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NOTE: There might be errors in the solution. If you find something which doesn't look right, please let me know

Partial solutions to problems: Lecture 5

Problem 1

The easiest way is to solve this problem using center of mass coordinates. Write the velocities in terms of \vec{v}_1^{CM} and \vec{v}_2^{CM} and use expressions from the lectures on celestial mechanics to write these in terms of the relative velocity \vec{v} which is the velocity of one object as observed from the other (remember that this is the frame from which Kepler's law is valid). Finally write the velocity in terms of the period P.

Problem 2

1. $32' = 2\pi/60/60/360 = 0.0093$ radians. The radius (for *small radial* values of θ) is given as $r = \theta d = 0.8 \text{Mpc}$.
2. We assume all galaxies are like the milky way, and that there are 200 billion stars in each galaxy. Assuming each star to have a mass of $2 \cdot 10^{30} \text{kg}$, we end up with an assumed mass of the 100 galaxies:

$$M_{\text{total}} = 100 \text{galaxies} \cdot 2 \cdot 10^{11} \text{stars} \cdot 2 \cdot 10^{30} \text{kg} = 4 \cdot 10^{43} \text{kg}$$

which is the *luminous* mass of the cluster.

3. Result given in the question.
4. We will use the following Python code:

```
from scitools.all import *
```

```
#Function that reads galaxydata from file
```

```
def read_data():
```

```
...
```

```
...
```

```
return ang_x, ang_y, dist, obs_lambda
```

```
#Function that calculates the radial velocity of the cluster of galaxies,
```

```
#and writes the result to screen
```

```
def rad_vel_cluster():
```

```
    v_gal = ...
```

```
    #Radial velocity of galaxies - use Doppler's formula
```

```
    v_clus = sum(..)/len(..) #Compute velocity of the cluster (peculiar velocity)
```

```
    v_rel = #Compute the relative velocity of the galaxies (velocity
```

```
    #relative to center of the cluster - center of mass system)
```

```

    print '...'
    return v_gal, v_clus, v_rel

#Function that plots (and stores) the cluster as seen from a telescope
#as an eps-file
def plot_cluster():
    plot(ang_x, ang_y, 'o')          #Plot galaxy as a ring
    ...

#Function that calculates the mass of a galaxy in the cluster
def mass_galaxy():
    #Find the position of the galaxy in the xyz-coord.frame
    dist_galaxy = ...
    rad_x = ...                    #Convert from arcmin to radians
    rad_y = ...
    x = ...                        #Trigonometry
    y = ...
    z = ...

    #Calculate the mass of the galaxy by eq. (7)
    # --> m = m_1/(G*m_2)
    m_1 = ...                      #Calculate SUM(v_i^2) (Relative velocity)
    step1 = range(len(dist_galaxy))
    m_2 = 0.0
    for i in step1:
        step2 = seq(i+1, len(dist_galaxy)-1)
        for j in step2:
            m_2 += ...             #Calculate 1/r_ij, use sqrt()-function
    mass = ...                      #Calculate mass by eq. (7)
    print '...'

#----
#MAIN

#Constants
...

#Variables
...

#Calculations/Call functions
ang_x, ang_y, dist, obs_lambda = read_data()
v_gal, v_clus, v_rel = rad_vel_cluster()
plot_cluster()
mass_galaxy()

```

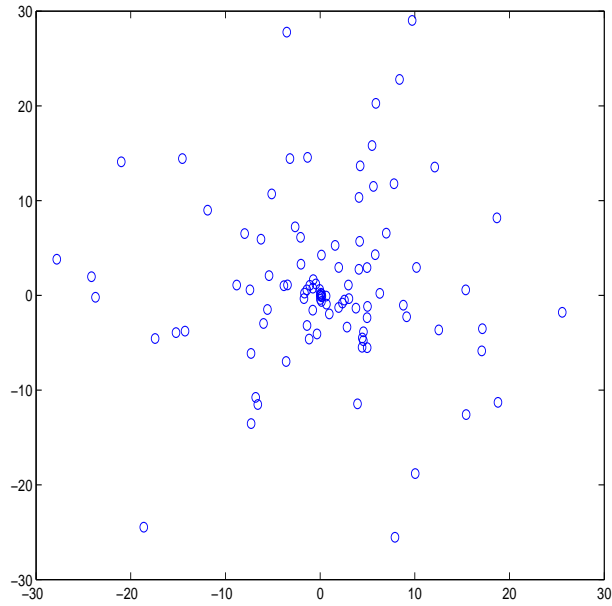


Figure 1: The galaxies in the cluster. The unit on the axes is in arcminutes.

- (a) $v_{\text{pec}} = 1.24 \cdot 10^6$ m/s.
- (b) The plot is shown in figure 1
- (c) $m \approx 8.7 \cdot 10^{41} \text{kg}$, about 2.17 times the estimated luminous mass. This suggests that more than 50% of the galaxy consists of something unknown and non-luminous.
- (d)

$$\langle \sin^2 i \rangle = \frac{\int_0^{\pi/2} \sin^2 i \, di}{\int_0^{\pi/2} di} = \frac{1}{2}$$

- (e) Using the result from the previous question, we see that the results from 2.4.c should be **doubled**: The “more correct” mass is $m \approx 1.7 \cdot 10^{42} \text{kg}$, about 4.4 times the estimated galaxy mass found in 2.2. This means that only about 20% of the total mass of a galaxy is represented as luminous matter, the rest consists of something else and more sinister, namely “dark matter”.