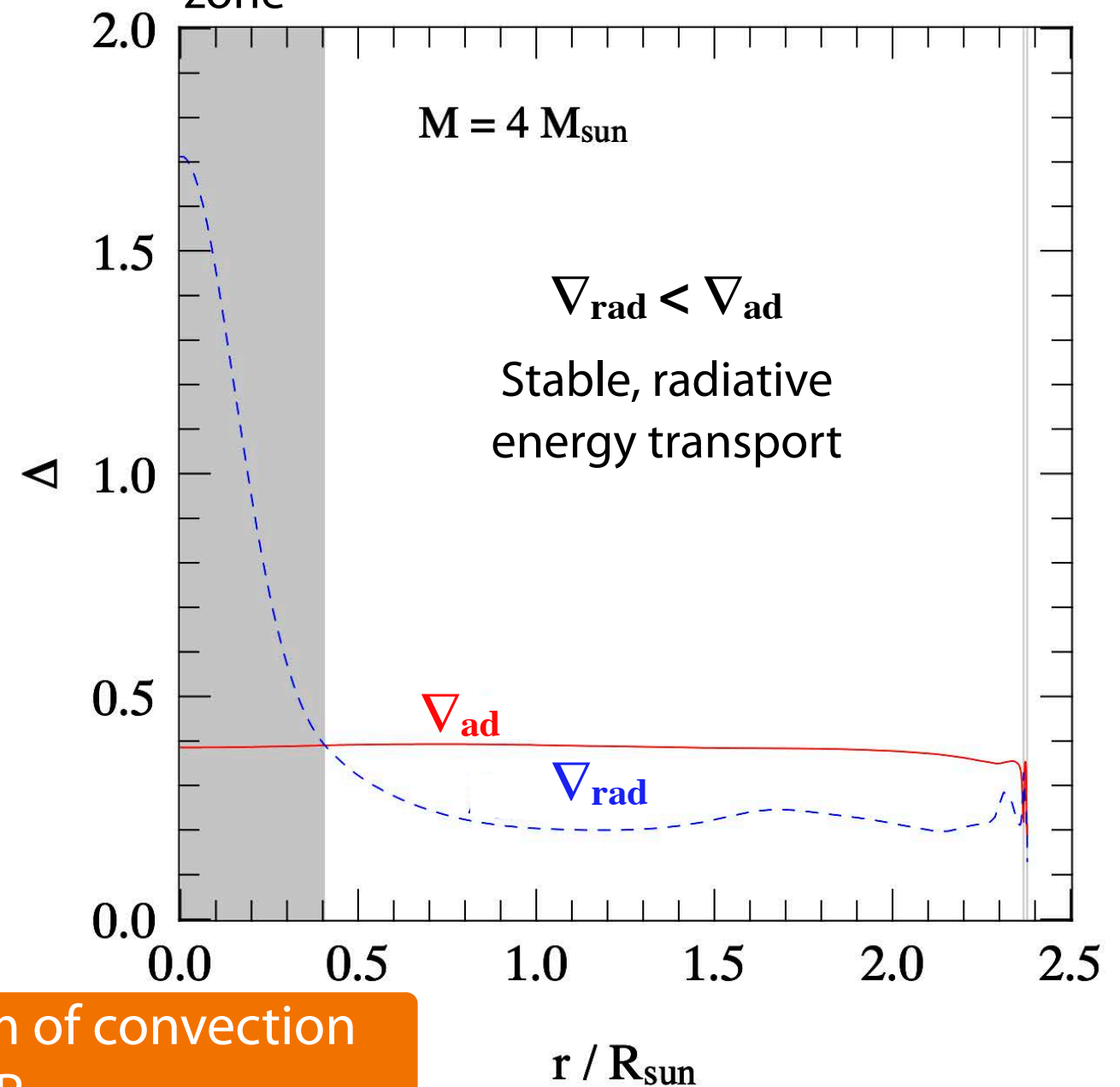
$M = 1 M_{\text{sun}}$

Outer
convection
zone



$M = 4 M_{\text{sun}}$

Inner
convection
zone



Stellar interiors

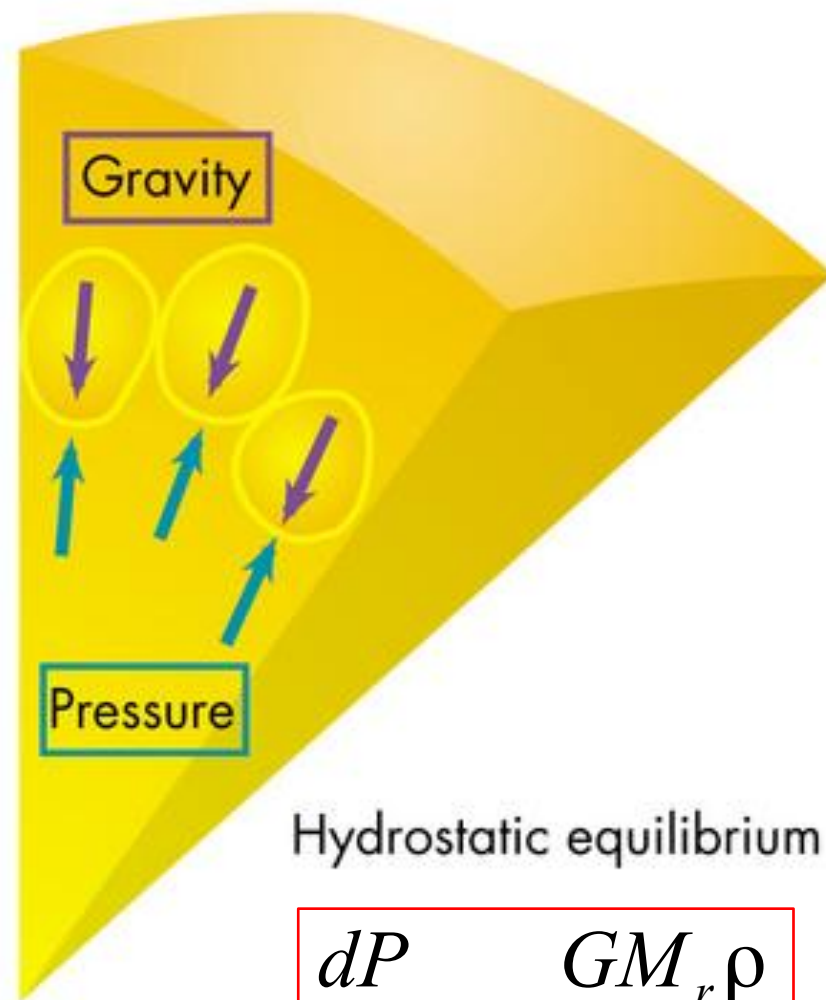


Equations of stellar structure

Stellar interior

Hydrostatic Equilibrium

The outward **pressure force** balances the inward **gravitational force** everywhere inside the Sun.

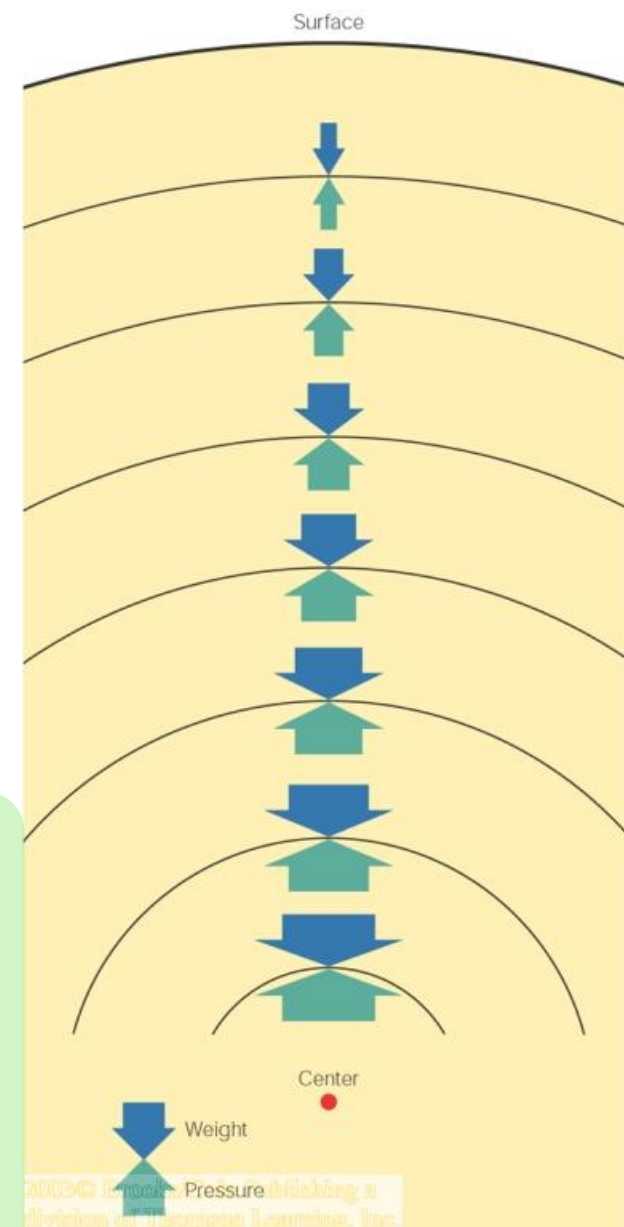


Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$

- Outward pressure of hot gas in the center balances the inward force due to gravity.
 - ➔ At any given radius balancing the weight of all layers above
 - ➔ Imbalance at some radius will result in corresponding adjustment of the stratification
 - ➔ Determines the interior structure (stratification)

- **Main Sequence in the Hertzsprung-Russell diagram is a narrow strip as it requires stability over long enough time**



Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following “ingredients”
- Variables and their stratification (function of radius) P, ρ, T, M_r, L_r
- The following equations :

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2 \rho$
Equation of state	$P = \frac{\rho k T}{\mu m_H}$
Energy “generation”	$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa \rho}{16\pi ac} \frac{L_r}{r^2 T^2}$

A “small” problem:
We cannot observe the interior of stars directly
 but rely mostly on few measurable properties ($M, L, \dots$), assumptions, (tested) physical laws and material properties (often with substantial uncertainties)

- And these boundary conditions:
 - In the centre ($r=0$): $M_r(0) = 0, \quad L_r(0) = 0$
 - At the surface ($r=R$): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\text{eff}}^4$
- Note that the chemical composition affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

Stellar interior

Equations of stellar structure

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Solution

Standard Model

assumes
[X:Y:Z]

=
[0.73:0.25:0.015]

- And these boundary conditions:

- In the centre ($r=0$): $M_r(0) = 0, \quad L_r(0) = 0$
- At the surface ($r=R$): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\text{eff}}^4$

- Note that the **chemical composition** affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1\text{H})$	$X(^2\text{He})$	$X(^4\text{He})$	$X(^7\text{Be})$	$X(^{12}\text{C})$	$X(^{14}\text{N})$	$X(^{16}\text{O})$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
0.00078	0.0197	1.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
0.00135	0.0237	1.55E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
0.00214	0.0277	1.54E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
0.00320	0.0317	1.53E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60758	1.41E-11	2.43E-05	6.14E-03	8.71E-03
0.00456	0.0358	1.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
0.00625	0.0400	1.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
0.00832	0.0442	1.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
0.01080	0.0484	1.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
0.01373	0.0528	1.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
0.01715	0.0572	1.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
0.02109	0.0616	1.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
0.02560	0.0662	1.43E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
0.03071	0.0708	1.42E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
0.03645	0.0756	1.40E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
0.04287	0.0804	1.38E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
0.05000	0.0853	1.37E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
0.05769	0.0902	1.35E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
0.06538	0.0948	1.33E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
0.07308	0.0992	1.31E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
0.08077	0.1033	1.30E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
0.08846	0.1073	1.28E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
0.09615	0.1111	1.27E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.14E-03
0.10385	0.1147	1.25E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
0.11154	0.1182	1.24E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
0.11923	0.1217	1.22E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
0.12692	0.1250	1.21E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
0.13462	0.1283	1.20E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
0.14231	0.1315	1.18E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.35E-05	5.77E-03	9.15E-03
0.15000	0.1346	1.17E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.32E-05	5.77E-03	9.15E-03
0.15600	0.1370	1.16E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	1.24E-05	5.76E-03	9.15E-03
0.16200	0.1393	1.15E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	1.20E-05	5.75E-03	9.15E-03
0.16800	0.1417	1.14E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	1.17E-05	5.73E-03	9.15E-03
0.17400	0.1440	1.13E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	1.15E-05	5.70E-03	9.15E-03
0.18000	0.1462	1.12E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.12E-05	5.65E-03	9.15E-03
0.18600	0.1485	1.12E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.10E-05	5.62E-03	9.15E-03
0.19200	0.1507	1.11E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	1.08E-05	5.60E-03	9.15E-03
0.19800	0.1529	1.10E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	1.07E-05	5.58E-03	9.15E-03
0.20400	0.1551	1.09E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	1.05E-05	5.57E-03	9.15E-03
0.21000	0.1572	1.08E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	1.04E-05	5.55E-03	9.15E-03
0.21600	0.1594	1.08E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	1.03E-05	5.54E-03	9.15E-03
0.22200	0.1615	1.07E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.02E-05	5.53E-03	9.15E-03
0.22800	0.1636	1.06E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.03E-05	5.52E-03	9.15E-03
0.23400	0.1657	1.05E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.00E-05	5.51E-03	9.15E-03
0.24000	0.1678	1.04E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.00E-05	5.50E-03	9.15E-03
0.24600	0.1699	1.04E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.00E-05	5.49E-03	9.15E-03
0.25200	0.1719	1.03E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	1.00E-05	5.48E-03	9.15E-03
0.25800	0.1740	1.02E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	1.00E-05	5.47E-03	9.15E-03
0.26400	0.1760	1.02E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	1.00E-05	5.46E-03	9.15E-03
0.27000	0.1781	1.01E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	1.00E-05	5.45E-03	9.15E-03
0.27600	0.1801	1.00E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	1.00E-05	5.44E-03	9.15E-03
0.28200	0.1821	9.95E+06	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	1.00E-05	5.43E-03	9.15E-03
0.28800	0.1841	9.88E+06	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	1.00E-05	5.42E-03	9.15E-03
0.29400	0.1861	9.81E+06	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	1.00E-05	5.41E-03	9.15E-03
0.30000	0.1881	9.74E+06	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	1.00E-05	5.40E-03	9.15E-03
0.31000	0.1914	9.63E+06	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	5.28E-14	1.00E-05	5.39E-03	9.15E-03
0.32000	0.1948	9.52E+06	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.72E-14	1.00E-05	5.38E-03	9.15E-03
0.33000	0.1981	9.41E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	4.20E-14	1.00E-05	5.37E-03	9.15E-03
0.34000	0.2014	9.31E+06	3.53E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.71E-14	1.00E-05	5.36E-03	9.15E-03
0.35000	0.2047	9.20E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	3.20E-14	1.00E-05	5.35E-03	9.15E-03
0.36000	0.2080	9.10E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.73E-14	1.00E-05	5.34E-03	9.15E-03

Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following “ingredients”
- Variables and their stratification (function of radius) P, ρ, T, M_r, L_r
- Solution: Standard Model (for the Sun)**
assumes chemical composition $[X:Y:Z] = [0.73:0.25:0.015]$

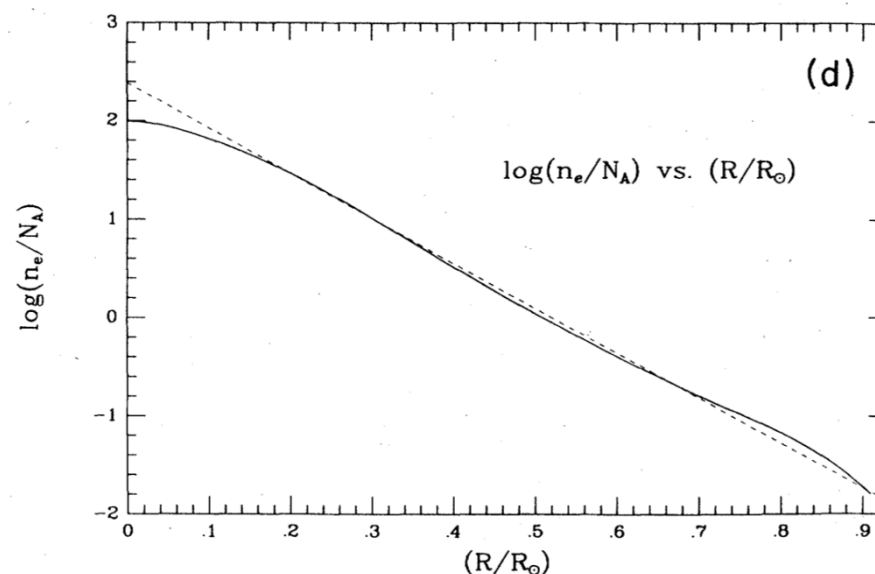
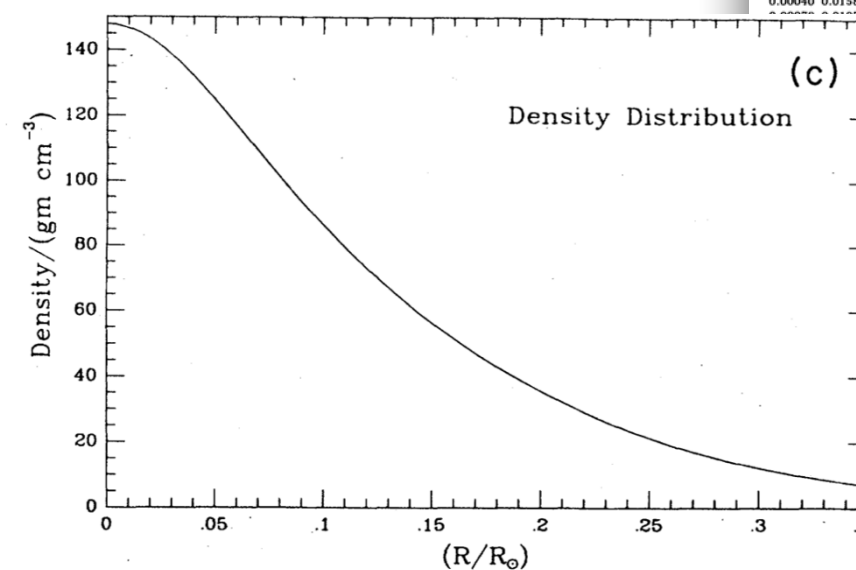
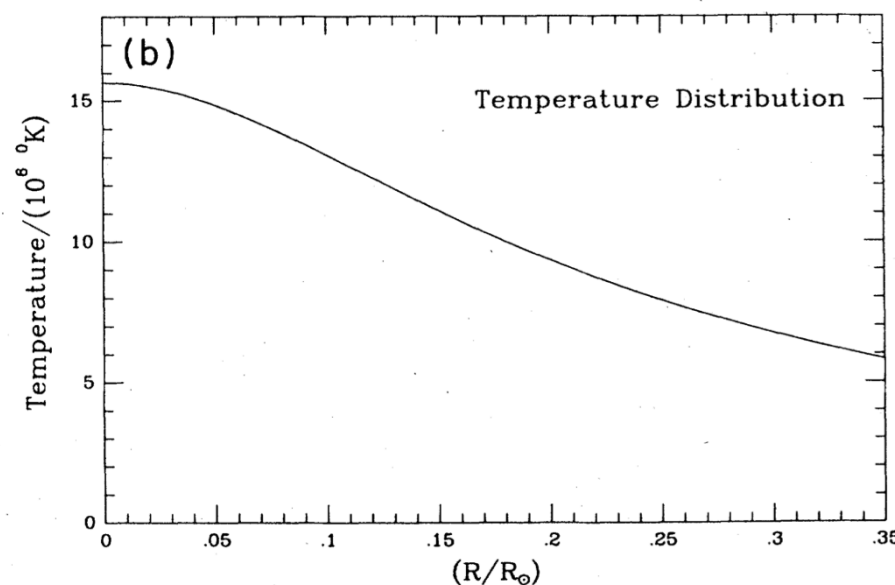
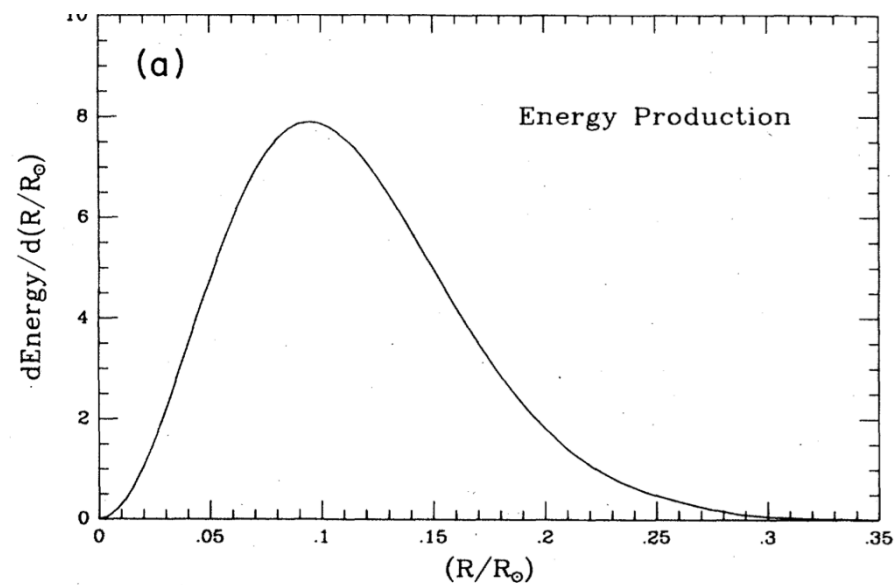


TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1\text{H})$	$X(^2\text{He})$	$X(^4\text{He})$	$X(^7\text{Be})$	$X(^{12}\text{C})$	$X(^{14}\text{N})$	$X(^{16}\text{O})$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
0.00100	0.0200	1.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
0.00200	0.0250	1.54E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
0.00400	0.0312	1.53E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
0.00800	0.0375	1.52E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60585	1.40E-11	2.42E-05	6.13E-03	8.71E-03
0.01600	0.0438	1.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
0.03200	0.0500	1.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
0.06400	0.0562	1.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
0.12800	0.0625	1.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
0.25600	0.0688	1.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
0.51200	0.0750	1.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
1.02400	0.0812	1.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
2.04800	0.0875	1.44E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
4.09600	0.0938	1.43E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
8.19200	0.1000	1.42E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
16.38400	0.1062	1.41E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
32.76800	0.1125	1.40E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
65.53600	0.1188	1.39E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
131.07200	0.1250	1.38E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
262.14400	0.1312	1.37E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
524.28800	0.1375	1.36E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
1048.57600	0.1438	1.35E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
2097.15200	0.1500	1.34E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.15E-03
4194.30400	0.1562	1.33E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
8388.60800	0.1625	1.32E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
16777.21600	0.1688	1.31E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
33554.43200	0.1750	1.30E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
67108.86400	0.1812	1.29E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
134217.72800	0.1875	1.28E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.45E-05	5.77E-03	9.15E-03
268435.45600	0.1938	1.27E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.68E-05	5.77E-03	9.15E-03
536870.91200	0.2000	1.26E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	1.64E-05	5.76E-03	9.15E-03
1073741.82400	0.2062	1.25E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	1.60E-05	5.75E-03	9.15E-03
2147483.64800	0.2125	1.24E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	1.47E-05	5.73E-03	9.15E-03
4294967.29600	0.2188	1.23E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	1.37E-05	5.70E-03	9.15E-03
8589934.59200	0.2250	1.22E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.29E-05	5.65E-03	9.15E-03
17179869.18400	0.2312	1.21E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.24E-05	5.67E-03	9.15E-03
34359738.36800	0.2375	1.20E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	1.20E-05	5.67E-03	9.15E-03
68719476.73600	0.2438	1.19E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	1.17E-05	5.64E-03	9.15E-03
137438953.47200	0.2500	1.18E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	1.14E-05	5.61E-03	9.15E-03
274877906.94400	0.2562	1.17E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	1.11E-05	5.58E-03	9.15E-03
549755813.88800	0.2625	1.16E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	1.08E-05	5.55E-03	9.15E-03
1099511627.77600	0.2688	1.15E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.05E-05	5.52E-03	9.15E-03
2199023255.55200	0.2750	1.14E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.03E-05	5.49E-03	9.15E-03
4398046511.10400	0.2812	1.13E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.01E-05	5.46E-03	9.15E-03
8796093022.20800	0.2875	1.12E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.00E-05	5.43E-03	9.15E-03
17592186044.41600	0.2938	1.11E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.00E-05	5.40E-03	9.15E-03
35184372088.83200	0.3000	1.10E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	1.00E-05	5.37E-03	9.15E-03
70368744177.66400	0.3062	1.09E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	1.00E-05	5.34E-03	9.15E-03
140737488355.32800	0.3125	1.08E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	1.00E-05	5.31E-03	9.15E-03
281474976710.65600	0.3188	1.07E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	1.00E-05	5.28E-03	9.15E-03
562949953421.31200	0.3250	1.06E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	1.00E-05	5.25E-03	9.15E-03
1125899906842.62400	0.3312	1.05E+07	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	1.00E-05	5.22E-03	9.15E-03
2251799813685.24800	0.3375	1.04E+07	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	1.00E-05	5.19E-03	9.15E-03
4503599627370.49600	0.3438	1.03E+07	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	1.00E-05	5.16E-03	9.15E-03
9007199254740.99200	0.3500	1.02E+07	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	1.00E-05	5.13E-03	9.15E-03
18014398509481.98400	0.3562	1.01E+07	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	5.28E-14	1.00E-05	5.10E-03	9.15E-03
36028797018963.96800	0.3625	1.00E+07	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.72E-14	1.00E-05	5.07E-03	9.15E-03
72057594037927.93600	0.3688	9.99E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	4.27E-14	1.00E-05	5.04E-03	9.15E-03
144115188075855.87200	0.3750	9.98E+06	3.51E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.81E-14	1.00E-05	5.01E-03	9.15E-03
288230376151711.74400	0.3812	9.97E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	3.39E-14	1.00E-05		
576460752303423.48800	0.3875	9.96E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.73E-14	1.00E-05	1.16E-03	9.15E-03

Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following “ingredients”
- Variables and their stratification (function of radius) P, ρ, T, M_r, L_r
- The following equations :

Hydrodynamic equilibrium	$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} - \rho \frac{d^2r}{dt^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2 \rho$
Equation of state	$P = \frac{\rho kT}{\mu m_H}$
Energy “generation”	$\frac{dL_r}{dr} = 4\pi r^2 \rho \left[\epsilon - T \frac{dS}{dt} \right]$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\pi ac} \frac{L_r}{r^2 T^2}$

More general, **time-dependent form**

Stellar structure is time-dependent and will adjust

→ S: entropy

→ $T (dS/dt)$ is the energy of collapse expressed in terms of the entropy change

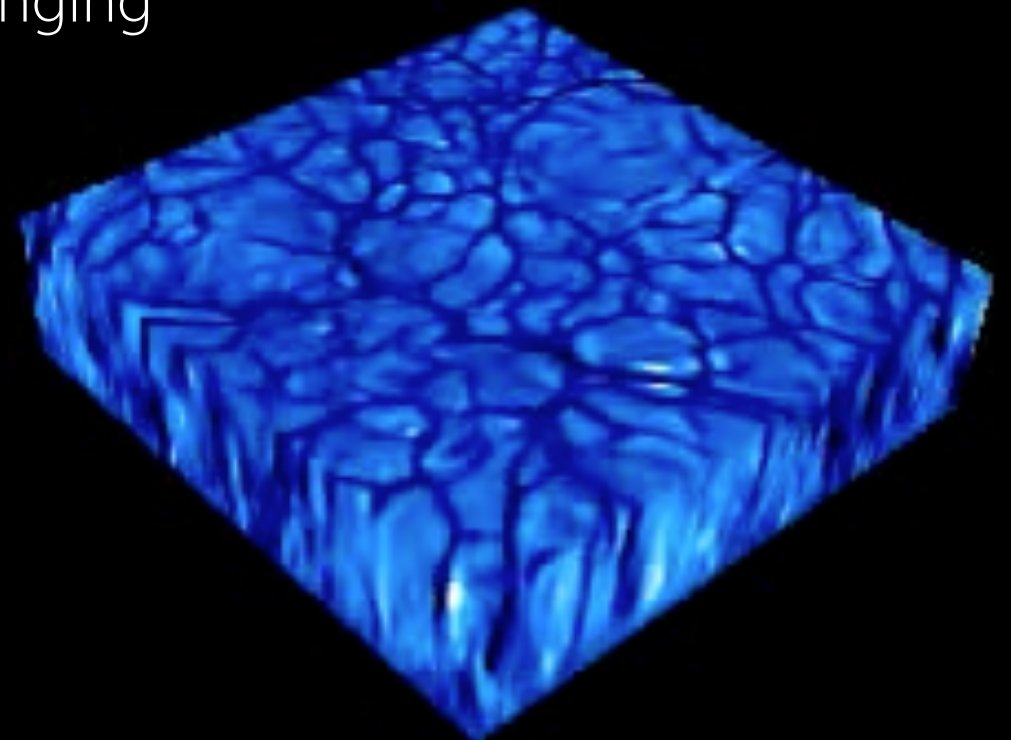
- And these boundary conditions:
 - In the centre ($r=0$): $M_r(0) = 0, \quad L_r(0) = 0$
 - At the surface ($r=R$): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\text{eff}}^4$
- Note that the mean molecular weight μ and the opacity κ are now functions of radius and time (due to fusion in the core)

Energy transport — Convection

Energy transport

Mixing length theory

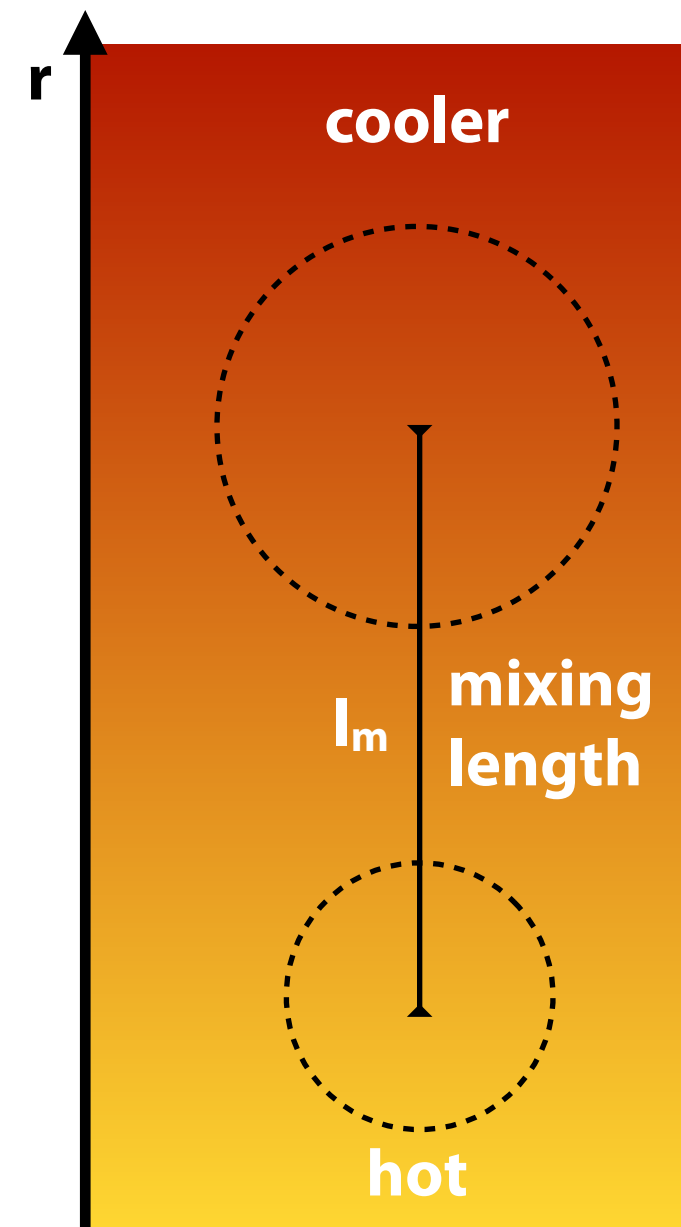
- In the context of stellar (interior) structure and evolution, we want to know about convection zones:
 - **How much energy** can be transported by convection?
 - What is the **temperature gradient** ?
- But: Detailed theory of convection and its practical application still challenging today
- Addressed with numerical simulations but very challenging and computationally expensive
 - ➡ Prohibitive to use as part of stellar evolution calculations.
 - ➡ Simpler approach needed
 - ➡ **Mixing Length Theory (MLT)**



Energy transport

Mixing length theory

- Simplified picture of convective energy transport:
 1. Gas element rises (or sinks) over a radial distance
 2. Gas element dissolves (becomes part of the new environment) and releases excess heat
- Mixing length l_m
- If gas element sinks, it absorbs deficit energy from environment
- Mixing length l_m is an unknown free parameter!
- Assumption: l_m on the order of local pressure scale height H_P



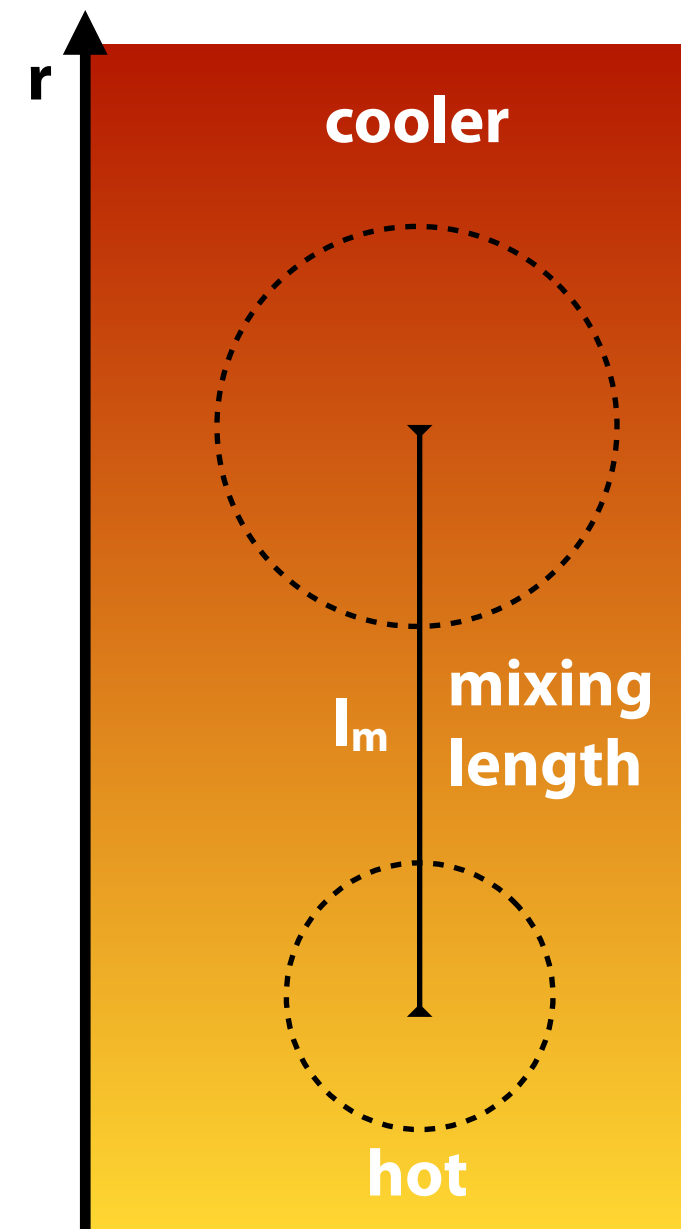
- **Pressure scale height:** (in a stratified medium) the radial distance over which the pressure changes by a factor $1/e$

$$H_P = \left| \frac{dr}{d \ln P} \right| = \frac{P}{\rho g}$$

Energy transport

Mixing length theory

- Assumption: $l_m = \alpha H_P$
 - Valid in hydrostatic equilibrium.
 - Reasonable as gas element expands while rising
-
- Assume spherical surface inside the convection zone
 - 1/2 covered by rising blobs
 - 1/2 covered by sinking blobs
 - ➡ Expanding rising blobs would cover most of the area after rising 1-2 pressure scale heights.
 - ➡ **Net energy transport upwards** (down the gradient)



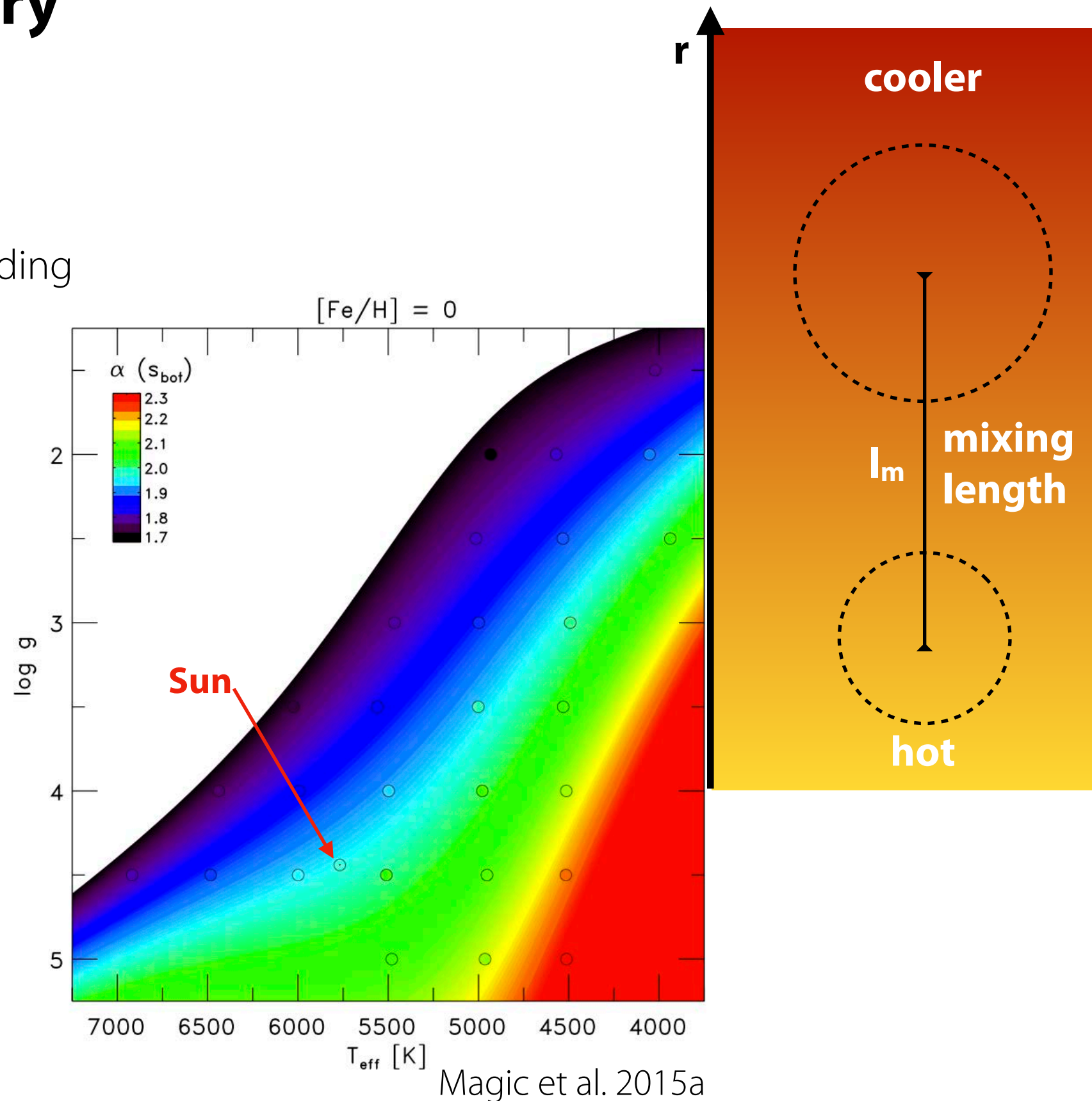
- Pressure scale height:** (in a stratified medium)
the radial distance over which the pressure changes by a factor 1/e

$$P(r) = P_0 e^{-(r/H_P)} \implies H_P = \left| \frac{dr}{d \ln P} \right| = \frac{P}{\rho g}$$

Energy transport

Mixing length theory

- Detailed numerical model calculations
(in comparison to observation)
to derive/calibrate corresponding
mixing length
- Mixing length (via parameter
depends on
 - Effective temperature T_{eff}
 - Grav. Acceleration $\log g$
 - Metallicity $[Fe/M]$
- Values for α typically ~ 2



Energy transport

The convective energy flux

- Gas element after rising a distance $\delta r = l_m$:
 ➡ Mean excess temperature ΔT between element and surrounding:

$$\Delta T = T_e - T_s = \left[\left(\frac{dT}{dr} \right)_e - \frac{dT}{dr} \right] \ell_m = \Delta \left(\frac{dT}{dr} \right) \ell_m$$

- dT/dr : temperature gradient in surrounding
 - $(dT/dr)_e$: variation of temperature with radius r for the gas element while rising and expanding adiabatically
 - $\Delta(dT/dr)$: difference between the two gradients.
- Rewrite the equation above with gradients $\nabla \equiv \left(\frac{\partial \ln T}{\partial \ln P} \right)$ and $H_P = \left| \frac{dr}{d \ln P} \right| = \frac{P}{\rho g}$, deriving and using the following equation

$$\frac{dT}{dr} = T \frac{d \ln T}{dr} = T \frac{d \ln T}{d \ln P} \frac{d \ln P}{dr} = -\frac{T}{H_P} \nabla \quad \text{and} \quad \left(\frac{dT}{dr} \right)_e = -\frac{T}{H_P} \nabla_{\text{ad}}$$

➡

$$\Delta T = T \frac{\ell_m}{H_P} (\nabla - \nabla_{\text{ad}})$$

