

Figure 1: The galaxies in the cluster. The unit on the axes is in arcminutes.

**The following people have participated in creating these solutions:**  
**Nicolaas E. Groeneboom, Magnus Pedersen Lohne, Karl R. Leikanger**  
*NOTE: There might be errors in the solution. If you find something which does't look right, please let me know*

## Partial solutions to problems: Lecture 5

### Problem 1

The easiest way is to solve this problem using center of mass coordinates. Write the velocities in terms of  $\vec{v}_1^{\text{CM}}$  and  $\vec{v}_2^{\text{CM}}$  and use expressions from the lectures on celestial mechanics to write these in terms of the relative velocity  $\vec{v}$  which is the velocity of one object as observed from the other (remember that this is the frame from which Kepler's law is valid). Finally write the velocity in terms of the period P.

### Problem 2

1.  $32' = 2\pi/60/60/360 = 0.0093$  radians. The radius (for *small radial* values of  $\theta$ ) is given as  $r = \theta d = 0.8 \text{Mpc}$ .
2. We assume all galaxies are like the milky way, and that there are 200 billion stars in each galaxy. Assuming each star to have a mass of  $2 \cdot 10^{30} \text{kg}$ , we end up with an assumed mass of the 100 galaxies:

$$M_{\text{total}} = 100 \text{galaxies} \cdot 2 \cdot 10^{11} \text{stars} \cdot 2 \cdot 10^{30} \text{kg} = 4 \cdot 10^{43} \text{kg}$$

which is the *luminous* mass of the cluster.

3. Result given in the question.
  - (a)  $v_{\text{pec}} = 1.24 \cdot 10^6$  m/s.
  - (b) The plot is shown in figure ??
  - (c)  $m \approx 8.7 \cdot 10^{41} \text{kg}$ , about 2.17 times the estimated luminous mass. This suggests that more than 50% of the galaxy consists of something unknown and non-luminous.
  - (d)

$$\langle \sin^2 i \rangle = \frac{\int_0^{\pi/2} \sin^2 i \, di}{\int_0^{\pi/2} di} = \frac{1}{2}$$

- (e) Using the result from the previous question, we see that the results from 2.4.c should be **doubled**: The "more correct" mass is  $m \approx 1.7 \cdot 10^{42} \text{kg}$ , about 4.4 times the estimated galaxy mass found in 2.2. This means that only about 20% of the total mass of a galaxy is

represented as luminous matter, the rest consists of something else and more sinister, namely “dark matter”.