The following people have participated in creating these solutions: Nicolaas E. Groeneboom, Magnus Pedersen Lohne, Karl R. Leikanger *NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know*

Partial solutions to problems: Lecture 15-16

Problem 1

1. A similar calculation is made in the text.
2. Use that $\lambda = 1/\nu = 1/\Delta t$ and use the result from the previous question.
3. Taylor expand
$$\frac{1}{1-x} \approx 1 + \frac{1}{2}x$$
4. (a) $M_\odot = 1482\text{m}$
   (b) $M/r = 2.1 \times 10^{-6}$
   (c) $\Delta\lambda/\lambda \approx M/r = 2.1 \times 10^{-6}$
   (d) No, too small change.
   (e) Here we need to reverse the argument when deducing the gravitational Doppler shift. We obtain $\Delta\lambda/\lambda = 1 - \sqrt{1 - \frac{2M}{r}} \approx 6.3 \times 10^{-12}$.
   (f) No.
5. There is a huge redshift of the spectral line. Since all Doppler effects are corrected for, this means that the redshift may be due to the difference in observed time periods (and thereby frequency and wavelength) in the large gravitational field close to the black hole. We have $\Delta\lambda = 2150nm - 600nm = 1550nm$. Using the formula from the above exercise (we cannot use the simple formula since this is a large redshift which probably origintes from a point $r \approx 2M$:
$$\frac{r}{M} = 2\left(1 - \frac{1}{\left(\frac{\Delta\lambda}{\lambda} + 1\right)^2}\right)^{-1} \approx 2.2$$
6. Assume that stars radiate at a wavelength $\lambda = 500\text{nm}$ (the Sun radiaties most of its energy at wavelenghts close to this). Using the inverse formula for the gravitational redshift from the problem above, we have
$$\frac{\Delta\lambda}{\lambda} = 0.92$$
The observed wavelength of star light would be $\lambda' = \lambda - \lambda \times \Delta\lambda/\lambda = 500nm \times (1 - 0.92) \approx 35nm$

Problem 2

Following very similar calculations in the text, this one should be straight forward so no solutions are given.

Problem 3

1. A similar calculation is shown in the text in the chapter on GPS.
2. $M/r \approx 10^{-9}$, $v_{\text{Airplane}} \approx 9.3 \times 10^{-7}$ and $v_{\text{Earth}} = 2\pi r/(24h) \approx 1.5 \times 10^{-6}$ which are all small and thus allows for Taylor expansions.
3. We Taylor expand in $x = -(2M/(r + \Delta r) + v^2_{\text{Airplane}})$

$$\sqrt{1+x} \approx \left(1 + \frac{1}{2}x\right)$$

and in $y = -(2M/r + v^2_{\text{Earth}})$

$$\frac{1}{\sqrt{1+y}} \approx 1 - \frac{1}{2}y.$$

Thus

$$\frac{\sqrt{1+x}}{\sqrt{1+y}} = \left(1 + \frac{1}{2}x\right)\left(1 - \frac{1}{2}y\right) \approx 1 + \frac{1}{2}x - \frac{1}{2}y,$$

since xy is much smaller than x and y. Inserting for x and y we obtain the expression in the text.

4. $\Delta\tau/\Delta t \approx 1 + 2 \times 10^{-12}$. The difference is thus $2 \times 10^{-12} \times 50 \times 365 \times 24 \times 3600 \approx 3ms$ during their lifetime.

Problem 4

1. We use the expression from the previous problem and find $\Delta\tau/\Delta t \approx 1 + 5 \times 10^{-10}$. After time Δt we have a deviation of 1 kilometer:

$$1km = c\Delta t \times 5 \times 10^{-10}$$

giving $\Delta t \approx 1.9h$

2. (a) We have Kepler's 3rd law giving ($m_{\text{mars}} = 6.4 \times 10^{23}$kg and $r_{\text{mars}} =$ 3397km)

$$r = \left(\frac{GMP^2}{4\pi^2}\right)^{1/3} - r_{\text{mars}} \approx 9240km$$

(b) The velocity of the satellite with respect to the center of Mars is thus $v = 2\pi r/12h \approx 1.8\text{km/s}$, the velocity of a person on the ground on Mars is (the orbital period of Mars is 24.6 hours) $v = 2\pi r/P \approx 0.24\text{km/s}$. Using the formula above we get $\Delta\tau/\Delta t \approx 1 + 8 \times 10^{-11}$ giving $\Delta t \approx 12\text{hours}$.