AST1100 Lecture Notes

3 Extrasolar planets

1 Detecting extrasolar planets

Most models of star formation tell us that the formation of planets is a common process. We expect most stars to have planets orbiting them. Why then, has only a very few planets (about ten by fall 2010) around other stars been seen directly? There are two main reasons for this:

1. The planet's orbit is often close to the star. If the star is far away from us, the angular distance between the star and the planet is so small that the telescope cannot separate the two objects.

2. The light from the star is much brighter than the starlight reflected from the planet. It is very difficult to detect a faint signal close to a very bright source.

How large is the angular distance on the sky between Earth and Sun seen from our closest star, Proxima Centauri 4.22 light years away? Look at the geometry in figure 1. The distance r is 4.22 light years, the distance Sun-Earth d is 150×10^6 km. Using the small angle formula from geometry (and this is indeed a very small angle),

$$d = r\theta$$

we find $\theta = 0.00021°$ (check!). In astrophysics we usually specify small angles in terms of *arcminutes* and *arcseconds*, denoted $'$ and $''$. There are 60 arcminutes in one degree and 60 arcseconds in one arcminute. Thus the angular distance between Sun and Earth as seen from Proxima Centauri is $0.77''$. From the ground, the best resolution a normal telescope can reach is $0.4''$ under very good atmospheric conditions (actually using so-called adaptive optics better resolutions may be attained). This means that two objects with a smaller angular distance on the sky cannot be separated by the telescope. So the green men on a planet orbiting our nearest star would just

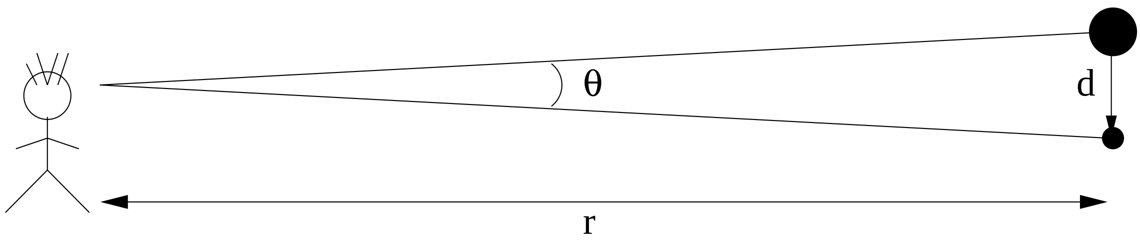

Figure 1: The angular extension of a distant planet with diameter d.

be able to see the Earth with the best telescopes under very good atmospheric conditions (provided the atmosphere on this planet is similar to the Earth's)! The Hubble Space Telescope which is not limited by the atmosphere can reach a resolution of $0.1''$. For the people on a planet orbiting a star located 100 light years away from Earth, the angular distance between the Earth and the Sun would be $0.03''$. From this planet, our green friends would be unable to see the Earth using the Hubble Space Telescope! A huge advance in optics and telescope technology is needed in the future in order to resolve planets which are orbiting close to their mother star.

Still, about 500 planets orbiting other stars have been detected (by fall 2010). The reason for this can be found in the previous lecture: In a star-planet system, the planet **and** the star are orbiting their common center of mass. Thus, the star is moving in an elliptical orbit. If the velocity of the star can be measured, then a regular variation of the star's velocity as it orbits the center of mass should be detected. This is the way most of the extrasolar planets have been discovered so far.

One way to measure the velocity of a star is by the Doppler effect, that electromagnetic waves (light) from the star change their wavelength depending on whether the star is moving towards us or away from us. When the star is approaching, we observe light with shorter wavelength, the light is *blueshifted*. On the contrary, when the star is receding, the light is *redshifted*. By measuring the displacement of spectral lines in the stellar spectra (more details about this in a later lecture), we can measure velocities of stars by the impressive precision of $1m/s$, the walking speed of a human being. In this way, even small variations in the star's velocity can be measured. Recall the formula for change in wavelength due to the Doppler effect

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v_r}{c},$$

where λ is the observed wavelength and λ_0 is the wavelength seen from the rest frame of the object emitting the wave. There is one drawback of this method: only radial velocity can be measured. Tangential velocity, movements perpendicular to the line of sight, does not produce any Doppler effect. The orbital plane of a planet (which is the same as the orbital plane of the star) will have a random orientation. We will therefore only be able to measure one component of the star's velocity, the radial velocity.

In figure 2 we have plotted the situation. The angle i is called the *inclination* of the orbit. It is simply the angle between the line of sight and a line perpendicular to the orbital plane (see figure 2). When the inclination $i = 90°$, the plane of the orbit is aligned with the line of sight and the velocity measured from the Doppler effect is the full velocity. For an inclination $i = 0°$, there is no radial component of the velocity and no Doppler effect is seen. A regular variation in a star's radial velocity could be the sign of a planet orbiting it.

We will in the following assume circular orbits (i.e. the eccentricity $e = 0$). This will make calculations easier, the distance from the center of mass a is always the same and more importantly, the velocity v is the same for all points in the orbit. In figure 3 we show how the radial velocity changes during the orbit of the star around the center of mass. If the inclination is $i = 90°$, then the radial velocity v_r equals the real velocity v in the points B and D in the figure. For other inclinations, the radial velocity v_r in points B and D is given by

$$|v_r| = v \sin i. \tag{1}$$

This is found by simple geometry, it is the component of the velocity vector taken along the line of sight (do you see this?). NOTE: The velocity v discussed here is the orbital velocity of the star, i.e. the velocity of the star with respect to the center of mass. Normally the star/planet system, i.e. the center of mass, has a (approximately) constant velocity with respect to the observer. This velocity v_{pec} is called the peculiar velocity and must be subtracted in order to obtain the velocity with respect to the center of

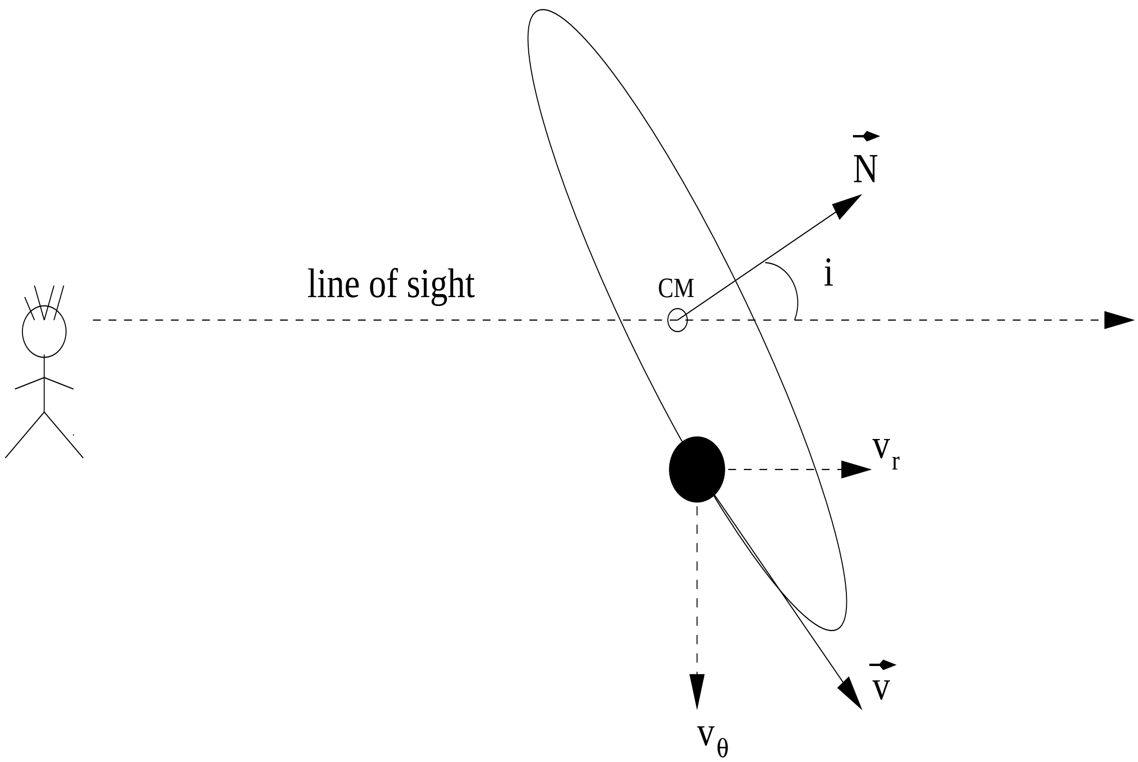

Figure 2: Inclination: The angle between the line of sight and the normal $\vec{N}$ to the orbital plane is called the inclination i. The maximum radial velocity of the star equals $v \sin i$.

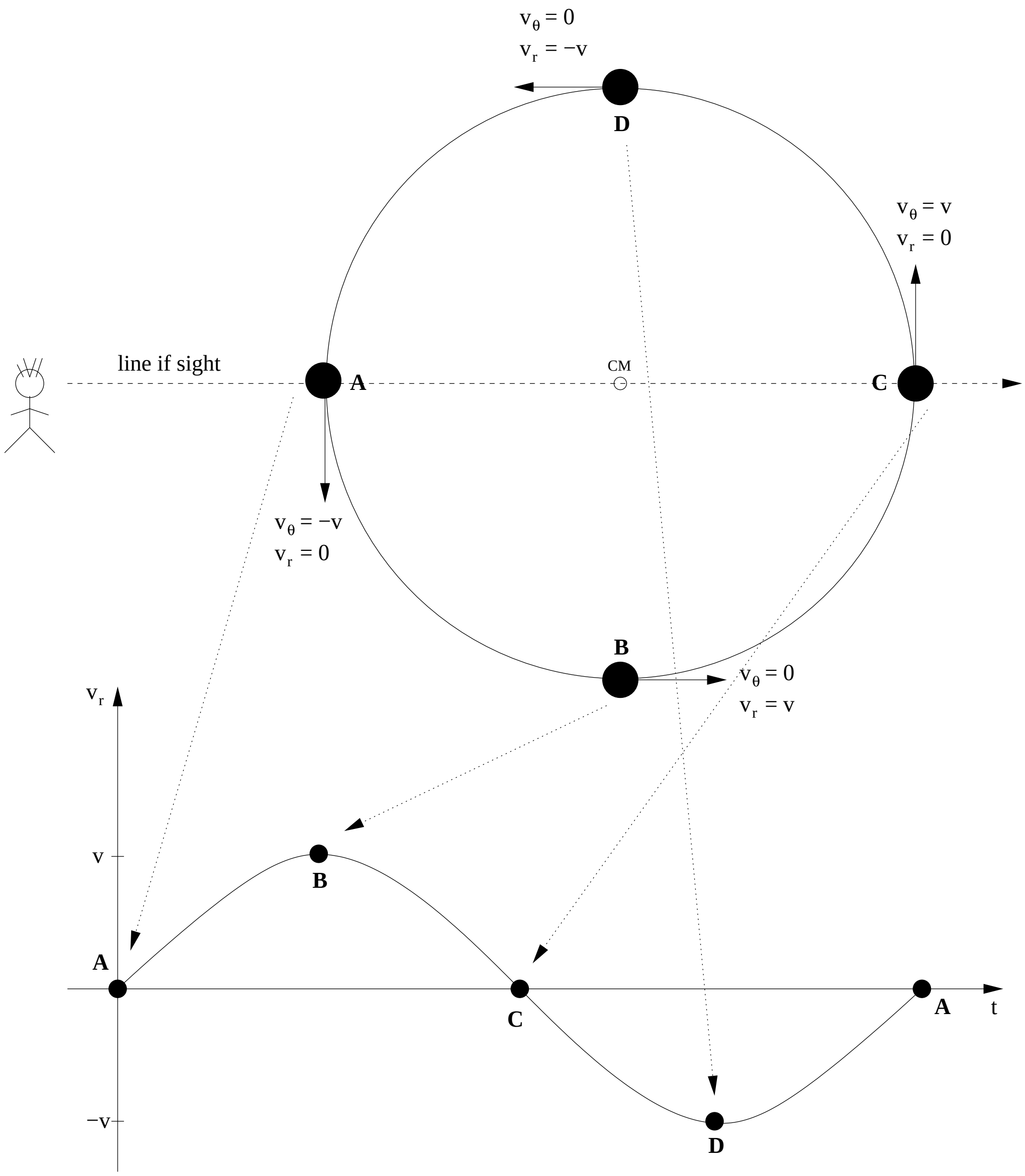

Figure 3: The velocity curve of a star orbiting the common center of mass with a planet. The points where the component of the velocity vector along the line of sight is zero (A and C) as well as the points where the radial component equals the full velocity (B and D) are indicated. In the figure, we have assumed an inclination of 90°.

mass. Recall from the previous lecture that the velocity of the star can be decomposed into the velocity of the center of mass (peculiar velocity) and the velocity of the star with respect to the center of mass (which is the one we need).

2 Determining the mass of extrasolar planets

We know that Kepler's third law connects the orbital period P, the semimajor axis a (radius in the case of a circular orbit) and mass m of the planet/star (do you remember how?). From observations of the radial velocity of a star we can determine the orbital period of the star/planet system. Is there a way to combine this with Kepler's laws in order to obtain the mass of the planet? The goal of this section is to solve this problem. We will deduce a way to determine the mass of an extrasolar planet with as little information as possible.

In the following we will use m_*, a_*, v_* for mass, radius of the orbit and velocity of the star in its orbit around the center of mass. Similarly we will use m_p, a_p and v_p for the corresponding quantities regarding the planet. The constant velocities may be expressed as,

$$v_* = \frac{2\pi a_*}{P} \qquad v_p = \frac{2\pi a_p}{P}. \tag{2}$$

Note again that this is velocity with respect to center of mass, any peculiar velocity has been subtracted. In the lecture notes for lecture 1-2, section 5, we found expressions for the position of the two bodies m_1 and m_2 taken in the center of mass frame, $\vec{r}_1^{CM}$ and $\vec{r}_2^{CM}$. Before reading on, look back at these lecture notes now and make sure you remember how these expressions were obtained!

Did you check those lecture notes? Ok, then we can continue. Take these masses to be the star and the planet. Using these expressions, we obtain (check!)

$$\frac{|\vec{r}_*^{CM}|}{|\vec{r}_p^{CM}|} = \frac{m_p}{m_*} = \frac{a_*}{a_p},$$

where the expressions for the semimajor axes a_1 and a_2 from lecture 1-2 were used. Using equation (2), we also have that

$$\frac{a_*}{a_p} = \frac{v_*}{v_p} = \frac{v_{*r}/\sin i}{v_{pr}/\sin i} = \frac{v_{*r}}{v_{pr}},$$

where equation (1) was used. NOTE: Here, the radial velocities v_{*r} and v_{pr} refer to the velocity at the point B in figure 3, the point for which the radial velocity is maximal. We may use these two equation to eliminate the unknown velocity of the planet

$$v_{pr} = v_{*r}\frac{m_*}{m_p}. \tag{3}$$

We will now return to Kepler's third law,

$$m_* + m_p = \frac{4\pi^2 a^3}{P^2 G},$$

where we have used the exact expression for Kepler's third law, derived in problem 2 in lecture notes 1-2. From section 5 in those notes, we also had that

$$a = a_* + a_p,$$

the semimajor axis a (of the orbit of the planet seen from the star or vice versa) equals the sum of the semimajor axes of the orbits of the planet and star about the center of mass. We can now express these in terms of velocities (equation 2)

$$a = \frac{P}{2\pi}(v_* + v_p).$$

Inserting this into Kepler's third law, we have

$$m_* + m_p = \frac{P}{2\pi G}(v_* + v_p)^3.$$

Normally we are only able to measure radial velocities, not the absolute velocity. We thus use equation (1) as well as equation (3) to obtain

$$m_* + m_p = \frac{P}{2\pi G}\frac{(v_{*r} + v_{pr})^3}{\sin^3 i} = \frac{P v_{*r}^3}{2\pi G \sin^3 i}\left(1 + \frac{m_*}{m_p}\right)^3.$$

Assuming that the star is much more massive than the planet (which is normally the case, for instance $m_{\text{Jupiter}}/m_{\text{Sun}} \sim 10^{-3}$) we get

$$m_* = \frac{P v_{*r}^3}{2\pi G \sin^3 i}\frac{m_*^3}{m_p^3},$$

which solved for the mass of the planet (which is the quantity we are looking for) gives

$$m_p \sin i = \frac{m_*^{2/3} v_{*r} P^{1/3}}{(2\pi G)^{1/3}}.$$

Normally, the mass of the star is known from spectroscopic measurements. The radial velocity of the star and the orbital period can both be inferred from measurements of the Doppler effect. Thus, the expression $m_p \sin i$ can be calculated. Unfortunately, we normally do not know the inclination angle i. Therefore, this approach for measuring the planet's mass can only put a lower limit on the mass. By setting $i = 90°$ we find $m_p^{\min}$. If the inclination angle is smaller, then the mass is always greater than this lower limit by a factor of $1/\sin i$. In the next section however, we will discuss a case in which we can actually know the inclination angle.

3 Measuring the radius and the density of extrasolar planets

If the inclination is close to $i \sim 90°$, the planet passes in front of the stellar disc and an eclipse occurs: The disc of the planet obscures a part of the the light from the star. Be looking at the light curve of the star, a dip will occur with regular intervals corresponding to the orbital period. In figure 4 we show a typical light curve. When the disc of the planet enters the disc of the star, the light curve starts falling. When the entire disc of the planet is inside the disc of the star, the light received from the star is now constant but lower than before the eclipse. When the disc of the planet starts to leave the disc of the star, the light curve starts rising again. When such a light curve is observed for a star where a planet has been detected with the radial velocity method described above, we know that the inclination of the orbit is close to $i = 90°$ and the mass estimate above is now a reliable estimate of the planet's mass rather than a lower limit.

In these cases, where the effect of the eclipse can be seen, the radius of the planet may also be measured. If we know the time of first contact (time t_0 in figure 4), the time when the disc of the planet has fully entered the disc of the star (time t_1) as well as the velocity of the planet with respect to the star, we can measure the radius of the planet. If the radius of the planet is

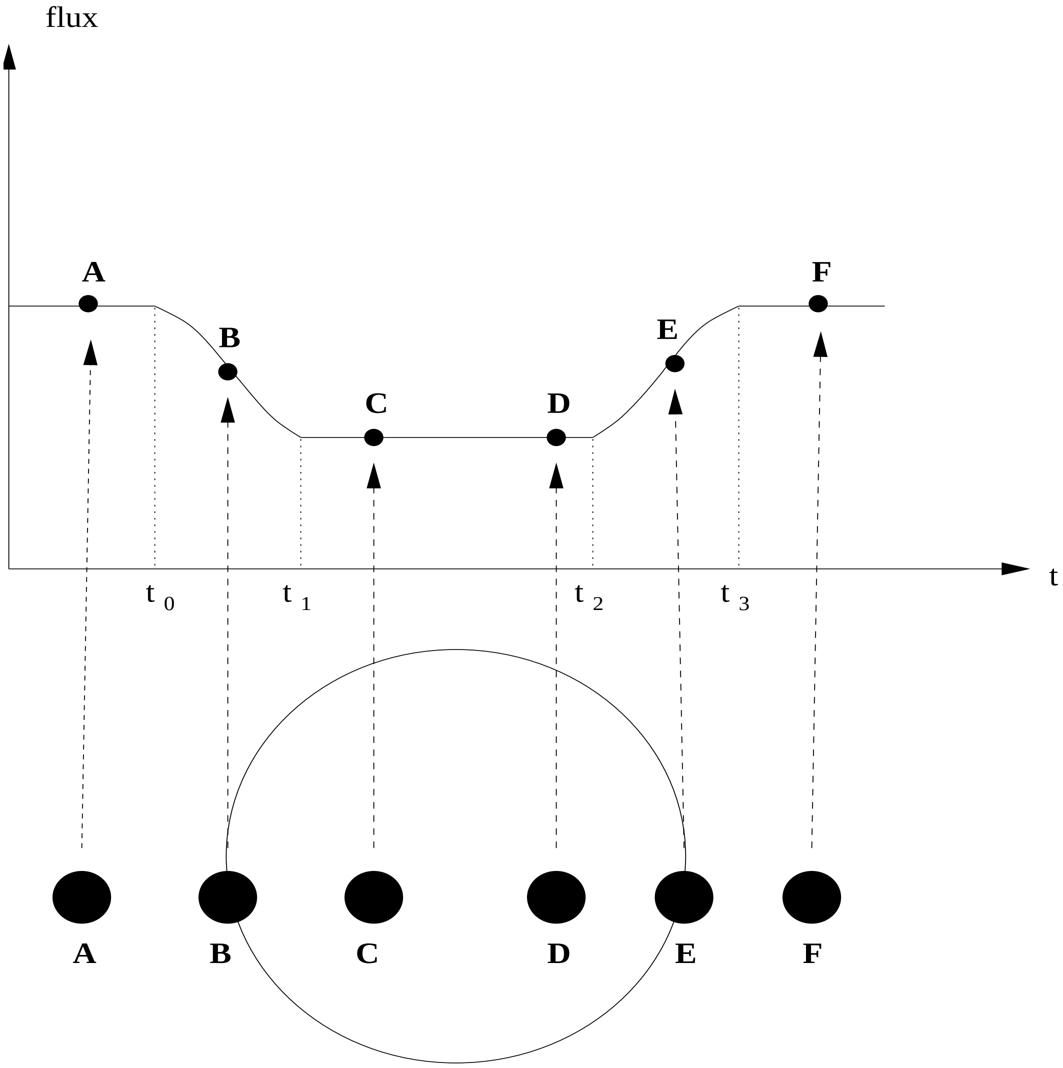

Figure 4: The lower part of the figure shows a planet (small filled circle) eclipsing a star (large open circle). The upper part shows a plot of the flux variation with time at the different points during the eclipse. The moments at which the eclipse starts $t = t_0$ and ends $t = t_3$ as well as the moments when the full disc of the planet enters $t = t_1$ and leaves $t = t_2$ the star are indicated.

R_p, then it took the disc of the planet with diameter $2R_p$ a time $t_1 - t_0$ to fully enter the disc of the star. The planet moves with a velocity $v_* + v_p$ with respect to the star (the velocity v_p is only the velocity with respect to the center of mass). Using simply that distance equals velocity times interval, we have

$$2R_p = (v_* + v_p)(t_1 - t_0)$$

As we have seen, we can obtain t_1 and t_0 from the light curve. We can also obtain the velocity of the planet (the velocity of the star is measured directly by the Doppler effect) by using equation (3),

$$v_p = v_* \frac{m_*}{m_p}.$$

Here the mass of the planet m_p has been calculated since we know that the inclination is $i \sim 90°$. Thus, the radius R_p of the planet is easily obtained. Combining the measured mass and radius of the planet we get an estimate of the mean density

$$\rho_p = \frac{m_p}{4/3\pi R_p^3}.$$

We can use this to determine whether the detected planet is *terrestrial planet* with a solid surface like the inner planets in the solar system, or a *gas planet* consisting mainly of gas and liquids like the outer planets in our solar system. The terrestrial planets in our solar system have densities of order $4-5$ times the density of water whereas the gas planets have densities of order $0.7-1.7$ times the density of water. If the detected planet is a terrestrial planet, it could also have life.

Finally, note that also the radius R_* of the observed star can be obtain by the same method using the time it takes for the planet to cross the disc of the star,

$$2R_* = (v_* + v_p)(t_2 - t_0).$$

We have discussed two ways of discovering extrasolar planets,

- by measuring radial velocity
- by measuring the light curve

In the following problems you will also encounter a third way,

- by measuring tangential velocity

For very close stars, the tangential movement of the star due to its motion in the orbit about the center of mass may be seen directly on the sky. The velocity we measure in this manner is the projection of the total velocity onto the plane perpendicular to the line of sight. There are two more methods which will briefly be discussed in later lectures,

- by gravitational lensing
- by pulsar timing

4 Exercise to be presented on the blackboard: The atmosphere of extrasolar planets

In figure 5 we show observations of the radial velocity of a star over a large period of time. We assume that these data is a collection of data from several telescopes around the world. Real data contain several additional complicated systematic effects which are not included in this figure. For instance, changes in the velocity of the Earth need to be corrected for in velocity measurements. Here we assume that these corrections have already been made. Even if this plot does not show you all the complications of real life, it does give an impression of how data from observations may look like and how to use them to say something about extrasolar planets. You see that this is not a smooth curve, several systematic effects as for instance atmospheric instabilities give rise to what we call 'noise'.

1. Does this star move towards us or away from us? Use the figure to give a an estimate of the peculiar velocity.
2. Use the curve to find the maximum radial velocity v_{r*} of the star (with respect to the center of mass) and the orbital period of the planet.
3. Spectroscopic measurements have shown the mass of the star to be 1.1 solar masses. Give an estimate of the lower bound for the mass of the planet. The result should be given in Jupiter masses.
4. In figure 6 we show a part of the light curve (taken at the wavelength 600nm) of the star for the same period of time. Explain how this curve helps you to obtain the real mass of the planet, not only the lower bound, and give an estimate of this mass.

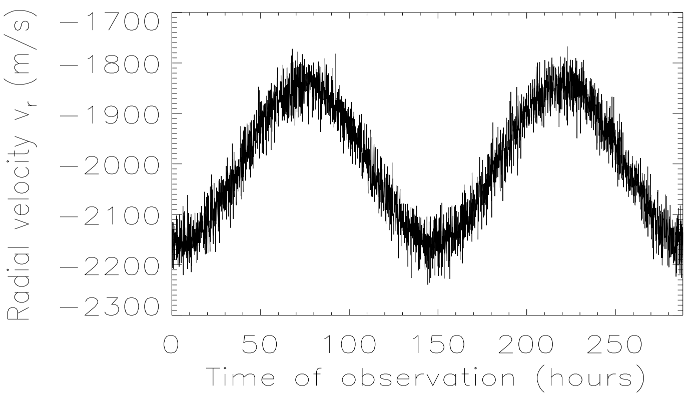

Figure 5: Velocity measurements of a star

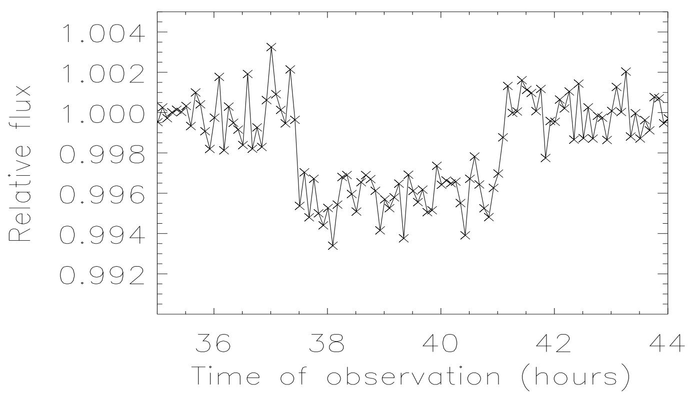

Figure 6: The light curve of a star at 600nm. There are 5 minutes between each cross.

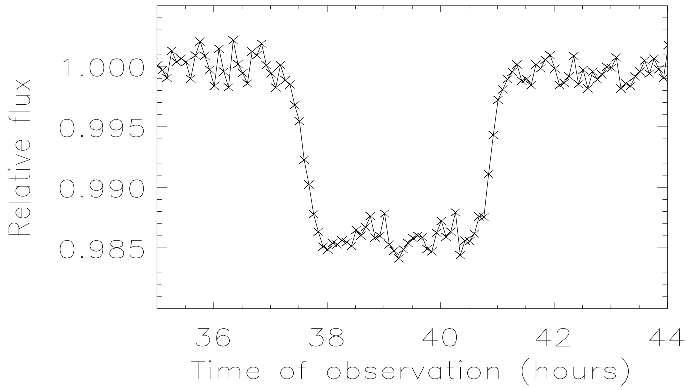

Figure 7: The light curve of a star at 1450nm. There are 5 minutes between each cross.

5. Use the light curve to find the radius of the planet. Note: there are 5 minutes between each cross in the plot.
6. In figure 7 we show a part of the light curve taken at the same time as the previous light curve but at a wavelength 1450nm which is an absorption line of water vapor. Use the figure to determine if this planet may have an atmosphere containing water vapor and estimate the thickness of the atmosphere.

5 Problems

Problem 1 (10-20 min.)

1. The current precision in measurements of radial velocities by the Doppler effect is $1m/s$. Can a Jupiter like planet orbiting a star similar to the Sun at a distance from the mother star equal to the Sun-Jupiter distance be detected? (use www or other sources to find the necessary data)
2. What about an Earth like planet in orbit at a distance $1AU$ from the same star?

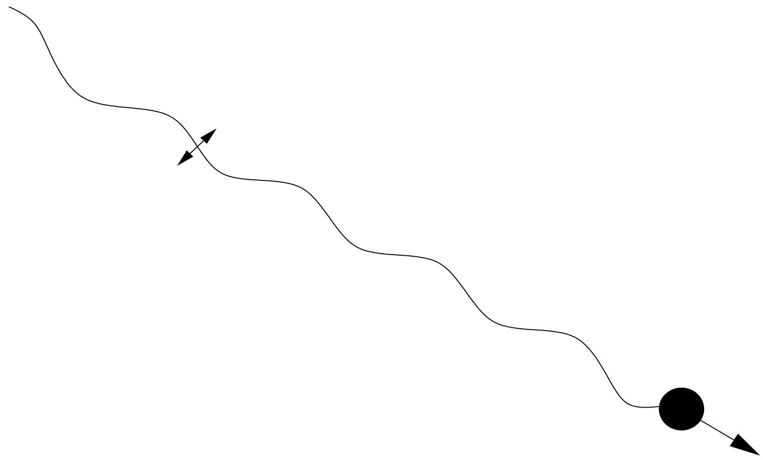

Figure 8: The transversal wobbling of a nearby star due to its orbital motion about the common center of mass with a planet. The angular extension of the orbit is indicated by two small arrows.

3. Using the radial velocity method, is it easier to detect planets orbiting closer or further away from the star?
4. In what distance range (from the mother star) does an Earth like planet need to be in order to be detected with the radial velocity method? (again use a star similar to the Sun). Compare with the distance Sun-Mercury, the planet in our solar system which is closest to the Sun.

Problem 2 (20-30 min.) For stars which are sufficiently close to us, their motion in the orbit about a common center of mass with a planet may be detected by observing the motion of the star directly on the sky. A star will typically move with a constant velocity in some given direction with respect to the Sun. If the star has a planet it will also be wobbling up and down (see figure 8). We will now study the necessary conditions which might enable the observation of this effect.

1. The Hubble Space Telescope (HST) has a resolution of about $0.1''$ How close to us does a star similar to the Sun with a Jupiter like planet (at the distance from the mother start equal to the Sun-Jupiter distance) need to be in order for the HST to observe the tangential wobbling of the star.

2. What about an Earth like planet at the distance of one AU from the same star?

3. The closest star to the Sun is Proxima Centauri at a distance of 4.22 l.y.. How massive does a planet orbiting Proxima Centauri at the distance of 1 AU need to be in order for the tangential wobbling of the star to be observed?

4. What about a planet at the distance from Proxima Centauri equal to the Sun-Jupiter distance?

5. If we can measure the tangential velocity component of a star, we can get an estimate of the mass of the planet not only a lower limit. Show that the exact mass of the planet can be expressed as

$$m_p = \left(\frac{m_*^2 P}{2\pi G}\right)^{1/3} v_{t*}$$

(tangential velocity v_{t*} here is measured when the radial velocity is zero)

Problem 3 (45 min. - 1 hour) In figure 9 we show observations of the radial velocity of a star over a large period of time. We assume that these data is a collection of data from several telescopes around the world. Real data contain several additional complicated systematic effects which are not included in this figure. For instance, changes in the velocity of the Earth need to be corrected for in velocity measurements. Here we assume that these corrections have already been made. Even if this plot does not show you all the complications of real life, it does give an impression of how data from observations may look like and how to use them to say something about extrasolar planets. You see that this is not a smooth curve, several systematic effects as for instance atmospheric instabilities give rise to what we call 'noise'.

1. This plot shows a curve with a wave like shape, can you explain the shape of the curve?

2. Use this plot to give an estimate for the the 'perculiar velocity' of the star. 'Peculiar velocity' is a term used to describe the average motion of the star with respect to us, not taking into account oscillations from planets.

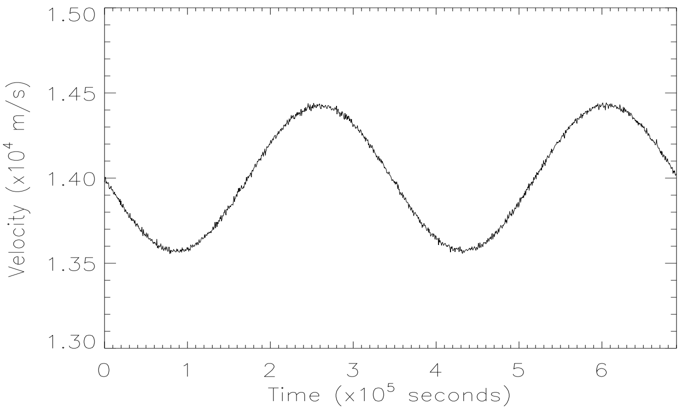

Figure 9: Velocity measurements of a star

3. Use the curve to find the maximum radial velocity v_{r*} of the star (with respect to the center of mass) and the orbital period of the planet.
4. Spectroscopic measurements have shown the mass of the star to be 1.3 solar masses. Give an estimate of the lower bound for the mass of the planet. The result should be given in Jupiter masses.
5. In figure 10 we show the light curve of the star for the same period of time. Explain how this curve helps you to obtain the real mass of the planet, not only the lower bound, and give an estimate of this mass.
6. In figure 11 we have zoomed in on a part of the light curve. Use the figure to give a rough estimate of the density of the planet.
7. Is this a gas planet or a terrestrial planet?

Problem 4 (4-5 hours) At the following link you will find some files containing simulated velocity and light curves of 10 stars:

http://folk.uio.no/frodekh/AST1100/lecture3/

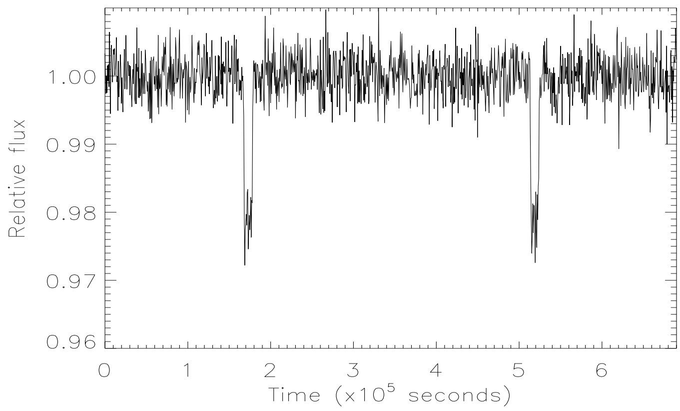

Figure 10: The light curve of a star

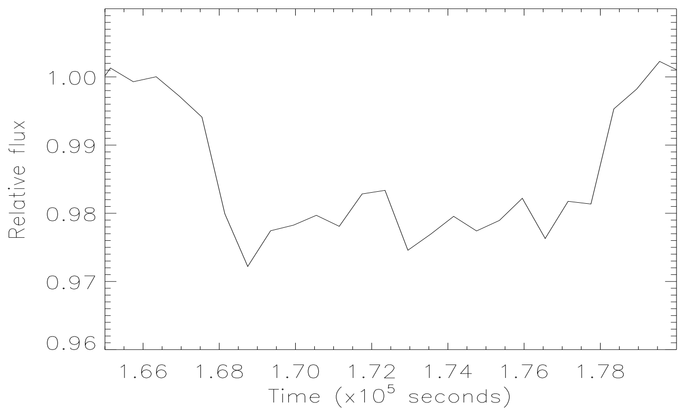

Figure 11: The light curve of a star

Real data contain several additional complicated systematic effects which are not included in these files. For instance, changes in the velocity of the Earth need to be corrected for in velocity measurements. Here we assume that these corrections have already been made. Even if these data do not show you all the complications of real life, they will still give an impression of how data from observations may look like and how to use them to say something about extrasolar planets. Each file contains three rows, the first row is the time of observation, counted in seconds from the first observation which we define to be $t = 0$. We assume that these data is a collection of data from several telescopes around the world, studying these stars intensively for a given period of time (the length of this observing period is different for each star). The second row gives the observed wavelength λ of a spectral line (The $H\alpha$ line) at $\lambda_0 = 656.3nm$ in $nm = 10^{-9}m$. You need to use the Doppler formula to obtain radial velocities yourself. You will see that this is not a smooth curve, several systematic effects, i.e. atmospheric instabilities give rise to what we call 'noise'. As you will see, this noise makes exact observations difficult. The third row shows the measured flux relative to the maximum flux for the given star. Again, also these data contain noise.

Use Python, Matlab or other software/programming languages to solve the following problems:

1. Estimate the peculiar velocity (the mean velocity of the star with respect to Earth) for each of the 10 stars, taking the mean of the velocity over all observations. Plot the velocity curves (subtract the mean velocity from the velocity for each observation) and light curves for the ten stars. Which of the stars appear to have a planet orbiting? Which of these planets are eclipsing their mother star?

2. The mass of the stars have been measured by other means, these are 0.8, 2.8, 0.5, 0.5, 1.8, 0.7, 1.6, 2.1, 7 and 8 solar masses for star 0-9 respectively. Can you, by looking at the velocity curves (velocity as a function of time), find the lower limit for the mass of the planet for the stars where you detected a planet. Find the numbers for the periods and max radial velocities by eye.

3. If you, by looking at the light curve, discovered that some of the planets are actually eclipsing the star, can you also estimate the radius and density of these planets. Again, you will need to estimate the time of

eclipse by eye. Does any of the planets you have detected have the possibilities for life (at least in the form that we know life) ?

4. You have made estimates of mass and radius using 'by-eye' measurements. This is not the way that astrophysicists are working. Often, advanced signal processing methods are employed in order to get the best possible estimates. Also, scientific measurements always have uncertainties. The detailed methods for analyzing these data are outside the scope of this course, but you will encounter this in more advanced courses in astrophysics. Here we will show you a simple way to obtain estimates which are more exact than the 'by-eye' observations above. A similar method will be used in other problems in this course. The key to this method is the method of 'least squares'. We will use this to obtain more accurate periods and max radial velocities from the velocity measurements. We will model the velocity curves as cosine curves in the following way,

$$v_r^{\mathrm{model}}(t) = v_r \cos(2\pi/P(t - t_0)), \tag{4}$$

where $v_r^{\mathrm{model}}(t)$ is the theoretical model of the radial velocity as a function of time, v_r is the maximal radial velocity, P is the period of revolution and t_0 is some point for which the radial velocity is maximal (you see that if $t = t_0$ then the cosine term equals one). The unknown parameters in this model are v_r, P and t_0. Only the two first parameters, v_r and P, are necessary in order to estimate the mass of the planet, but we need to estimate all three in order to have consistent estimates of the first two. We will now try to find a combination of these three parameters, such that equation (4) gives a good description of the data. To do this, you need to write a computer code which calculates the difference, or actually the square of the difference, between the data and your model for a large number of values for the three parameters t_0, P and v_r. You need to define a function (an array in you computer) $\Delta(t_0, P, v_r)$ given as

$$\Delta(t_0, P, v_r) = \sum_{t=t_0}^{t=t_0+P} (v_r^{\mathrm{data}}(t) - v_r^{\mathrm{model}}(t, t_0, P, v_r))^2$$

This function gives you the difference between the data and your model for different values of t_0, P and v_r. What you want to find is the function which best fits your data, that is, the model which gives the minimum difference between the data and your model. You simply want to find for which parameters t_0, P and v_r that the function $\Delta(t_0, P, v_r)$ is minimal. How do you find the parameters t_0, P and v_r which minimizes Δ ? In this case it is quite easy, try to follow these steps:

(a) Choose one of your stars which clearly has a planet orbiting.

(b) Look at your data: You know that for $t = t_0$, the velocity is maximal. Look for the first peak in the curve and define a range in time around this curve for which you think that the exact peak must be. Define a minimum possible t_0 and a maximum possible t_0 (being sure that exact peak is somewhere between these two values). Then define a set of, say 20 (you choose what is more convenient in each case) values of t_0 which are equally spaced between the minimum and maximum value.

(c) Do the same for v_r, try to find a minimum and a maximum v_r which are such that you see by eye that the real exact v_r is between these two values. Then divide this range into about 20 equally spaced values (maybe less depending on the case).

(d) Do the same thing for the period. Look at the time difference between two peaks, and find a set of possible periods.

(e) Now, calculate the function Δ for all these values of t_0, P and v_r which you have found to be possible values. Find which of these about 20^3 combination of values which gave the smallest Δ, thus the smallest difference between data and model. These values are now your best estimates of P and v_r.

(f) Calculate the mass of the planet again with these values for P and v_r and compare with your previous 'by-eye' estimates. How well did you do in estimating 'by-eye' ?

(g) Now repeat the procedure to estimate the exact mass for two other stars with planets and compare again with your 'by-eye' estimates.