Exercises for AST1100, Introduction and Microscopic Physics

1. For some point in space P, show that, for any arbitrary closed surface surrounding P, the intgral over a solid angle about P gives

$$\Omega_{\rm tot} = \oint d\Omega = 4\pi.$$

2. (a) Using the Rayleigh criterion; the resolution θ of a circular lens diameter D is given by $\theta = 1.22(\lambda/D)$ for light of wavelength λ, estimate the angular resolution limit of the human eye at 5500 Å (550 nm). Assume that the diameter of the pupil is 5 mm.

   (b) Compare your answer to the angular diameters of the Moon and Jupiter. Data for the Moon and Jupiter may surely be found on `www.google.com`.

   (c) What can you conclude about the ability to resolve the Moon's disk and Jupiter's disk with the unaided eye?

3. (a) Using the Rayleigh criterion, estimate the theoretical diffractioni limit for the angular resolution of a typical 8-inch amateur telescope at 5500 Å. Express your answer in arc seconds.

   (b) Estimate the minimum size of a crater on the Moon that can be resolved by an 8-inch telescope. Express your answer in km.

   (c) Is this resolution limit likely to be acheived? Why or why not?

4. Prove that the total energy conservation

$$E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + (-\frac{Gm_1m_2}{r}) = \text{constant}$$

   for two isolated point masses is a consequence of Newton's second law of motion and Newton's law of gravitation.

   Hint: Take the dot product of the equation of motion for particle 1 with

$$\mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt} \tag{1}$$

   and add the result to the equivalent equation for particle 2.

5. Calculate the ratio of the electric force to the gravitational force between two protons.

6. Formulate Gauss' law for the electrostatic force law.

7. Combining de Broglies expression for the wavelength of an electron with the assumption that only an integral number of these wavelengths can orbits around protons give an expression for the possible sizes of the hydrogen atom.

   Hint: Use Newton's second law for the motion of an electron in a circular orbit around a proton to relate v and r.

8. Show that the energies corresponding to the orbits computed above are given by the follwing expression.

$$E_n = -\frac{1}{n^2}\left[\frac{m_e e^4}{8\epsilon_0^2 h^2}\right].$$

9. Show the the wavelength λ for emission/absorption between two energy levels in the hydrogen atom may be written as

$$\lambda = (\frac{2hc\epsilon_0}{e^2})(\frac{4\epsilon_0^2 h^2 n^2}{m_e e^2})(\frac{n'^2}{n'^2 - n^2}). \tag{2}$$

What is the wavelength λ_{32} of the transition $n = 3$ to $n = 2$? What is the wavelength λ_{21} of the transition $n = 2$ to $n = 1$? Is either of these visible to the nake eye? And if so, which colour should we expect to see from the transition?

10. A white dwarf is a very dense star, with its ions and electrons packed extermely close together. Each electron may be considered to be located within a region of the size $\Delta x \approx 1.5 \times 10^{-12}$ m. Use Heisenberg's uncertainty priciple to estimate the minimum speed of the electron. Do you think the effects of relativity will be important for these stars?

11. Each quantum state of the hydrogen atom is labeled by a set of four quantum numbers $[n, l, m_l, m_s]$.

    (a) List the sets of quantum numbers for the hydrogen atom having $n = 1, n = 2$, and $n = 3$.

    (b) Show that the degeneracy of level n is $2n^2$.