Proposed solution
UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Exam in AST2110 — The Universe

Day of exam: Friday 8th June 2007

Exam hours: 09.00 – 12.00

This examination paper consists of 4 pages.

Appendices: None

Permitted materials: Rottmann: "Matematisk formelsamling"
Øgrim og Lian: "Størrelser og enheter i fysikk og teknikk"
or Angell og Lian: "Fysiske størrelser og enheter"
Approved calculator
Two A4 pages (can be written on both sides) with your own notes

Make sure that your copy of this examination paper is complete before answering.

Exercise 1

A star is at a distance of 223 pc from the Earth. Assume that it is spherical with radius 5.16×10^9 m and that it has an effective

temperature of 28,000 K. In the following you may find use of the following quantities:
$\sigma = 5.6704 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
$L_\odot = 3.839 \times 10^{26}\,\mathrm{W}$
$M_{\mathrm{Bol},\odot} = 4.74$
$1\,\mathrm{pc} = 3.0856 \times 10^{16}\,\mathrm{m}$
$1\,\mathrm{AU} = 1.495985 \times 10^{11}\,\mathrm{m}$.

Calculate the following quantities:

a) The luminosity of the star (in W and in solar luminosities).

$$L = 4\pi r^2 \sigma T^4 = 1.17 \times 10^{31}\,\mathrm{W} = 30380 L_\odot.$$

b) The absolute bolometric magnitude of the star.

$$M = M_\odot - 2.5 \log_{10}\left(\frac{L}{L_\odot}\right) = -6.47$$

c) The distance modulus of the star.

$m - M = 5 \log_{10} d - 5 = 6.74$

d) The apparent bolometric magnitude of the star.

$m = (m - M) + M = 6.74 - 6.47 = +0.27$

e) The radiant flux at the surface of the star.

$F = \sigma T^4 = 3.5 \times 10^{10}\,\mathrm{W\,m^{-2}}$

f) The radiant flux from the star at the distance of the Earth from the star. Compare this with the solar irradiance on the Earth.

$F_{\mathrm{Earth}} = F(r_*/d_*)^2 = F(5.19\times10^9/(223\times3.0856\times10^{16}))^2 = 2.0 \times 10^{-8}\,\mathrm{W\,m^{-2}}$.

The radiant flux from the Sun on the Earth (the solar constant) is given by $F_{\odot,\mathrm{Earth}} = L_\odot/(4\pi\times(1\,\mathrm{AU})^2) = 1365\,\mathrm{W\,m^{-2}}$**. I.e., the flux from the star on the Earth is** 1.5×10^{-11} **times the flux from the Sun on the Earth.**

Exercise 2

In this exercise we will study the rotation of the Galaxy in the vicinity of the Sun. Jan Oort showed that for stars near the Sun, $v_r \sim Ad\sin 2l$ and $v_t \sim Ad\cos 2l + Bd$, where the Oort constants are

$$A = -\frac{1}{2}\left[\left(\frac{d\Theta}{dR}\right)_{R_0} - \frac{\Theta_0}{R_0}\right],$$

$$B = -\frac{1}{2}\left[\left(\frac{d\Theta}{dR}\right)_{R_0} + \frac{\Theta_0}{R_0}\right].$$

a) Explain what the quantities in these equations are, and draw a figure and give a brief explanation of what the starting point is for reaching these results. A full derivation of the results is not necessary.

See page 909 in the textbook

b) Draw a diagram that sketches the transversal velocity of stars in the vicinity of the Sun as a function of their galactic longitude.

See figure 24.33 in the textbook, the dashed line

c) Assume that all the mass in the Galaxy is in its centre. Use Kepler's 3rd law to find an analytic expression for $d\Theta/dR$ and determine the Oort constants expressed by Θ_0 and R_0.

Kepler's third law says that T^2R^3 is a constant if all the mass is in the centre. Hence, $T \propto R^{3/2}$. The orbital velocity is therefore $\Theta \propto 2\pi R/T \propto R^{-1/2}$. We therefore have that $\Theta = \Theta_0(R_0/R)^{1/2}$. Then $d\Theta/dR = -(1/2)\Theta_0 R_0^{1/2} R^{-3/2}$ and $(d\Theta/dR)_{R_0} = (-1/2)\Theta_0/R_0$. Therefore $A = (3/4)\Theta_0/R_0$ and $B = -(1/4)\Theta_0/R_0$.

d) Assume that the mass density in the Galaxy is $\propto R^{-2}$. Repeat exercise c in this case.

Use the law of centripetal acceleration in circular motion with constant speed, $a = v^2/r$. The centripetal force is the gravitational force, i.e., $ma = mv^2/r = F = GmM(r)/r^2$, where $M(r)$ is the mass inside r. We see that $\Theta(R) = v = [GM(R)/R]^{1/2}$ (this we could also have used in c). Now $M(R) = 4\pi \int_0^R R^2 \rho(R) dR$. In this case, $\rho(R) \propto R^{-2}$ and $M(R) \propto R$. Therefore $\Theta = \text{constant} = \Theta_0$ and $(d\Theta/dR)_{R_0} = 0$. Then $A = (1/2)\Theta_0/R_0$ and $B = -(1/2)\Theta_0/R_0$.

e) Assume that the mass density in the Galaxy is constant with radius out to distances from the centre larger than the distance of the Sun. Repeat exercise c in this case.

If the desnity is constant at ρ_0, $M(R) = (4/3)\pi\rho_0 R^3$ and $\Theta \propto R$, i.e., $\Theta = \Theta_0(R/R_0)$. Then $(d\Theta/dR)_{R_0} = \Theta_0/R_0$, $A = 0$ and $B = -\Theta_0/R_0$.

f) $R_0 = 8\,\mathrm{kpc}$ and $\Theta_0 = 220\,\mathrm{km\,s^{-1}}$. What would the Oort constants be in the three cases from exercises c, d and e? Observations show that the Oort constants are $A = 14.8 \pm 0.8\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -12.4 \pm 0.6\,\mathrm{km\,s^{-1}\,kpc^{-1}}$. What does this say about the distribution of mass in the Galaxy?

In case c, we would have $A = 20.6\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -6.8\,\mathrm{km\,s^{-1}\,kpc^{-1}}$, in case d, we would have $A = 13.8\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -13.8\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and in case e we would have $A = 0.0\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -27.5\,\mathrm{km\,s^{-1}\,kpc^{-1}}$. It is clear that case d gives a much better fit to the data than both case c and e. Therefore we can assume that around the distance of the Sun from the centre of the Galaxy, the density of mass is approximately $\propto R^{-2}$.

Exercise 3

This exercise is given 50% of the weight of exercises 1 and 2. Give short answers to the following questions:

a) What is synodic period?

The orbital period of a planet relative to the Earth

b) What is meant with the Roche limit?

The radius for which tidal forces are similar to the gravitational force. An object coming within the Roche limit will be torn apart by tidal forces.

c) For which classes of galaxies does the surface brightness follow de Vaucouleurs' $r^{1/4}$ law?

For elliptical galaxies, except dwarf ellipticals/spheroidals, also for the bulge in spiral galaxies.

d) Sketch Hubble's tuning fork diagram.

See figure 25.1 in the textbook.

e) What is the relation between the scale factor of the Universe and redshift?

$1 + z = R(t_0)/R(t)$.