The following people have participated in creating these solutions: Nicolaas E. Groeneboom, Magnus Pedersen Lohne, Karl R. Leikanger
NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know

Partial solutions to problems: Lecture 20

Problem 1

1. We use that $M \propto T^2$, that is,

$$M = CT^2$$

where we obtain C by using what we know about the sun: $C = M/T^2 \approx 6 \cdot 10^{22}$. The small star with $M = 0.5M_{sun}$ has temperature

$$T = \sqrt{0.5 * M/C} \approx 4000K$$

while the larger star $M = 5M_{sun}$

$$T = \sqrt{5 * M/C} \approx 13000K$$

and the largest star $M = 40M_{sun}$

$$T = \sqrt{40 * M/C} \approx 36000K$$

2. We use Wien's displacement law $T = 2.9 \cdot 10^6 nmK/\lambda = 14400K$. From this, $M = CT^2 = 6.2M_{sun}$.

Problem 2

We use that the radiation pressure is given as $P\tilde{T}^4$ such that the hydrostatic equation reads

$$\frac{dP}{dR} = \frac{dP}{dT}\frac{dT}{dR} \propto T^3 \frac{T}{R} \propto g\rho \propto \frac{M^2}{R^5}$$

in other words,

$$T^4 \propto \frac{M^2}{R^4} \tag{0.1}$$

using the fact (equation 2 in the lecture notes) that

$$L \propto \frac{R^4 T^4}{M} = M$$

when equation (0.1) inserted

Problem 5

1. The hydrostatic equation says that

$$\frac{dP}{dr} \propto g\rho \propto \frac{M^2}{R^5}$$

but assuming that $P \propto r^n$ for any n, we find that

$$\frac{dP}{dr} \propto R^{n-1} = \frac{R^n}{R} \propto \frac{P}{R}$$

such that

$$P \propto \frac{M^2}{R^4}$$

2. For an ideal gas,

$$P \propto \rho T = \frac{M}{R^3} T = \frac{M^2}{R^4}$$

such that

$$T \propto \frac{M}{R}$$

3. The efficiency of the *pp*-chain is

$$\epsilon_{pp} = \epsilon_{0,pp} X_H^2 \rho T_6^2 \approx 0.005$$

while

$$\epsilon_{CNO} = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} \approx 0.009$$

so the CNO-cycle dominated this star

4. For the triple-α process to occur, we need a helium-abundant core, that is, $X_{he} = 1$. In this case, we find

$$\epsilon_{03\alpha} \rho^2 \left(\frac{T_6}{100}\right)^{41} = \epsilon_{0CNO} X_H X_{CNO} \rho 18^{20}$$

where we used that $T_8 = T_6/100$. Solving for T_6 yields

$$T_6 = \left(\frac{\epsilon_{0CNO} X_H X_{CNO}}{\epsilon_{03\alpha} \rho} \cdot 100^{41} \cdot 18^{20}\right)^{1/41} \approx 131\,\text{million K} \tag{0.2}$$

5. We use that

$$R \propto \frac{M}{T} = C \cdot \frac{1}{T}$$

for M constant when the core is contracting and C is a constant. We determine C by using what we know about the star on the main sequence ($R = 0.2R_{sun}, T_{sun} = 18MK$), so $C = RT$. For a core temperature to reach 131 million K, the radius of the core needs to be

$$R = \frac{C}{T} = 0.2R_{sun} 18 \frac{1}{131} = 0.0275 R_{sun}$$

6. A more suitable density can be found as such

$$\rho_{before}(0.2R_{sun})^3 = \rho_{after}(0.0275R_{sun})^3$$

such that

$$\rho_{after} = \rho_{before}(\frac{0.2}{0.0275})^3 = 384\rho_{before}$$

Using equation (0.2) we obtain a new core temperature of $T \approx 114$ million K.