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*NOTE: There might be errors in the solution. If you find something which
doesn't look right, please let me know*

Partial solutions to problems: Lecture 4

Problem 1

1. We choose to consider the orbit as seen from one of the galaxies. The radius of the orbit is then $a = \theta \cdot 220Mpc = 200kpc$. Use Newton's form of Kepler's third law and insert $P = 2\pi a/v$ to eliminate the period. Remember that seen from this frame of reference the velocity of the second galaxy is $v = 200km/s$. The result then yields $m = 9.2 \times 10^{11}M_{\odot}$
2. $2 \times 10^{11}M_{\odot}$
3. 3.6

Problem 2

Here is an outline of a Python code which can be used to solve the problem:

```
from scitools.all import *

#GALAXY-CLASS
class galaxy:
    #Constructor
    def __init__(self, name, gal_dist, vel):
        #Declare self-variables/lists
        self.name = name
        self.filename = name + '.txt'
        self.gal_dist = gal_dist
        self.gal_vel = vel
        self.ang_dist = []
        self.rad_vel = []
        self.rel_vel = []
        ...
        ...
        ...

    #Calculations/Call functions
    self.read_file()
    self.dist_observation()
    self.rel_velocity()
    self.rot_curve()
    self.density_profile()
    self.constants()
```

```

#Function that reads data from self.filename, and store in self-lists
#Remember to convert from lists to arrays
def read_file(self):
    ...

#Function that calculates the distance from the center of galaxy to the
#point of observation (emitter)
def dist_observation(self):
    radians = ... #Convert angular distance from arcsec to radians
    self.dist = ... #Use simple trigonometry to calculate the the distance

#Function that calculates the relative velocity (velocity relative to the
#center of the galaxy) for given observed wavelength
def rel_velocity(self):
    self.rad_vel = ... #Use Doppler's formula
    self.rel_vel = self.rad_vel - self.gal_vel

#Function that plots (stores) the rotation curve of the galaxy
def rot_curve(self):
    ... #Plot relative velocity vs distance from center of galaxy

#Function that calculates and plots the density profile of the galaxy
def density_profile(self):
    self.density = ... #Use eq. (5)

    #Convert to solarmass/pc^3
    ...

    #Plot density vs distance
    ...

#Function that determines the constants ro_0 and R in equation (3)(lecturenotes)
def constants(self):
    ro_0 = float(raw_input('...')) #Read ro_0 from screen - ro_0 = ro(r=0)

    #For example - use least-square method to find R
    best_R = ...
    best_delta = ...
    R = linspace(0.01, 20, 100) #Array of R-values from 0.01 to 20, 100 points

    for i in range(100):
        mod_density = ... #Array of model-density-values - use eq. (6) and R[i]
        delta = sum((...)**2) #Compute error

```

```

        if delta < best_delta:    #Check if R[i] is a better value
            ...                    #Update best-variables

#Plot the density profile for the datapoints and the model in the same plot,
#and store in an eps-file
...

#Write ro_0 and best value of R to file - write all galaxy-values in one file
...

#----
#MAIN

#Make galaxy objects
galaxy0 = galaxy('galaxy0', 32.0*10**3, 120)
galaxy1 = galaxy('galaxy1', 4.0*10**3, -75)
galaxy2 = galaxy('galaxy2', 12.0*10**3, 442)

```

As in the problem of the previous lecture, the class approach is the best way to build up the program. In this problem we have one file for each galaxy, and it is therefore natural to make a galaxy class.

To calculate the density profile from observed velocity, use the following formula;

$$\rho(r) = \frac{v(r)^2}{4\pi Gr^2} \quad (0.1)$$

A model which fits the observed density profile well over most distances r is given by;

$$\rho(r) = \frac{\rho_0}{1 + \left(\frac{r}{R}\right)^2} \quad (0.2)$$

We will now show the results for galaxy1:

1. In figure 1 we show the velocity curve
2. In figure 2 we show the density
3. In the same figure we have overplotted the best fit model with $\rho_0 = 0.25M_{\odot}/pc^3$ and $R = 5.5kpc$.

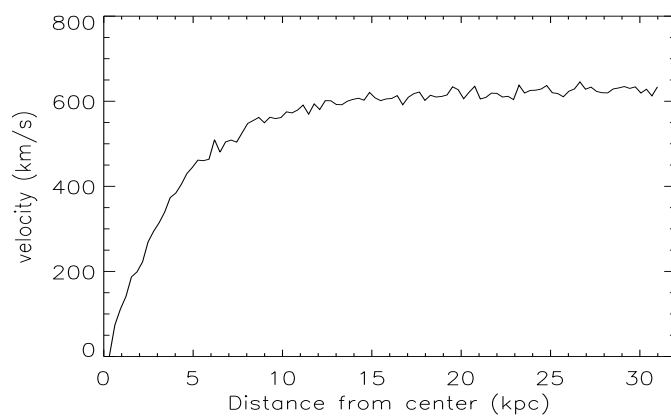


Figure 1: Velocity curve of galaxy1

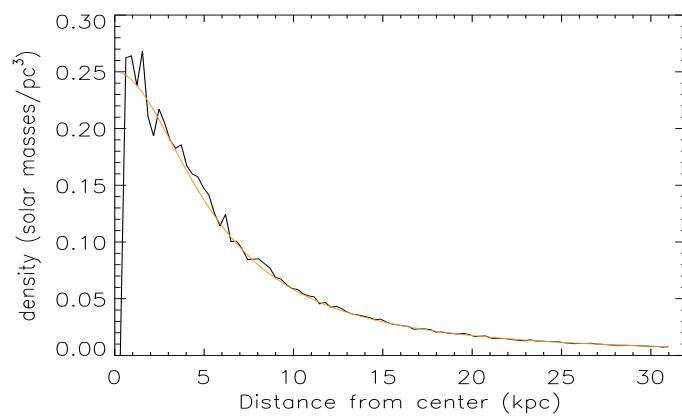


Figure 2: Density curve of galaxy1. The coloured line shows the best fit model curve.