The following people have participated in creating these solutions: Nicolaas E. Groeneboom, Magnus Pedersen Lohne, Karl R. Leikanger
NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know

Partial solutions to problems: Lecture 22

Problem 1

We will not show the details of the energy flow here as you can easily find that by studying the details in the text, but we I will focus on the most important point to be learned in this exercise: The energy in the supernova explosion comes mainly from gravitational potential energy. Check that you understand why!

Problem 2

1. The derivation is identical to that of a white dwarf, except for m_e replaced by m_n. The radius (when inserted into the equation for the radius of a white dwarf, but with m_n instead) yields about $1km$ for a $1.4M_{sun}$ star which is too small but gives a rough idea of the size of a neutron star. More detailed calculations are necessary to obtain a more correct result.

2. The density of a neutron star of $1.4M_\odot$ with radius $r = 10$km is

$$\rho = \frac{1.4M_\odot}{4/3\pi(10km)^3} = 6.7 \times 10^{17} kg/m^3$$

If we compress the Earth to this density we find

$$6.7 \times 10^{17} kg/m^3 = \frac{M_{\text{Earth}}}{4/3\pi R^3}$$

giving $R = 129$m.

3. We use that $M = \frac{4}{3}\pi R^3 \rho$, or $\rho = \frac{3}{4\pi R^3}M$. A uranium atom thus has density

$$\rho_{ur} = \frac{3}{4\pi} 200 m_p (7fm)^{-3} \sim 2.3 \cdot 10^{17} kg/m^3$$

quite similar to that of an neutron star.

4. We have that $R = 10km \times G/c^2 = 1.3 \times 10^{31} kg = 6.7M_\odot \approx 5M$ which is very close to the Schwarzschild radius $R = 2M$. Hence GR is needed!

5. As $L = I\omega$ and $I = (2/5)MR^2$ for a sphere, preservation of angular momentum gives

$$L_{before} = L_{after} \rightarrow R^2_{before}\omega_{before} = R^2_{after}\omega_{after}$$

or in terms of periods $P_{\text{after}} = (R_{\text{after}}/R_{\text{before}})^2 * P_{\text{before}}$ giving 2.6 minutes for a Earth-size $R = 6000$km white dwarf and 4.4×10^{-4} seconds for a $R = 10$km neutron star. The rotation slows down with time due to energy/angulat momentum losses through the magnetic field. The neutron stars we observed have already been slowed down with respect to the angular velocity they had immediately after the SN explotion.

6. With an orbital period of $1\mu s = 10^{-3}s$, what is the maximum radius of an object with this speed? The orbital velocity is given by $\omega = 2\pi/P$, and the speed at the surface is given as $v = c = R \cdot \omega$. Then $R = c/\omega = c \cdot 1\mu s/2\pi \approx 48km$.

Problem 3

1. The luminosity of a star is given as

$$L = \sigma T^4 4\pi R^2$$

   letting $R = s = v\Delta t$, we obtain the desired result. The approximation is given by assuming that the radius of the original star is zero, as its supernova radius dominates completely (typically : the radius of the sun compared to the radius of the solar system)

2. By direct insertion:

$$L = 2.9 \cdot 10^9 L_{sun}$$

3.

$$M = M_{sun} - 2.5 \log_{10}(L/L_{sun}) = -18$$

4. From previous exercises, recall that

$$d = 10 \cdot 10^{\frac{m-M}{5}} pc = 5.8Mpc$$