AST1100 Lecture Notes

4 Stellar orbits and dark matter

1 Using Kepler's laws for stars orbiting the center of a galaxy

We will now use Kepler's laws of gravitation on much larger scales. We will study stars orbiting the center of galaxies. Our own galaxy, the Milky Way, contains more than 2×10^{11} stars. The diameter of the galaxy is about 100 000 light years and the Sun is located at a distance of about 25 000 light years from the center. It takes about 226 million years for the Sun to make one full revolution in its orbit.

The Milky way is a spiral galaxy where most of the stars are located in the *galactic disc* surrounding the center of the galaxy and in the galactic bulge, a spherical region about 10 000 light years in diameter located at the center (see figure 1). We will apply Newton/Kepler's laws to stars in the outer parts of a galaxy, at a large distance r from the center. For these stars, we may approximate the gravitational forces acting on the star to be the force of a mass $M(r)$ (which equals the total mass inside the orbit of the star) located at the center of the galaxy. Kepler's third law (Newton's modified version of it, see lecture notes 1-2, problem 2) for this star reads

$$P^2 = \frac{4\pi^2}{G(M(r)+m_*)}r^3,$$

where we assume a circular orbit with radius r. The orbital velocity of the star at distance r is (check!)

$$v(r) = \frac{2\pi r}{P} = \frac{2\pi r}{\sqrt{4\pi^2 r^3/(G(M(r)+m_*))}} \approx \sqrt{\frac{GM(r)}{r}}. \tag{1}$$

where we used Kepler's third law and assumed that the total mass inside the star's orbit is much larger than the mass of the star, $M(r) >> m_*$.

The density of stars is seen to fall off rapidly away from the center of the galaxy. Observations indicate that the stellar density decreases as $1/r^{3.5}$. Therefore, for stars in the outer parts of the galactic disc, we may consider the amount of mass inside the orbit to be the total mass M of the galaxy (since there is not much more material outside the star's orbit which can contribute to the total mass), that is to say $M(r) \to M$ asymptotically for large values of r. In this case expression (1) above can be written as

$$v(r) = \sqrt{\frac{GM}{r}}.$$

Thus, we expect the orbital velocity of stars in the outer parts of the galaxy to fall off as $1/\sqrt{r}$ with the distance r from the galactic center.

By measuring the Doppler effect, we can estimate the velocity of stars orbiting a galaxy at different distances r from the center. A huge number of observations show that the *galactic rotation curve*, the curve showing the orbital velocity as a function of distance r, is almost flat for large r for a large number of galaxies. Instead of falling off as $v \propto 1/\sqrt{r}$, the orbital velocity turns out not to decrease with distance (see figure 2). This came as a big surprise when it was first discovered. There must be something wrong about the assumptions made above. The main assumption made in our derivation was that the density of stars traces the mass density in the galaxy. Using the fact that the density of stars falls of rapidly for large r, we also assumed the total mass density to fall off similarly. This is true if the only constituents of the galaxy were stars. However, if there are other objects in the galaxy which do not emit light, which we cannot see, and which has a different distribution of mass than the stars, the assumptions leading to the $\propto 1/\sqrt{r}$ relation does not hold. One could explain this discrepancy between theory and data if there was an additional invisible matter component, *dark matter*.

2 Modeling the mass density field of a galaxy

Assuming that there is indeed an unknown matter component which has a different density profile $\rho(r)$ than the stars, we could make an attempt to find out how this dark matter is distributed in the galaxy. How can we map the

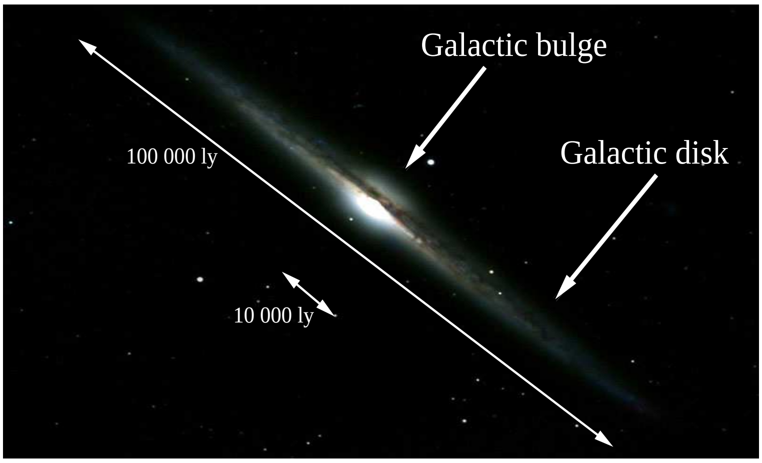

Figure 1: Dimensions of a typical galaxy

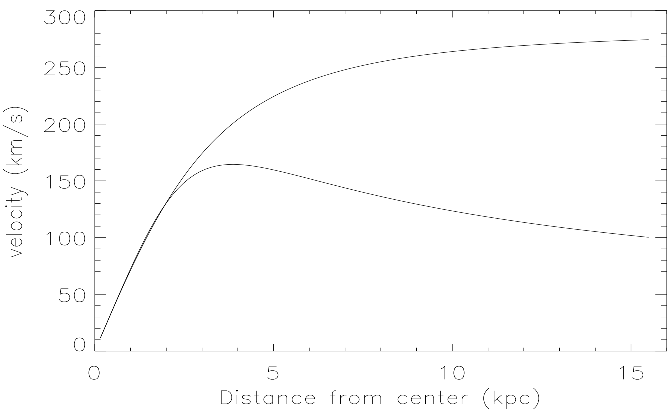

Figure 2: Models of galactic rotation curves. The lower curve is the curve expected from Kepler's laws (taking into account that M(r) is a function of r for lower radii), the upper curve is a model for the observed velocity curve.

matter distribution of invisible matter? We can simply look at its gravitational effect on visible matter. We have already seen traces of such an effect: the invisible matter changes the rotation curve of stars in the galaxies. Is there a way to use the rotation curve $v(r)$ to estimate the density profile $\rho(r)$ of the dark matter?

In the lack of better models, we will assume the distribution of dark matter to be spherically symmetric about the center of the galaxy. Thus, we assume that the density can be written as a function of distance r to the center only. We know that the total mass dM of a spherical shell of infinitesimal thickness dr at a distance r from the center of the galaxy can be written as

$$dM = 4\pi r^2 \rho(r) dr.$$

The surface of a spherical shell at distance r is $4\pi r^2$, the volume of the same shell of thickness dr is $4\pi r^2 dr$. Multiplying with the density $\rho(r)$ we obtain the total mass of the shell given in the previous expression. We now look back at equation (1), write it in terms of $M(r)$ and take the derivative of $M(r)$ with respect to r

$$\frac{dM}{dr} = \frac{v(r)^2}{G}.$$

Here we used the fact that $v(r)$ (taken from observations) seems independent of r such that $dv/dr \approx 0$ for large distances from the center. This is strictly not a necessary assumption, for any power law in the velocity $v(r) \propto r^n$ (where n is an arbitrary index) this expression holds up to a constant factor (check by taking the derivative of $M(r)$ setting $v(r) \propto r^n$). Thus, the following expressions will be valid for more general forms of the velocity $v(r)$ and is therefore also valid for more central regions.

We now have two equations for dM/dr. Setting these two expressions equal, we obtain

$$\rho(r) = \frac{v(r)^2}{4\pi G r^2}. \tag{2}$$

This is a simple expression for the matter density in the galaxy at distance r from the center, expressed only in terms of the rotational velocity $v(r)$. Note that for spherical symmetry, this expression holds also for small values of r. One could think that for stars close to the center, the matter outside the star's orbit would also contribute to the gravitational forces. However, it can be shown that the gravitational forces from a spherical shell add to zero everywhere inside this shell. Thus, simply by a set of Doppler measurements of orbital velocities at different distances r in the galaxy we are able to obtain a map of the matter distribution in terms of the density profile $\rho(r)$.

Recall that observations have shown the rotation curve $v(r)$ to be almost flat, i.e. independent of r, at large distances from the center. Looking at equation (2) this means that the total density in the galaxy falls of like $1/r^2$. Recall also that observations have shown the density of stars to fall off as $1/r^{3.5}$. Thus, the dark matter density falls of much more slowly than the density of visible matter. The dark matter is not concentrated in the center to the same degree as visible matter, it is distributed more evenly throughout the galaxy. Moreover, the density $\rho(r)$ which we obtain by this method is the total density, i.e.

$$\rho(r) = \rho(r)^{\mathrm{LM}} + \rho(r)^{DM},$$

the sum of the density due to luminous matter (LM) and the density due to dark matter (DM). Since the density of luminous matter falls off much more rapidly $\rho(r)^{\mathrm{LM}} \propto r^{-3.5}$ than the dark matter, the outer parts of the galaxy must be dominated by dark matter.

What happens to the mass density as we approach the center? Doesn't it diverge using $\rho(r) \propto r^{-2}$? Actually, it turns out that the rotation curve $v(r) \propto r$ close to the center. Looking at equation (2) we see that this implies a constant density in the central regions of the galaxy. A density profile which fits the observed density well over most distances r is given by

$$\rho(r) = \frac{\rho_0}{1 + (r/R)^2}, \tag{3}$$

where ρ_0 and R are constants which are estimated from data and which vary from galaxy to galaxy. For small radii, $r << R$ we obtain $\rho = \rho_0 =$ constant. For large radii $r >> R$ we get back $\rho(r) \propto r^{-2}$.

Before you proceed, check that you now understand well why we think that dark matter must exist! Can you imagine other possible explanations of the strange galactic rotation curves without including dark matter?

3 What is dark matter?

Possible candidates to dark matter:

- planets and asteroids?
- brown dwarf stars?
- something else?

From our own solar system, it seems that the total matter is dominated by the Sun, not the planets. The total mass of the planets only make up about one part in 1000 of the total mass of the solar system. If this is the normal ratio, and we have no reason to believe otherwise, then the planets can only explain a tiny part of the invisible matter. Brown dwarf stars (more about these in later lectures) are stars which had too little mass to start nuclear reactions. They emit thermal radiation, but their temperature is low and they are therefore almost invisible. Observations of brown dwarfs in our neighborhood indicates that the number density is not large enough to fully explain the galactic rotation curves.

We are left with the last option, 'something else'. Actually, different kinds of observations in other areas of astrophysics (we will come back to this in the lectures on cosmology) indicate that the dark matter must be *non-baryonic matter*. Non-baryonic matter is matter which does not (or only very weakly) interact with normal visible matter in any other way than through gravitational interactions. From particle physics, we learn that the particle of light, the photon, is always created as a result of electromagnetic interactions. Non-baryonic matter does not take part in electromagnetic interactions (or only very weakly), only gravitational interactions, and can therefore not emit or absorb photons. Theoretical particle physics has predicted the existence of such non-baryonic matter for decades but it has been impossible to make any direct detections in the laboratory since these particles hardly interact with normal matter. We can only see them through their gravitational interaction on huge structures in the universe, such as galaxies. This is one example of how one can use astrophysics, the science of the largest structures in the universe, to study particle physics, the science of the smallest particles in the universe.

Dark matter is usually divided into two groups,

1. warm dark matter (WDM): light particles with high velocities ($v \approx c$)
2. cold dark matter (CDM): massive particles with low velocities ($v << c$)

One candidate to WDM are the *neutrinos* although these actually belong to baryonic matter. Neutrinos are very light particles which are associated with the electron and other elementary particles. When an electron is created in a particle collision, a neutrino is normally created in the same collision. Until a few years ago, neutrinos were thought not to have mass. Only some recent experiments have detected that they have a small but non-zero mass. Neutrinos, even if they are baryons, react only weakly with other particles and are therefore difficult to detect. One has been able to show that neutrinos do not make an important contribution to the total mass of galaxies. Nowadays, the most popular theories for dark matter are mostly theories based on CDM. Many different candidates for CDM exist in theoretical particle physics, but so far one has not been able to identify which particle might be responsible for the dark matter in galaxies.

Dark matter has been seen in many other types of observations as well. For instance by observing the orbits of galaxies about a common center of mass in clusters of galaxies, a similar effect has been seen: the orbits cannot be explained by including only the visible matter. Traces of dark matter has also been seen through observations of gravitational lenses (which we will come back to later) as well as other observations in cosmology.

4 Problems

Problem 1 (45 min. - 1 hour) Two galaxies with similar sizes orbit a common center of mass. Their distance from us has been estimated to 220 Mpc (one parsec=3.26 light years, 1 Mpc=10^6 parsecs). Their angular separation on the sky has been measured to $3.1'$. Their velocity with respect to the center of mass has been estimated to $v = 100 km/s$ for both galaxies, one approaching us the other receding. Assume circular orbits. Assume that the velocities of the galaxies only have a radial component such that the given velocity is the full velocity of the galaxies.

1. What is the mass of the galaxies? (**hint:** here you need to go back to the two-body problem. First calculate the radius of the orbit and then

use equations from the lectures on celestial mechanics. You will need to play a little with the equations.)

2. The size of the galaxies indicate that they contain roughly the same number of stars as the Milky Way, about 2×10^{11}. The average mass of a star in these two galaxies equals the mass of the Sun. What is the total mass of one of the galaxies counting only the mass of the stars?

3. What is the ratio of dark matter to luminous matter in these galaxies? This is an idealize example, but the result gives you the real average ratio of dark to luminous matter observed in the universe.

Problem 2 (90 min. - 2 hours) In the following link I have put three files with simulated (idealized) data taken from three galaxies:

http://folk.uio.no/frodekh/AST1100/lecture4/

Each file contains two columns, the first column is the position where the observation is made given as the angular distance (in arcseconds) from the center of the galaxy. These data are observations of the so-called 21 cm line. Neutral hydrogen emits radiation with wavelength 21.2 cm from a so-called forbidden transition in the atom. Radiation at this wavelength indicates the presence of neutral hydrogen. Galaxies usually contain huge clouds of neutral hydrogen. Measurements of the rotation curves of galaxies are usually made measuring the Doppler effect on this line at different distances from the center. The second column in these files is just that, the received wavelength of the 21.2 cm radiation. Again you need to use the Doppler formula to translate these wavelengths into radial velocities.

The three galaxies are estimated to be at distances 32, 4 and 12 Mpc. The total velocity of the galaxies has been measured to be 120, -75 and 442 km/s (positive velocity for galaxy moving away from us).

1. Make a plot of the rotation curves of these galaxies, plot distance in kpc and velocity in km/s.

2. Make a plot of the density profile of the galaxies (assuming that equation (2) is valid for all distances), again plot the distance in kpc and the density in solar masses per parsec^3.

3. Finally, assume that the density profile of these galaxies roughly follow equation (3). Find ρ_0 and R for these three galaxies (in the units you used for plotting). **hints:** Looking at the expression for the density, it is easy to read ρ_0 off directly from the plot of the density profile. Having ρ_0 you can obtain R by trial and error, overplotting the density profile equation (3) for different R on top of your profile obtained from the data.