Exercises for AST1100, Galaxies, Clusters of Galaxies, and Hubble’s Law

1. Using our rough estimate for the density of dark matter in the Milky Way (explaining the constant velocity $V$ of the disk's rotation)

$$\rho(r) = \frac{V^2}{4\pi Gr^2},$$

estimate the mass density of dark matter in the solar neighborhood. Express your answer in kg/m$^3$, M$_S$/pc$^3$, and M$_S$/pc$^3$. How does this answer compare with the mass stellar mass density in the solar neighborhood?

2. The Sun is currently located 30 pc north of the Galactic midplane and is moving away from it with a velocity $w_S = 7$ km/s. The $z$ component of the gravitational acceleration vector is directed toward the midplane so the Sun's peculiar velocity must be decreasing. Eventually the direction of motion will reverse and the Sun will pass through the midplane heading in the opposite direction. At that time the direction of the $z$ component of the gravitational acceleration will also reverse, ultimately causing the Sun to move northward again. This oscillatory behaviour above and below the midplane has a well defined period and amplitude that we will estimate in this exercise.

Assume that the disk of the Milky Way has a radius that is much larger than its thickness. In this case, as long as we confine ourselves regions near the midplane, the disk appears infinite in the $z = 0$ plane. Consequently the gravitational acceleration is always oriented in the $\pm z$-direction. We will neglect the radial acceleration component in this problem.

(a) By constructing an appropriate surface and employing Gauss’ formulation of Newton’s law of gravity

$$\oint g \cdot dA = 4\pi GM_{in},$$

derive an expression for the gravitational acceleration vector at a height $z$ above the midplane, assuming that the Sun always remains inside the disk of constant density $\rho$ (i.e. the Sun does not move far compared to the disk’s scale height $h_z$ in the $z$ direction).

(b) Using Newton’s second law, show that the motion of the Sun in the $z$-direction can be described by a differential equation of the form

$$\frac{d^2z}{dt^2} + kz = 0.$$

Express $k$ in terms of $\rho$ and $G$. This is just the familiar equation for simple harmonic motion.
(c) Find general expressions for $z$ and $w$ as functions of time.

(d) If the total mass density in the solar neighborhood (including stars, gas, dust, and dark matter) is $0.15 \, M_S/pc^3$, estimate the oscillation period.

(e) By combining the current determinations of $z_S$ and $w_S$, estimate the amplitude of the solar oscillation and compare your answer with the vertical scale height of the thin disk.

(f) Approximately how many vertical oscillations does the Sun execute during one orbital period around the Galactic center?

3. Using Shapley’s assumption that M101 has a diameter of 100 kpc, and adopting van Maanen’s flawed observation that M101 rotates with an angular rotation rate of 0.02 arcsec/yr, estimate the speed of a point at the edge of the galaxy and compare it to the characteristic rotation speed of the Milky Way.

4. (a) The mass density of stars in the neighborhood of the Sun is approximately $0.05 \, M_S/pc^3$. Assuming that the mass density is constant and that all of the stars are main sequence M stars, estimate the fraction of the Galactic disk’s volume that is occupied by stars.

(b) Suppose that an intruder star (a main sequence M star) travels perpendicularly through the Galactic disk. What are the odds of the intruder colliding with another star during its passage through the disk? Take the thickness of the disk to be approximately 1 kpc.

5. Estimate how long a Galaxy in the Coma cluster would take to travel from one side of the cluster to the other. Assume that the galaxy moves with a constant speed equal to the cluster’s radial velocity dispersion $\sigma_r = 977$ km/s. The Coma cluster has a diameter roughly equal to the diameter of the Virgo cluster. How does this time compare to the Hubble time $t_H$? What can you conclude about whether the galaxies in the Coma cluster are gravitationally bound?

6. Use Newtonian physics to calculate the values of the average density and “surface gravity” for a $10^8 M_S$ black hole. Compare these values with those for the Sun.

7. Hubble’s constant was measured to be roughly 500 km/s/Mpc by Hubble himself (see figure 25.6 in Modern Astrophysics if you have a copy of the book handy). Modern values range between 40 and 100 km/s/Mpc, though a value of close to 70 km/s/Mpc has recently seemed to be gaining favour. Estimate roughly the age of the Universe using these values.