Konstanter og uttrykk som kan være nyttige:

Lyshastigheten: $c = 2.98 \times 10^8$ m/s
Plancks konstant: $h = 6.626 \times 10^{-34}$ J s
Gravitasjonskonstanten: $G = 6.673 \times 10^{-11}$ N m^2/kg^2
Boltzmanns konstant: $k = 1.38 \times 10^{-23}$ J/K
Stefan Boltzmann konstant: $\sigma = 5.670 \times 10^{-8}$W/m^{2}K^4.
Elektronets hvilemasse: $m_{\mathrm{e}} = 9.1 \times 10^{-31}$ kg
Protonets hvilemasse: $m_{\mathrm{p}} = 1.6726 \times 10^{-27}$ kg
Nøytronets hvilemasse: $m_{\mathrm{n}} = 1.6749 \times 10^{-27}$ kg
Wiens forskyvnigslov: $\lambda_{\mathrm{max}} T = 0.0029$ m K
1 eV (elektronvolt) = 1.60×10^{-19} J
Solmassen: $M_\odot = 2 \times 10^{30}$ kg
Solradien: $R_\odot = 6.98 \times 10^8$ m.
Solas tilsynelatende magnitude: $m = -26.7$
Solas luminositet: $L_\odot = 3.827 \times 10^{26}$W
Massen til Jupiter: 1.9×10^{27} kg
Temperaturen på solens overflate: 5780 K
Astronomisk enhet: 1AU = 1.5×10^{11} m
Hubblekonstanten: $H_0 = 71$ km/s/Mpc
lysår: 1 ly = 9.47×10^{15} m
parsec: 1 pc = 206 265 AU = 3.27 ly

Formler vi har brukt/utledet i kurset:

$$P^2 = a^3$$

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$$\ddot{\vec{r}} + m\frac{\vec{r}}{r^3} = 0$$

$$r = \frac{p}{1 + e\cos f}$$

$$p = h^2/m$$

$$p = a(1 - e^2) \quad \text{(ellipse)}$$

$$p = a(e^2 - 1) \quad \text{(hyperbel)}$$

$$p = 1/2a \quad \text{(parabel)}$$

$$\sum_{i=1}^{N} m_i \vec{r}_i = M\vec{R}$$

$$m_p \sin i = \frac{m_*^{2/3} v_{*r} P^{1/3}}{(2\pi G)^{1/3}}$$

$$< K >= -\frac{1}{2} < U >$$

$$U = -\frac{3GM^2}{5R}$$

$$B(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(kT)} - 1}$$

$$L = \frac{dE}{dt}$$

$$F = \frac{dE}{dAdt}$$

$$F = \sigma T^4$$

$$n(v)dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/(2kT)} 4\pi v^2 dv$$

$$\Delta\lambda_{FWHM} = \frac{2\lambda_0}{c}\sqrt{\frac{2kT\ln 2}{m}}$$

$$m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$$

$$m - M = 5\log_{10}\left(\frac{d}{10\text{pc}}\right)$$

$$U - B = M_U - M_B = m_U - m_B$$

$$B - V = M_B - M_V = m_B - m_V$$

$$M_V = -2.81\log_{10} P_d - 1.43$$

$$M_V = -3.53 \log_{10} P_d - 2.13 + 2.13(B - V)$$

$$v = H_0 r$$

$$\tau(\lambda) = \int_0^r dr' n(r') \sigma(\lambda, r')$$

$$m(\lambda) = M(\lambda) + 5 \log_{10} \left(\frac{d}{10\text{pc}} \right) + 1.086 \tau(\lambda)$$

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

$$\frac{\Delta\lambda}{\lambda} = \left(\sqrt{\frac{1+v}{1-v}} - 1 \right)$$

$$c_{\mu\nu} = \begin{pmatrix} \gamma_{\text{rel}} & -v_{\text{rel}}\gamma_{\text{rel}} & 0 & 0 \\ -v_{\text{rel}}\gamma_{\text{rel}} & \gamma_{\text{rel}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$< E_K >= \frac{3}{2} kT$$

$$N = \frac{M}{\mu m_H}$$

$$M_J = \left(\frac{5kT}{G\mu m_H} \right)^{3/2} \left(\frac{3}{4\pi\rho} \right)^{1/2} .$$

$$\rho(r) \frac{d^2 r}{dt^2} = -\rho(r) g(r) - \frac{dP(r)}{dr}$$

$$P = \frac{\rho k T}{\mu m_H}$$

$$P_r = \frac{1}{3} a T^4$$

$$\rho_r = a T^4$$

$$\Delta s^2 = \left(1 - \frac{2M}{r} \right) \Delta t^2 - \frac{\Delta r^2}{1 - \frac{2M}{r}} - r^2 \Delta\phi^2$$

$$\frac{M_{\text{m}}}{M_{\text{kg}}} = \frac{G}{c^2}$$

$$\Delta t_{\text{shell}} = \sqrt{1 - \frac{2M}{r}}\Delta t$$

$$\Delta r_{\text{shell}} = \frac{\Delta r}{\sqrt{1 - \frac{2M}{r}}}$$

$$\frac{E}{m} = \left(1 - \frac{2M}{r}\right)\frac{dt}{d\tau}$$

$$\frac{L}{m} = r^2\frac{d\phi}{d\tau}$$

$$\Delta t = \frac{E/m}{\left(1 - \frac{2M}{r}\right)}\Delta\tau$$

$$\Delta\phi = \frac{L/m}{r^2}\Delta\tau$$

$$\Delta r = \pm\sqrt{\left(\frac{E}{m}\right)^2 - \left[1 + \left(\frac{L/m}{r}\right)^2\right]\left(1 - \frac{2M}{r}\right)}\Delta\tau$$

$$\frac{V_{\text{eff}}(r)}{m} = \frac{1}{2}\frac{(L/m)^2}{r^2} - \frac{M}{r}$$

$$\frac{V_{\text{eff}}(r)}{m} = \sqrt{\left(1 - \frac{2M}{r}\right)\left[1 + \frac{(L/m)^2}{r^2}\right]}$$

$$\Delta r = \pm\left(1 - \frac{2M}{r}\right)\sqrt{1 - \left(1 - \frac{2M}{r}\right)\frac{(L/E)^2}{r^2}}\Delta t$$

$$r\Delta\phi = \pm\frac{L/E}{r}\left(1 - \frac{2M}{r}\right)\Delta t$$

$$b = \frac{L}{p}$$

$$V_{\text{eff}} = \frac{1}{r}\sqrt{1 - \frac{2M}{r}}$$

$$b_{\text{crit}} = 3\sqrt{3}M$$

$$\Delta\phi = \frac{4M}{R}$$

$$\theta_E = \sqrt{\frac{4M(d_s - d_L)}{d_L d_s}}$$

$$U = -\frac{1}{4\pi\epsilon_0}\frac{Z_A Z_B e^2}{r}$$

$$r_{AB} = \left(\frac{2}{kT}\right)^{3/2} \frac{n_A n_B}{\sqrt{\mu\pi}} \int_0^E dE e^{-E/kt}\sigma(E)$$

$$r_{AB} \propto X_A X_B \rho^{\alpha'} T^{\beta}$$

$$\varepsilon_{AB} = \varepsilon_0 X_A X_B \rho^{\alpha} T^{\beta}$$

$$\varepsilon_{pp} \approx \varepsilon_{0,pp} X_H^2 \rho T_6^4$$

$$\varepsilon_{0,pp} = 1.08 \times 10^{-12} \mathrm{Wm}^3/\mathrm{kg}^2$$

$$\varepsilon_{CNO} = \varepsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20}$$

$$\varepsilon_{0,CNO} = 8.24 \times 10^{-31} \mathrm{Wm}^3/\mathrm{kg}^2$$

$$\varepsilon_{3\alpha} = \varepsilon_{0,3\alpha} \rho^2 X_{He}^3 T_8^{41}$$

$$\varepsilon_{0,3\alpha} = 3.86 \times 10^{-18} \mathrm{Wm}^3/\mathrm{kg}^2$$

$$L \propto M^4$$

$$t \propto 1/M^3$$

$$M \propto T_{\mathrm{eff}}^2$$

$$P = \frac{1}{3}\int_0^{\infty} p\, v\, n(p)\, dp$$

$$n(\vec{p}) = n\left(\frac{1}{2\pi mkT}\right)^{3/2} e^{-p^2/(2mkT)}$$

$$n(E) = \frac{g(E)}{e^{(E-E_F)/(kT)} + 1}$$

$$n(\vec{p}) = \frac{1}{e^{(p^2-p_F^2)/(2mkT)} + 1}\frac{2}{h^3}$$

$$E_F = \frac{h^2}{8m_e}\left(\frac{3n_e}{\pi}\right)^{2/3}$$

$$P = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_e} n_e^{5/3}$$

$$P = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} n_e^{4/3}$$

$$< E_K >= \frac{3}{5} E_F$$

$$R_{\mathrm{WD}} \approx \left(\frac{3}{2\pi}\right)^{4/3} \frac{h^2}{20 m_e G}\left(\frac{Z}{A m_H}\right)^{5/3} M^{-1/3}$$

$$M_{\mathrm{Ch}} \approx \frac{\sqrt{3/2}}{2\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{Z}{A m_H}\right)^2 \approx 1.4 M_\odot$$

$$\Delta s^2 = \Delta t^2 - R^2(t)\left[\frac{\Delta r^2}{1 - kr^2} + r^2 \Delta\theta^2 + r^2 \sin^2\theta \Delta\phi^2\right]$$

$$H(t) = \frac{1}{R(t)}\frac{dR(t)}{dt}$$

$$z = \frac{R_0}{R(t)} - 1$$

$$\dot{R}^2(t) - \frac{8}{3}\pi G \rho(t) R^2(t) - \frac{\Lambda}{3} R(t)^2 = -k$$

$$\ddot{R}(t) = -\frac{4}{3}\pi G(\rho(t) + 3P(t))R(t) + \frac{\Lambda}{3} R(t)$$

$$\rho_C(t) = \frac{3H^2(t)}{8\pi G}$$

$$\Omega(t) = \frac{\rho(t)}{\rho_C(t)}$$

$$\frac{d}{dt}(\rho R^{3(1+w)}) = 0$$

$$\rho(t) = \rho_0\left(\frac{R_0}{R(t)}\right)^{3(1+w)}$$

$$R(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$q(t) = -\frac{1}{R(t)H^2(t)}\frac{d^2R(t)}{dt^2}$$

$$q(t) = \frac{1}{2}\Omega(t)$$

$$F = \frac{L}{4\pi d_L^2}$$

$$d_L = r(1+z)R_0$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}$$

$$P_\Lambda = -\frac{\Lambda}{8\pi G}$$

$$\frac{n_n}{n_p} = e^{(m_n - m_p)/kT}$$

$$\frac{n(t_1)}{n(t_2)} = e^{-\ln 2 (t_1 - t_2)/\tau}$$

$$d_L = \frac{1}{H_0 q_0^2}[q_0 z + (q_0 - 1)(\sqrt{1 + 2zq_0} - 1)]$$