AST1100 Lecture Notes

22: The end state of stars

We will continue the discussion on stellar evolution from lecture 20. The star has reached the asymptotic giant branch having a radius of up to 1000 times the original radius. The core consists of carbon and oxygen but the temperature is not high enough for these elements to fuse to heavier elements. Helium fuses to carbon and oxygen in a shell around the core. Hydrogen fuses to helium in another shell further out. In the outer parts of the star, the temperature is too low for fusion reactions to take place. In the low and medium mass stars, convection has been transporting heavy elements from the core to the surface of the star allowing observers to study the composition of the core and test stellar evolutionary theories by studying the composition of elements on the stellar surface. The core is still contracting trying to reach a new hydrostatic equilibrium. The further evolution is now strongly dependent on the mass of the star.

1 Low mass stars

We will soon find out how we define low mass stars, but for the moment we will only say that a typical low mass star is our Sun. The core of the star, consisting mainly of carbon and oxygen contracts until the density of electrons is so high that the core becomes electron degenerate. In the more massive 'low mass stars' nuclear fusion may to some extent burn these elements to heavier elements like neon and magnesium. But eventually the core temperature is not high enough fore further nuclear reactions and the core remains electron degenerate.

As the star contracts, the temperature in the outer parts of the star increases and the hydrogen burning again becomes more efficient than the helium burning in lower shells. The helium produced in the upper shells 'falls' down on lower shells where no helium burning takes place (helium burning takes place even further down in the hotter areas). After a while, the density in the lower helium rich shell becomes very high and partially degenerate. At a point, the temperature in the lower shell is high enough for an explosive ignition of helium and a helium shell flash occurs, similar to the helium core flash described in lecture 20, but less energetic. The flash lifts the hydrogen burning shell to larger distances from the center. Hydrogen burning ceases, the star contracts until the temperature again is high enough for hydrogen burning. The whole process repeats, the produced helium falls on to lower layers which finally start burning helium in another helium shell flash. The star is very unstable and the repeated helium flashes result in huge mass losses from the star. The outer layers of the star are blown away in the helium flashes (this is one of the theories describing the huge mass losses the star undergoes in this period). A huge cloud of gas and dust is remaining outside the core of the star. After a few millions years, all the outer layers of the star have been blown off and only the degenerate carbon/oxygen core remains. This star which now consist only of the remaining degenerate core is called a *white dwarf*. The surrounding cloud of gas which has been blown off is called a *planetary nebula* (these have nothing to do with planets)

As the star was blowing away the outer layers, the hotter inner parts of the star made up the surface. Thus, the surface temperature of the star was increasing, and the star was moving horizontally to the left from the asymptotic giant branch in the HR-diagram. Finally when the layers producing energy by nuclear fusion are blown off, the luminosity of the star decreases dramatically and the star ends up on the bottom of the HR-diagram as a white dwarf (see HR-diagram in figure 1). The degenerate white dwarf does not have any sources of energy production and will gradually cool off as the heat is lost into space. The white dwarf will move to the right in the HR-diagram becoming cooler and dimmer.

How large is a white dwarf star? We can use the equation of hydrostatic equilibrium to get an estimate of the radius R assuming uniform density of the white dwarf (look back at lecture 20 where we did a similar approximation in the equation of hydrostatic equilibrium):

$$\frac{P}{R} \approx \frac{GM}{R^2}\frac{M}{(4/3)\pi R^3} = \frac{3GM^2}{4\pi R^5} \tag{1}$$

The pressure P is now the degeneration pressure. Inserting the expression for the degeneration pressure from the previous lecture in this equation gives

$$\left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{20m_e}n_e^{5/3} = \frac{3GM^2}{4\pi R^4}. \tag{2}$$

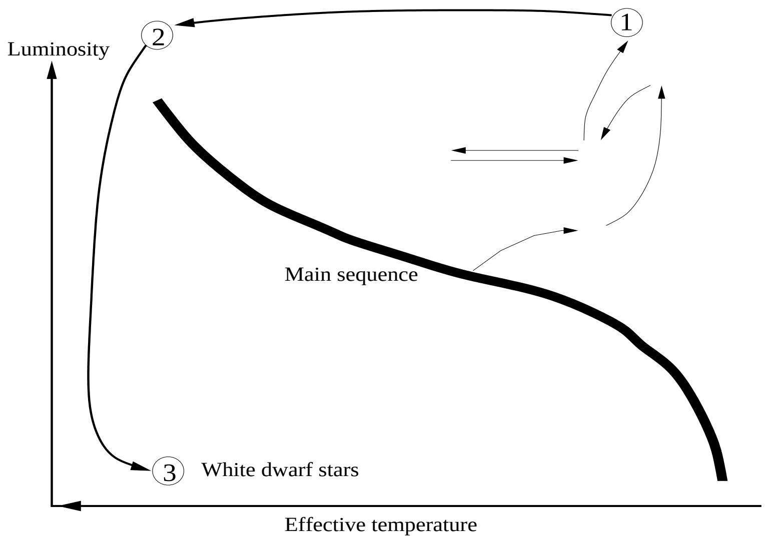

Figure 1: Motion in the HR-diagram for the last stages of stellar evolution. This is the path that a low mass $M < 8M_\odot$ star follows. From the asymptotic giant branch, the outer layers are blown off and the hotter inner parts become the new and hotter 'surface'. The effective temperature increases and the star moves from 1 to 2. As the layers were nuclear fusion takes place are blown off, the total luminosity decreases as the star is no more capable of producing energy. The star therefore falls down from 2 to 3 to the white dwarf area in the HR-diagram.

The electron number density n_e can be written in terms of the total gas density as

$$n_e = n_p = \frac{\rho_p}{m_H} = \frac{Z}{A}\frac{\rho}{m_H}.$$

Here Z is the number of protons per nucleus and A is the number of nucleons per nucleus. As the gas in total is neutral, the number of electrons in the gas equals the number n_p of protons. The number density, i.e., total number of protons per volume in the gas equals the mass density ρ_p of protons divided by the hydrogen mass (basically equal to the proton mass). The mass density of protons in a gas equals the mass density of the gas times the fraction of the mass in protons given by Z/A. A nucleus typically contains the same number of neutrons as protons such that the total number of nucleons is twice the number of protons and $Z/A = 0.5$. We will use this number in the calculations. Inserting this expression for n_e in the equation of hydrostatic equilibrium (equation 2) we have

$$\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_e} \left(\frac{Z}{Am_H}\right)^{5/3} \frac{M^{5/3}}{(4/3\pi)^{5/3}R^5} = \frac{3GM^2}{4\pi R^4},$$

or

$$R \approx \left(\frac{3}{2\pi}\right)^{4/3} \frac{h^2}{20m_eG} \left(\frac{Z}{Am_H}\right)^{5/3} M^{-1/3}.$$

For $M = M_\odot$ (where M is the mass which remained in the degenerate core, the star originally had more mass which was blown off) the radius of the white dwarf is similar to the radius of the Earth. A white dwarf is thus extremely dense, one solar mass compressed roughly to the size of the Earth. There is another interesting relation to be extracted from this equation. Multiplying the mass on the left side we have

$$MR^3 \propto MV = \text{constant},$$

where V is the volume of the white dwarf. Thus, if the mass of a white dwarf increases, the volume decreases. A white dwarf gets smaller and smaller the more mass it gets. It shrinks by the addition of mass. This can be understood by looking at the degenerate equation of state: When more mass is added to the white dwarf, the gravitational inward forces increase. This has to be balanced by an increased pressure. From the equation of state we see that since there is no temperature dependence, the only way to increase the pressure is by increasing the density. The density is increased by shrinking the size. So a white dwarf must shrink in order to increase the density and thereby the degeneration pressure in order to sustain the gravitational forces from an increase in mass.

Can a white dwarf shrink to zero size if we just add enough mass? Clearly when the density increases, the Fermi energy increases and the energy of the most energetic electrons increases. Finally the energy of the most energetic electrons will be so high that the velocity of these electrons will be close to the speed of light. In this case, the relativistic expression for the degeneration pressure is needed. We remember that the relativistic expression for the degeneration pressure went as $P \propto \rho^{4/3}$ instead of $P \propto \rho^{5/3}$ in the non-relativistic case. Inserting the relativistic expression in the equation of hydrostatic equilibrium we thus expect to obtain a different relation between radius and mass. Inserting the relativistic expression in equation 1 instead we obtain

$$\left(\frac{3}{\pi}\right)^{1/3}\frac{hc}{8}\left(\frac{Z}{Am_H}\right)^{4/3}\frac{M^{4/3}}{(4/3\pi)^{4/3}R^4}=\frac{3GM^2}{4\pi R^4}.$$

We see that the radius cancels out of the equation and we are left with a number for the mass of the relativistic white dwarf:

$$M \approx \frac{3}{16\pi}2^{-3/2}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{Z}{Am_H}\right)^2.$$

A more exact calculation taking into account non-uniform density would have given

$$M_{\text{Ch}} \approx \frac{\sqrt{3/2}}{2\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{Z}{Am_H}\right)^2 \approx 1.4M_\odot.$$

This is the *Chandrasekhar mass* M_{Ch} which gives the upper limit of the mass of a white dwarf. For a relativistic degenerate gas the pressure can only withstand the gravitational forces from a maximum mass of $M = 1.4M_\odot$. If the mass increases beyond that, the white dwarf will collapse. We will discuss this further in the next section. Having both the typical mass (or actually the upper bound on the mass) and the typical radius of a white dwarf we can find the typical density: a small needle made of white-dwarf material would weight about 50 kg.

So we are now closer to the definition of a 'low mass star'. A 'low mass star' is a star which has a mass low enough so that the remaining core after all mass losses has a mass less than the Chandrasekhar mass $1.4M_\odot$. The final result of stellar evolution for low mass stars is therefore a white dwarf. It turns out that stars which have up to about $8M_\odot$ when they reach the main sequence will have a core mass lower than the Chandrasekhar limit. We will now discuss the fate of stars with a main sequence mass larger than $8M_\odot$.

2 Intermediate and high mass stars

For stars of mass $M > 8M_\odot$, the evolution is different. The higher mass makes the pressure and thereby the temperature in the core higher than in the case of a low mass star. The forces of gravity are larger and therefore the pressure needs to by higher in order to maintain hydrostatic equilibrium. The carbon-oxygen core contracts, but before it gets degenerate the temperature is high enough for these elements to fuse to heavier elements. This sequence of processes which hydrogen and helium followed is repeated for heavier and heavier elements. When one element has been depleted, the core contracts until the temperature is high enough for the next fusion process to ignite while burning of the different elements takes place in shells around the core. This will continue until the core consists of iron $^{56}_{26}$Fe. We learned in the lecture on nuclear reactions that in order to produce elements heavier than iron, energy needs to be added, no energy is released. For elements heavier than iron, the mass per nucleon increases when the number of nucleons in the core increases. This is why no energy can be released in further fusion reactions. Nuclear processes which need energy input are difficult to make happen: The quantum mechanical probability for a nucleus to tunnel through the Coloumb barrier only to loose energy in the fusion process is very small. Thus, when the stellar core consists of iron, no more nuclear processes take place and the core starts contracting again. At this point, the star might look like figure 2. There are several layers of elements which resulted from previous nuclear fusion processes around the iron core. Fusion processes are still taking place in these shells.

At this point the temperature in the core is extremely high $T \sim 10^9 - 10^{10}K$ containing a dense gas of high energy photons. The core continues contracting and no more nuclear fusion processes are available to produce a pressure to withstand the forces of gravity pushing the core to higher and higher densities. The energy of the photons is getting so high that they start

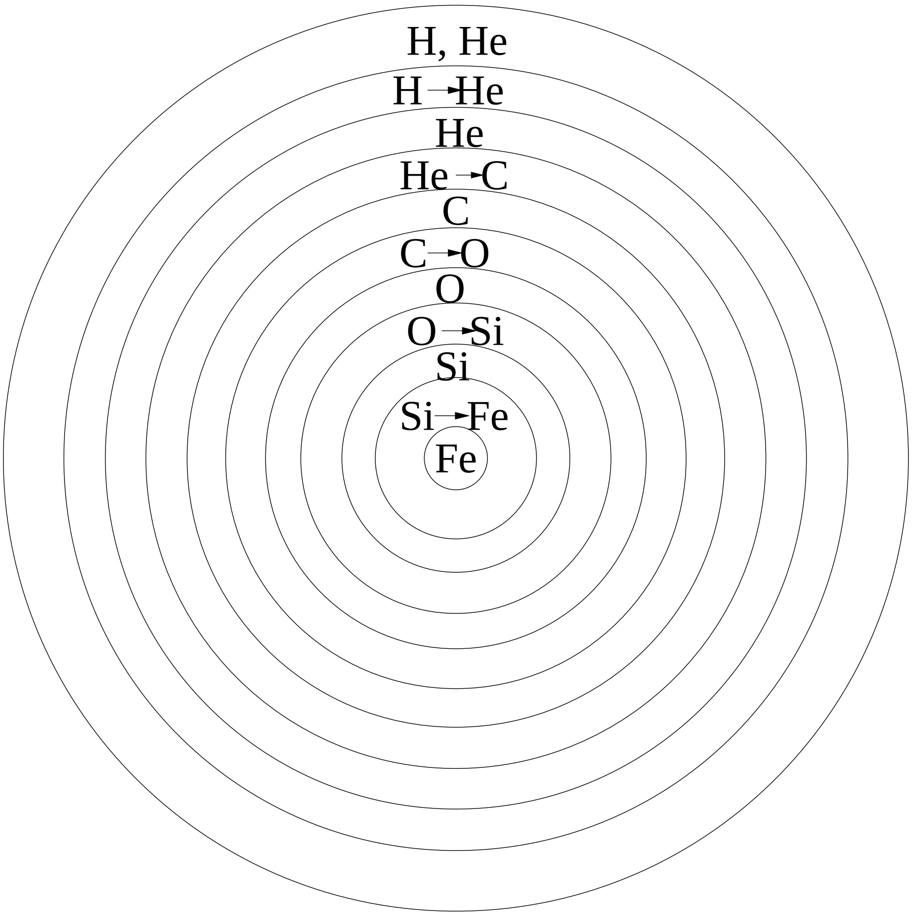

Figure 2: The structure of a star just hours before a supernova explosion. Energy production in the core has ceased as it now consists of Fe, the final product of nuclear fusion. Different elements are still burning in layers around the core.

splitting the iron atoms by the photo disintegration process

$$^{56}_{26}\mathrm{Fe} + ^{0}_{0}\gamma \rightarrow 13^{4}_{2}\mathrm{He} + 4^{1}_{0}\mathrm{n}$$

and helium atoms are further split into single protons and neutrons

$$^{4}_{2}\mathrm{He} + ^{0}_{0}\gamma \rightarrow 2^{1}_{1}\mathrm{p} + 2^{1}_{0}\mathrm{n}$$

reversing the processes which have been taking place in the core of the star for a full stellar life time. To split iron nuclei or other nuclei lighter than iron (with a few exceptions) requires energy. Again, by looking at the plot from the lecture on nuclear processes we see that nuclear fission processes only produce energy for elements heavier than iron. For lighter elements, the mass per nucleon increases when a core is split and energy is needed in the process. Thus, the photodisintegration processes take thermal energy from the core, energy which would contribute to the gas pressure preventing a rapid gravitational collapse of the core. When this energy is now taken away in the nuclear fission processes, the temperature and thereby the gas pressure goes down and the forces acting against gravity are even smaller. The core can now contract even faster. The result of the fast collapse is that the core is divided in two parts, the inner core which is contracting and the outer core being in free fall towards the rapidly contracting inner core.

The inner core becomes electron degenerate, but the degeneration pressure is not sufficient to withstand the weight from the mass around the core. But quantum physics absolutely forbids more than one electron to occupy one quantum state in momentum space, so how can the inner core continue contracting? Nature has found a solution: electron capture. Electrons and the free protons which are now available after the splitting of the nuclei combine to form neutrons

$$^{1}_{1}\mathrm{p} + ^{1}_{-1}\mathrm{e} \rightarrow ^{1}_{0}\mathrm{n} + ^{0}_{0}\nu$$

The final result is an inner core consisting almost entirely of neutrons. The neutron core continues collapsing until it becomes degenerate. This time, it is the neutrons and not the electrons which are degenerate. The stellar core is now so dense that all the quantum states of the neutrons are occupied and the core cannot be compressed further. The neutron degeneration pressure withstands the forces of gravity. Why can the neutron degeneration pressure withstand the forces of gravity when the electron degeneration pressure could not? The answer can be found in the expression (9) in the previous lecture for the degeneration pressure. Even if the neutron mass is much larger than the electron mass which should lead to a smaller degeneration pressure, the density is now much higher than it was in the electron degenerate gas. The higher density makes the total degeneration pressure larger.

When the inner core consisting mainly of neutrons becomes degenerate, the collapse is suddenly stopped, the core bounces back and an energetic shock wave is generated. This shock wave travels outwards from the core but is blocked by the massive and dense 'iron cap', the outer core, which is in free fall towards the inner core. The energy of the shock wave heats the outer core till temperatures large enough for photon disintegration and electron capture processes to take place. Thus, almost all the energy of the shock wave is absorbed in these energy demanding processes. So the huge amounts of energy carried outwards by the shock wave could in principle have blown the whole star apart, but the wave is blocked by the outer core where the energy is absorbed in photo disintegration and electron capture processes.

The final part of the story is truly remarkable. We see that the electron capture process releases neutrinos. So parts of the energy 'absorbed' in this process is reemitted in the form of energetic neutrinos. A large amount of electron capture processes are now taking place and computer simulations show that a neutrino sphere is created, an immensely dense wall of neutrinos is traveling outwards. Normally, we do not need to take neutrinos into account when studying processes in the stellar interiors because neutrinos hardly interact with matter at all and just leaves the star directly without influencing the star in any way. Now the outer core is extremely dense increasing the reaction probability of processes where neutrinos are involved. There is a huge amount of energetic neutrinos trying to pass through the very dense outer core. The combination of extreme densities and extreme neutrino fluxes makes the 'impossible' (or actually improbable) possible: a large part of the neutrinos reacts with the matter in the outer core. About 5% of the energy in the neutrinos is heating the outer parts just enough to allow the shock wave to continue outwards. The shock wave lifts the outer parts of the star away from the core, making the star expand rapidly. In short time, most of the star has been blown away to hundreds of AU away from the remaining inner core. This is a *supernova* explosion. The luminosity of the explosion is about $10^9 L_\odot$ which is the luminosity of a normal galaxy. The total luminsity of the supernova is thus similar to the total luminosity of an entire galaxy. And the energy released in photons is just a fraction of the energy released in neutrinos. An enormous amount of energy is released over very short time scales. Where does the energy from this explosion originally come from? We will discuss this in the exercises. Now we will look at the corpse of the dead star.

3 The fate of intermediate and high mass stars

For stars with mass $M < 25M_\odot$, the neutron degeneration pressure is high enough to withstand the forces of gravity and thereby to maintain hydrostatic equilibrium. Normally only $2 - 3M_\odot$ have remained in the core, the rest of the star was blown away in the supernova explosion. The remaining $2-3M_\odot$ star consists almost entirely of neutrons produced in the electron capture process taking place in the last seconds before the supernova explosion. It is a *neutron star*. As you will show in the exercises, the density of the neutron star is similar to the density in atomic nuclei. The neutron star is a huge atomic nucleus. In the exercises you will also show that the typical radius of a neutron star is a few kilometers. The mass of 2-3 Suns are compressed into a sphere with a radius of a few kilometers. The density is such that if you make a small needle out of materials from a neutron star it would weight about 10^6 tons.

It is thought that neutron stars have solid outer crusts comprised of heavy nuclei (Fe) and electrons. Interior to this crust the material is comprised mostly of neutrons, with a small percentage of protons and electrons as well. At a sufficiently deep level the neutron density may become high enough to give rise to exotic physical phenomena such as *super-fluidity* and perhaps even a *quark-gluon plasma* where one could find free quarks. To model the physics of the interior of neutron stars, unknown particle physics is needed. These neutron stars are therefore macroscopic objects which can be used to understand details of microscopic physics.

For white dwarfs we found that there is a an upper limit to the mass of the dwarf. Repeating these calculations reveals that there is a similar upper limit to the mass of neutron stars. Depending somewhat on the less known physics in the interiors of neutron stars, one has found this upper limit to be somewhere between 2 and 3 solar masses. If the collapsing core has a mass larger than about 3 solar masses, the gravitational forces will be higher than the neutron degeneration pressure. In this case, no known physical forces can withstand the forces of gravity and the core continues to contract and becomes a black hole. Using current theories of quantum physics and gravitation, the core shrinks to an infinitely small point with infinite mass densities. Infinite results in physics is usually a sign of physical processes which are not well understood. When the collapsing core becomes sufficiently small, both the general theory of relativity for large masses as well as quantum physics for small scales are needed at the same time. These theories are at the moment not compatible and a more general theory is sought in order to understand what happens at the center of the black hole.

4 Pulsars

When the stellar core is contracting, the rotational velocity of the core increases because of conservation of angular momentum. In order to maintain the angular momentum when the radius decreases, the angular velocity needs to increase. In the exercises you will calculate the rotational speed of the Sun if it had been compressed to the size of a neutron star. After the formation of the neutron star the rotational period is typically less than a second.

In 1967, Jocelyn Bell discovered a source of radio emission which emitted regular radio pulses. The pulses where found to be extremely regular, with exactly the same period between each pulse. The period between each pulse turned out to be about one second. Later, several of these regular radio emitters have been discovered, most of which with a period of less than a second. At first, no physical explanation for the phenomenon was found and the first radio emitter was called LGM-1 (Little Green Men). Later the name *pulsar* has been adopted. Today about 1500 pulsars are known, the fastest is called the millisecond pulsar due to the extremely short pulsation period.

Today the leading theory trying to explain the regular radio pulses from pulsars is that the pulsars are rotating neutron stars. In the exercises you will show that in order to explain pulsars in terms of a rotating object, the radius of the object needs to be similar to the radius of a neutron star. For larger object to rotate sufficiently fast, the outer parts of the object would need to rotate with a velocity larger than the velocity of light. The physics behind the process leading to the radio pulses of pulsars is still an an active field of research and the details are not well understood. In short, the current theory says that the neutron star has a strong magnetic field (which is created during the collapse of the stellar core) with the axis of the magnetic field lines shifted with respect to the axis of rotation (see figure 3). When the star rotates, the

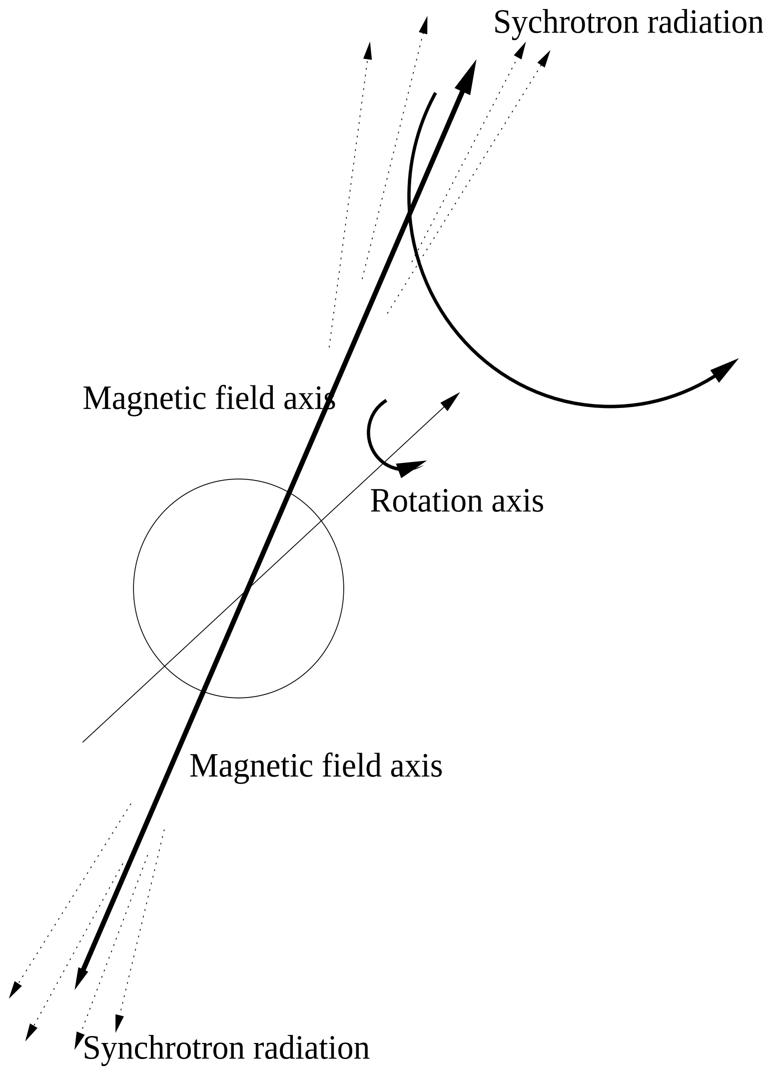

Figure 3: A rotating neutron star: The magnetic axis is not aligned with the rotation axis. Therefore the magnetic field lines are dragged around, producing synchrotron radiation as they accelerate electrons outside the neutron star. The synchrotron radiation is directed in the direction of the magnetic field lines. Every time the fields lines sweep over our direction we see a pulse of radiation.

field lines are sweeping out a cone around the rotation axis. Electrons in a hot electron gas around the neutron star are accelerated in the strong magnetic field lines from the neutron star. When electrically charged particles are accelerated, they emit electromagnetic radiation, synchrotron radiation. The electrons only feel the magnetic field when the magnetic poles (which are not aligned with the rotation axis) of the neutron star points almost directly in their direction. Thus, only the electrons in the gas where the magnetic field lines are pointing at the moment emit synchrotron radiation. This radiation is emitted radially outwards from the neutron star. If the Earth is along the line of such an emission, observers will see a pulse of synchrotron radiation each time the magnetic poles of the neutron star points in the direction of the Earth. We will therefore receive a pulse of radiation once every rotation period.

5 Generations of stars

In the Big Bang, mainly hydrogen and helium were produced. We have learned that heavier elements are produced in nuclear fusion processes in the interior of stars. But fusion processes produce energy only when the nuclei involved are lighter than iron. Fusion processes which produce elements heavier than iron need energy input and are therefore extremely difficult to make happen. How can it be that the Earth consists of large amounts of elements heavier than iron? Human beings contain elements heavier than iron. Where did they come from?

We learned that in the final stages of the stellar evolutionary process for high mass stars, the temperature in the core is very high and high energy photons are able to split nuclei. At these high temperatures, the iron nuclei around the core have so high thermal energy that even nuclear processes requiring energy may happen. Even the heavy nuclei have high enough energy to break the Coloumb barrier and fuse to heavier elements. All the elements in the universe heavier than iron have been produced close to the core of a massive star undergoing a supernova explosion. When the shock wave blows off the material around the core, these heavier elements are transported to the interstellar material. We remember than a star started its life cycle as a cloud of interstellar gas contracting due to its own weight. So the elements produced in supernova explosions are being used in the birth of another star. Parts of these elements go into the planets which are formed in a disc around the protostar.

The first stars which formed in the universe are called *population III* stars. These stars contained no heavier elements (in astrophysics, all the heavier elements are called 'metals', even if they are not metals in the normal sense). These stars have never been observed directly but theories for the evolution of the universe predict that they must have existed. The next generation, produced in part from the 'ashes' of the the population III stars are called *population II* stars. These stars have small traces of metals but are generally also metal poor stars. Finally, the *population I* stars is the latest generation of stars containing a non-zero abundance of metals. The Sun is a population I star. The exact details of stellar evolution are different depending on whether it is a population I, II or III star: computer simulations show that the metal content (which is usually very small even in population I stars) plays an important role in stellar evolutionary processes.

6 Type Ia supernovae

We learned in previous lectures that supernovae can be divided into type I and type II according to their spectra. The type I supernovae could further be divided into type Ia, Ib and Ic. The type I supernovae did not show any hydrogen lines in their spectra whereas type II supernovae show strong hydrogen lines. It is currently believed that type II as well as type Ib and Ic are core collapse supernovae discussed above. Type Ib and Ic do not have hydrogen lines because the outer hydrogen rich parts of the star have been blown off before the supernova explosion.

A type Ia supernova is believed to be a completely different phenomenon. There are several different theories trying to explain type Ia supernova, non of which are understood well. In one of the most popular theories, a type Ia supernova occurs in a binary system: A white dwarf star and a main sequence or giant star orbit a common center of mass. When the two stars are sufficiently close, the white dwarf starts accreting material from the other star. Material from the other star is accreted in a shell on the surface of the degenerate white dwarf. The temperature in the core of the white dwarf increases and nuclear fusion processes burning carbon and oxygen to heavier elements ignite. Since the white dwarf is degenerate, a process similar to the helium flash occur. Fusion processes start everywhere in the white dwarf at the same time and the white dwarf is completely destroyed in the following explosion. The exact details of these explosions are still studied along with other completely different theories in computer models. Hopefully, in the future, these models will be able to tell us exactly what happens in type Ia supernova explosions. As we will see in the lectures on cosmology, understanding type Ia supernovae is crucial for understanding the universe as a whole. We also remember that type Ia supernovae are used as standard candles to measure distances. It turns out that the luminosity is usually almost the same in most type Ia explosions. If these explosions really are white dwarf stars exploding, we can understand why the luminosity is almost the same in all supernovae of this kind: There is one common factor in all cases: the white dwarf stars usually have a mass close to the Chandrasekhar mass of $1.4M_{\odot}$.

7 An Example: SK 69^o202

Let us take as an example the blue giant that exploded as a supernova of type II in the Large Magellenic Cloud in 1987; the star SK 69^o202, later known as SN 1987a. The details of the star's life have been obtained through computer simulations. This originally $20M_{\odot}$ star's life may be summarized as follows:

1. H→He fusion for a period of roughly 10^7 yr with a core temperature $T_c \approx 40 \times 10^6$ K, a central density $\rho_c \approx 5 \times 10^3$ kg/m^3, and a radius $\approx 6\text{R}_{\odot}$.

2. He→C, O fusion for a period of roughly 10^6 yr with a core temperature $T_c \approx 170 \times 10^6$ K, a central density $\rho_c \approx 9 \times 10^5$ kg/m^3, and a radius $\approx 500\text{R}_{\odot}$. The core mass is $6\text{M}_{\odot}$. The star is in this phase a red super-giant.

3. C→Ne, Na, Mg fusion for a period of roughly 10^3 yr with a core temperature $T_c \approx 700 \times 10^6$ K, a central density $\rho_c \approx 1.5 \times 10^8$ kg/m^3, and a radius $\approx 50\text{R}_{\odot}$. The core mass is $4\text{M}_{\odot}$. From this stage and onward the star loses more energy through the emission of neutrinos than from the emission of photons. In addition, from this point onwards the evolution of the core is very rapid and the outer layers do not have time to adjust to the changes happening below: the star's radius remains unchanged.

4. Ne→O, Mg fusion for a period of some few years with a core temperature $T_c \approx 1.5 \times 10^9$ K, a central density $\rho_c \approx 10^{10}$ kg/m^3.

5. O→S, Si fusion for a period of some few years with a core temperature $T_c \approx 2.1 \times 10^9$ K. The neutrino luminosity is at this stage 10^5 greater than the photon luminosity.

6. S, Si→"Fe" (actually a mix of Fe, Ni, Co) fusion for a period of a few days with a core temperature $T_c \approx 3.5 \times 10^9$ K, a central density $\rho_c \approx 10^{11}$ kg/m^3. Si "melts" into α-particles, neutrons and protons which again are built into "Fe" nuclei. The core mass is now roughly 1.4M$_\odot$.

7. The core looses energy in the electron capture process and starts contracting rapidly. The energy in the core collapse is released and the star explodes as a supernova.

8 Problems

Problem 1 (20 - 30 min) Where does the huge amount of energy released in a supernova explosion originally come from? Does it come from nuclear processes or from other processes? Explain how the energy is released and how the energy is transferred between different types of energy until it is released as a huge flux of photons and neutrinos. Read the text carefully to understand the details in the processes and make a diagram of the energy flow.

Problem 2 (1 - 2 hours) In this exercise we will study the properties of a neutron star.

1. In the text we find an approximate expression for the radius of a white dwarf star using the equation of hydrostatic equilibrium and the degeneration pressure for electrons. Go back to the text and study how this was done in detail. Now, repeat the same exercise but for a neutron degenerate neutron star. What is the radius of a neutron star having $1.4M_\odot$? (The size of a neutron star is typically 10 km, your answer will not be correct but should be roughly this order of magnitude)

2. In the following, use a neutron star radius of 10km which is more realistic. What is the typical density of a neutron star? Express the result in the following way: To which radius R would you need to compress Earth in order to obtain densities similar to neutron star densities?

3. Compare the density in the neutron star to the densities in an atomic nucleus: Use the density you obtained in the previous question for the neutron star density. For the density of an atomic nucleus: Assume that a Uranium atom has about 200 nucleons and has a spherical nucleus with radius of about 7 fm (1 fm= $10^{-15}m$).

4. Express the radius of a $1.4M_{\odot}$ neutron star (use numbers from previous questions) in terms of the mass of the neutron star. How close are you to the Schwardschild radius when you are at the surface of a neutron star? Is general relativity needed when modeling a neutron star?

5. The Sun rotates about its axis once every 25 days. Use conservation of angular momentum to find the rotation period if the Sun is compressed to

   (a) a typical white dwarf star

   (b) a typical neutron star

   Compare your answer to numbers given in the text for the rotational period of a neutron star. If you found a faster period, can you find some possible reasons why the observed rotational period is slower? **hint1:** The angular momentum for a solid object is given by $L = I\omega$ where I is the moment of inertia. **hint2:** Assume that the Sun is a solid sphere and remember that the moment of inertia of a solid sphere is $(2/5)MR^2$

6. We will assume that we do not know what pulsars are, but we suspect that they might be rotating objects. The larger the radius of a rotating object, the faster the velocity of an object on the surface of the rotating object. The fastest observed pulsar is the millisecond pulsar. Assume that its rotational period is 1ms. What is the maximum radius R that the object can have without having objects at the surface of the object moving faster than light? What kind of astronomical objects could this possibly be? If you find several possibilities, try to find reasons to eliminate some of them.

Problem 3 (30 - 60 min) We learned in the lectures on distances in the universe that by observing the light curve of a supernova one could find the luminosity and thereby the absolute magnitude in order to obtain the distance. Here we will study a very simple model for a supernova in order to see if we can understand this relation between light curve and luminosity.

1. We have seen that in a supernova explosion, the outer shells of a star is basically lifted away from the central core. Assume that we can model the supernova in this way: the shell of gas simply expands spherically very fast outwards in all directions. It is equal to saying that the radius R of the star increases. Assume that we use Doppler measurements every day after the explosion of the supernova and find that the shell around the core expands with a constant velocity v. A time Δt after the explosion started we also measure the effective temperature of the shell by spectroscopic measurements and find it to have the temperature T. If we assume the shell to be a black body, show that the luminosity of the supernova at this point is given approximately by

$$L = 4\pi\sigma T^4 v^2 \Delta t^2.$$

   Which assumptions did you make in order to arrive at this expression?

2. A supernova is observed in a distant galaxy. Nobody had so far managed to measure the distance to this distant galaxy, but now there was a supernova explosion there and this allowed us to find the distance to the supernova and therefore also to the galaxy. You can now make all the assumptions from the previous question about the supernova. The supernova is observed every day after the explosion and the velocity of the shell is found to be moving at a constant velocity of 9500 km/s. After 42 days, the effective temperature is measured to be 6000 K. Find the luminosity of the supernova after 42 days expressed in solar luminosities ($L_\odot = 3.8 \times 10^{26}$W)

3. Find the absolute magnitude of the supernova after 42 days. **hint:** The absolute magnitude of the Sun is $M = 4.83$.

4. The apparent magnitude of the supernova after 42 days is $m = 10$. What is the distance to the supernova? Express your answer in Mpc.