

Lecture:

Interaction of radiation with matter-I

Chapter 2 in Leo:

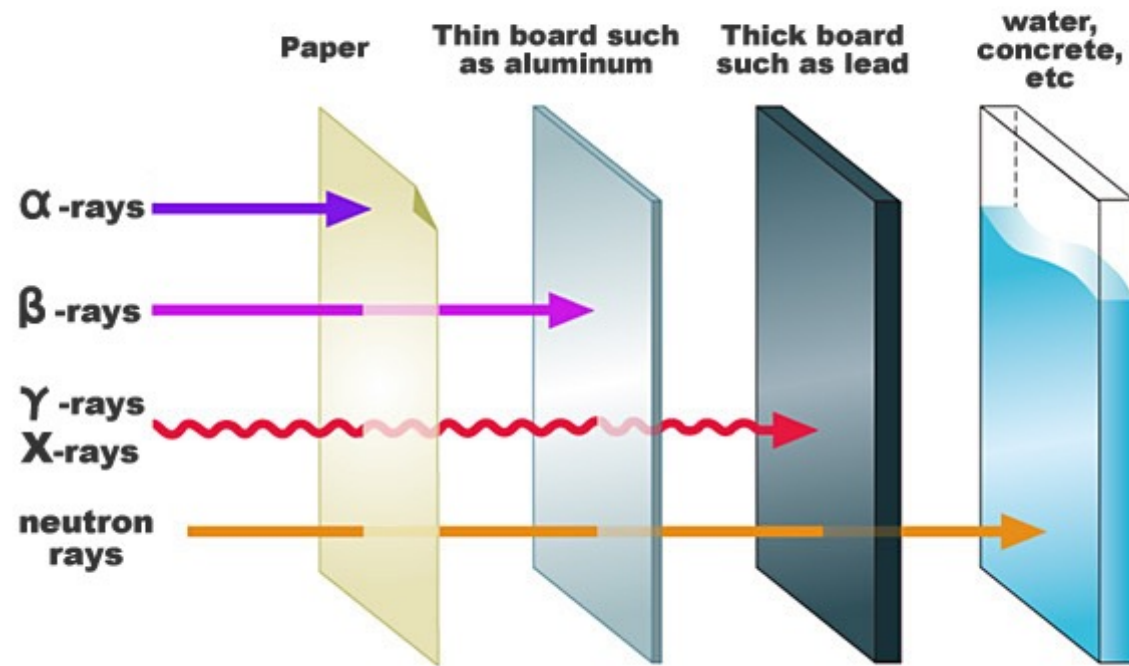
2.1 (self study and partially today)

2.2, 2.4 (today)

2.3 (by Alex & Trine)

Interaction with **matter**

Radiation types and the degree of penetration



Interaction with matter

Charged Particulate Radiations		Uncharged Radiations
Heavy charged particles (characteristic distance $\cong 10^{-5}$ m)	$\Leftarrow$	Neutrons (characteristic length $\cong 10^{-1}$ m)
Fast electrons (characteristic distance $\cong 10^{-3}$ m)	$\Leftarrow$	X-rays and gamma rays (characteristic length $\cong 10^{-1}$ m)

Classification:

See above

Common features:

Particles lose their energy traversing matter

Particles are deflected from their initial direction (if they are not fully absorbed in one instant)

Distinguished features:

Charged particles such as electrons, protons, alphas, lose their energy directly via *the Coulomb interaction mostly* with the electrons of the medium.

Uncharged particles are transparent to the Coulomb force. First, they need a “catastrophic” interaction which alters properties of the incident radiation in a single encounter. For example, gamma rays: *Photoelectric absorption, Compton scattering, pair production, etc.*

Interaction with matter

Charged Particulate Radiations		Uncharged Radiations
Heavy charged particles (characteristic distance $\cong 10^{-5}$ m)	←	Neutrons (characteristic length $\cong 10^{-1}$ m)
Fast electrons (characteristic distance $\cong 10^{-3}$ m)	←	X-rays and gamma rays (characteristic length $\cong 10^{-1}$ m)

Photons:

Interact with the electrons of the detector via PA,CS,PP.

Give their energy to electrons and now these energetic electrons interact with the electrons of the detector. (indirect)

Neutrons:

Interact with the nucleus of the detector material.

Give their energy to protons via scattering or generate alphas through nuclear reactions.

Now these protons or alphas will interact with the electrons of the detector (indirect).

Electrons and heavy charged particles → directly ionizing radiation

Photons, neutron → indirectly ionizing

Interaction with matter

Charged Particulate Radiations		Uncharged Radiations
Heavy charged particles (characteristic distance $\cong 10^{-5}$ m)	←	Neutrons (characteristic length $\cong 10^{-1}$ m)
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Detection efficiency:

Photons & Neutrons

If the interaction does not occur within the detector, these uncharged radiations (e.g., neutrons or gamma rays) can pass completely through the detector volume without revealing the slightest hint that they were ever there.

Therefore, detector efficiencies (sent/detected) are never 100% for detection of gamma rays and neutrons.

Heavy charged particles

How is the situation for heavy-charged particles? (Remember the paper example)

Answer: Heavy charged particles are detected with 100% of efficiency (i.e. fully).

Electrons

Electrons undergo significant multiple scattering and may escape the detector before depositing all of their energy. Consequently, their detection efficiency is generally lower than that for heavy charged particles.

Today

- Heavy charged particles
 - Name some heavy charged particles
 - Energy loss mechanism
 - Stopping power
 - Bethe-Bloch formula
 - Bethe-Bloch plot
 - Applications in NP, HEP, MP
- Electrons
 - Energy loss mechanism
 - Bremsstrahlung radiation
 - Critical energy

muons, pions, protons, alphas,
light nuclei (^{12}C , $Z=6$),
medium heavy nuclei (^{82}Se , $Z=34$),
heavy nuclei (^{238}U , $Z=92$)

HCPs and their detection are important in:

- NP for particle detection and identification and tagging/selecting of reaction channels
- HEP for designing of tracking detectors and particle identification.
- Radiation therapy

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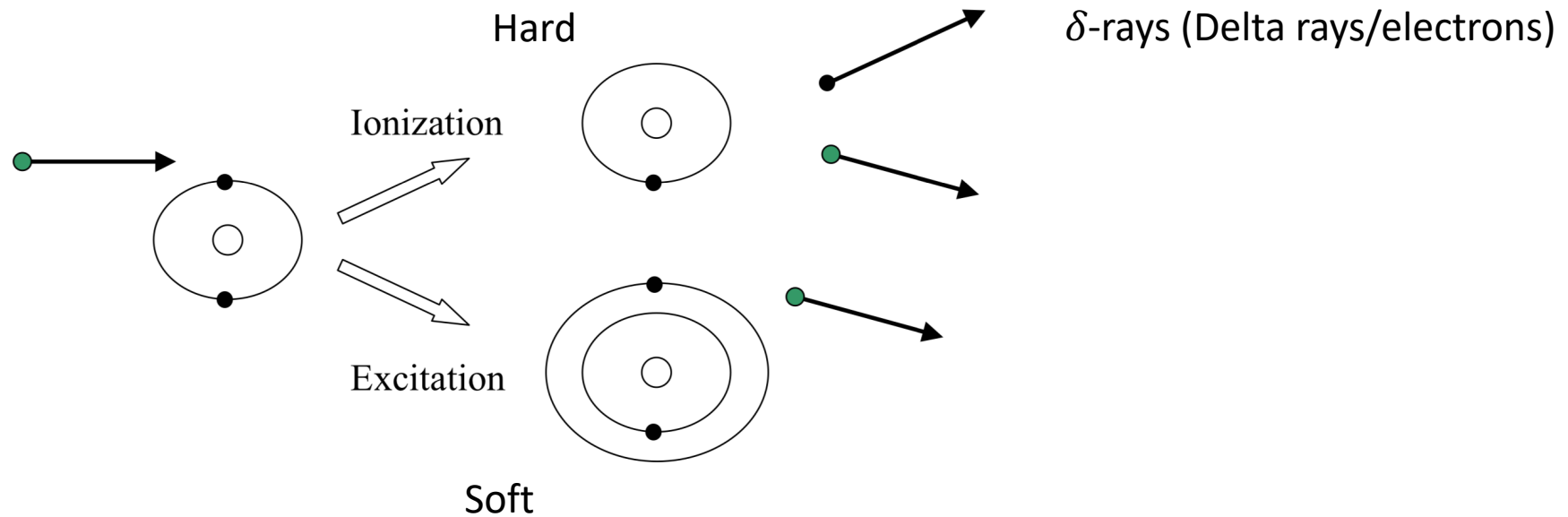
Main processes:

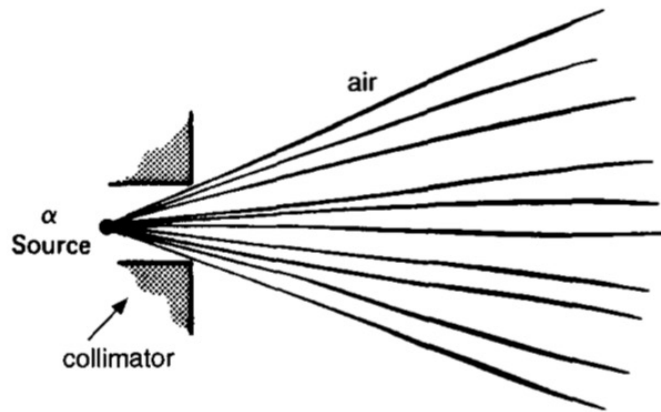
1. Elastic scattering from the nucleus (so-called Rutherford scattering)*
2. **Inelastic collision with the atomic electrons of the material (excitation and ionization)**

**Rutherford scattering does not cause the heavy charged particle to lose its energy significantly.*

Inelastic collision with the atomic electrons of the material (excitation and ionization)

Proton hitting atoms of the detector material





5 MeV alpha in 3 cm of air

$$m_e = 0.511 \text{ MeV}/c^2$$

$$M_{\text{proton}} = 938 \text{ MeV}/c^2$$

$$= M_\alpha \cong 4M_{\text{proton}}$$

- Small amount of energy in each collision
- Many collisions are necessary.
- A 10 MeV proton loses *all* of its energy in only 0.25 mm of copper!

How many collisions?

The simple expression to obtain the maximum energy (Q_{max}) transferred to an electron in a single collision from an alpha or proton is given as :

$$Q_{max} = \frac{4m_e E_i}{M}$$

E_i : Energy of incoming charged particle in MeV

M : Rest mass of the incoming charged particle in MeV

m_e : Rest mass of electron

Compare Q_{max} for $E_\alpha=10$ MeV and $E_p=10$ MeV.

How many collisions are necessary to stop these alpha and protons (E_i/Q_{max})?

Use approximate values of electron, proton and alphas rest masses:

$m_e = 0.5$ MeV,

$M_{proton} = 1000$ MeV

$M_{alpha} = 4000$ MeV

$E_i = 10$ MeV for both proton(E_p) and alpha (E_α) in the problem.

How many collisions?

$$Q_{max} = \frac{4m_e E_i}{M}$$

Use approximate values of electron, proton and alphas rest masses:

$m_e = 0.5 \text{ MeV}$,

$M_{\text{proton}} = 1000 \text{ MeV}$

$M_{\text{alpha}} = 4000 \text{ MeV}$

$E_i = 10 \text{ MeV}$ for both proton(E_p) and alpha (E_α) in the problem.

$$Q_{\max} = \frac{4m_e E_i}{M}$$

E_i: Energy of incoming charged particle in MeV

M: Rest mass of the incoming charged particle in MeV

m_e: Rest mass of electron

Proton:

$$Q_{\max} = (4 \times 0.5 \times 10) / 1000 = 0.02 \text{ MeV} = 20 \text{ keV}$$

$$\text{Collisions} = E_p / Q_{\max} = 10 / 0.02 = 500$$

Alpha

$$Q_{\max} = (4 \times 0.5 \times 10) / 4000 = 0.005 \text{ MeV} = 5 \text{ keV}$$

$$\text{Collisions} = E_{\alpha} / Q_{\max} = 10 / 0.005 = 2000$$

Conclusion: Alpha transfers 4 times less energy per collision and needs ~4 times more collisions.

Main processes:

1. Elastic scattering from the nucleus (so-called Rutherford scattering)
2. **Inelastic collision with the atomic electrons of the material (excitation and ionisation)**
3. Nuclear reactions (less probable, low cross sections)

The following ones are much more prominent for electrons (or highly energetic heavy charged particles) and will be discussed later.

4. *Cherenkov radiation emission (will be explained by Alex + Trine)*
5. *Bremsstrahlung (electrons)*

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 - Name some heavy charged particles
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 - Bethe-Bloch formula
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Now we know that the main energy loss mechanism of heavy charged particles is inelastic collision with the electrons of the material.

Inelastic collisions are statistical in nature, occurring with a certain quantum mechanical probability. However, number of collisions is large (since Q_{max} is small) hence the fluctuations in the total energy loss are small.

We can, therefore, consider an “average energy loss per unit of path length” ($-dE/dx$). This is called also “Stopping power”.

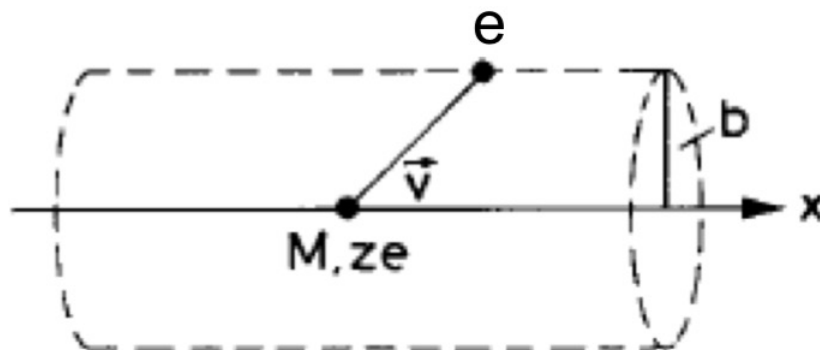
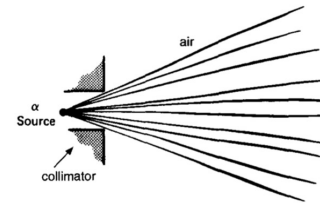
Stopping Power by Bohr's theory (classical case)

Assumptions:

- Electron is free and at rest.
 - ▶ Collision time is smaller than electron orbital time, i.e. incoming energy is sufficiently high. In this case shell electron is not bound but free and at rest.
 - ▶ During the interaction time it moves only slightly.
- Collision between charged particle and electron is head-on
- Charged particle does not deviate from its path after colliding with electron.
 - ▶ $M(\text{particle})$ is much heavier than m_e ($M \gg m_e$)

$E_\alpha = 5 \text{ MeV}$ $E_B(e^-) = 13.6 \text{ eV (H atom)}$ $E_K(e^-) = 3-4 \text{ eV}$
--

$m = 0.511 \text{ MeV}/c^2$ $M = M_{\text{proton}} = 938 \text{ MeV}/c^2$ $M = M_\alpha \cong 4M_{\text{proton}}$



b : Transverse distance (Impact parameter)
 Closest distance between the centroids of the two particles before the collision

1 Classical derivation N. Bohr (1913)

Energy and momentum transfer to a single electron is calculated in terms of transverse distance b

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{\max}}{b_{\min}}.$$

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \bar{v}}$$

$$b_{\min} = \frac{ze^2}{\gamma m_e v^2} \quad b_{\max} = \frac{\gamma v}{\bar{v}}.$$

Good for α particles and heavier nuclei .
Failed for light charged particles like electrons, positrons.

2 Quantum mechanical description Bethe (1930) & Bloch (1933)

The energy transfer is parametrised in terms of momentum transfer rather than the impact parameter. This, of course, is more realistic since the momentum transfer is a measurable quantity whereas the impact parameter is not.

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

Good for heavy-charged particles like alphas, protons, muons, pions, kaons and heavier nuclei
BUT also good for light charged particles like electrons, positrons

Today

- Heavy charged particles
 - Name some heavy charged particles
 - Energy loss mechanism
 - Stopping power
 - Bethe-Bloch formula
 - Bethe-Bloch plot
 - Applications in NP, HEP, MP

**VIF: Very Important
Formula**

Bethe (1930) & Bloch (1933)

At relativistic speeds

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 \right]$$

Bethe (1930) & Bloch (1933)

At relativistic speeds corrections



$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

Bethe (1930) & Bloch (1933)

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

Universal coefficients

$$2\pi N_a r_e^2 m_e c^2 = 0.1535 \text{ MeVcm}^2/\text{g}$$

r_e : classical electron
radius = $2.817 \times 10^{-13} \text{ cm}$

m_e : electron mass

N_a : Avogadro's
number = $6.022 \times 10^{23} \text{ mol}^{-1}$

I : mean excitation potential

Z : atomic number of absorbing
material

A : atomic weight of absorbing material

ρ : density of absorbing material

z : charge of incident particle in
units of e

$\beta = v/c$ of the incident particle

$\gamma = 1/\sqrt{1-\beta^2}$

δ : density correction

C : shell correction

$W_{\max}$: maximum energy transfer in a
single collision.

**Absorber
material**

**Incoming heavy charged
particle**

Bethe (1930) & Bloch (1933)

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

General properties of BB Formula

- Z/A term is nearly constant ($\sim 1/2$) for all absorbing material
- dE/dx is independent of the mass of the incident particle
- dE/dx depends on the charge (z^2) of the incident particle
- If we treat the slowly-varying logarithmic term as constant, dE/dx will be proportional to $1/\beta^2$

Particle with higher charge releases more energy
Fast particle releases less energy

Bethe (1930) & Bloch (1933)

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

density correction

Shell correction

$$W_{\max} \simeq 2m_e c^2 (\beta\gamma)^2$$

$\beta = v/c$

since $m_e \ll M$

I: Mean excitation potential $\sim h\bar{\nu}$

$$\frac{I}{Z} = 12 + \frac{7}{Z} \text{ eV} \quad Z < 13$$

$$\frac{I}{Z} = 9.76 + 58.8 Z^{-1.19} \text{ eV} \quad Z \geq 13$$

$\bar{\nu}$: average orbital frequency of the electrons of the material.

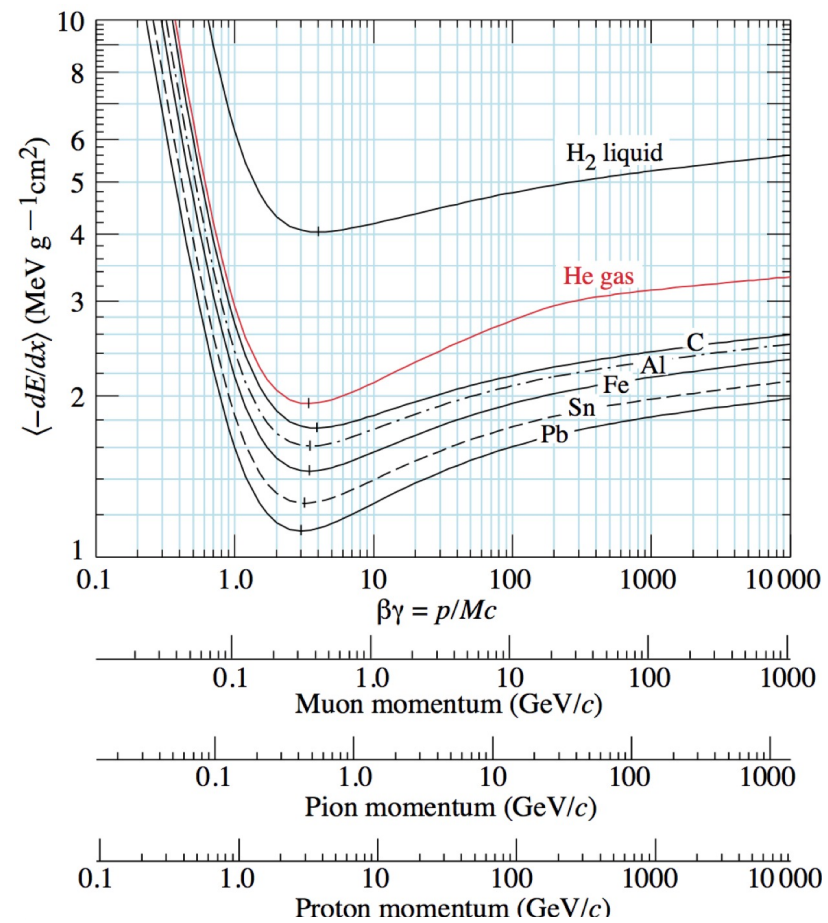
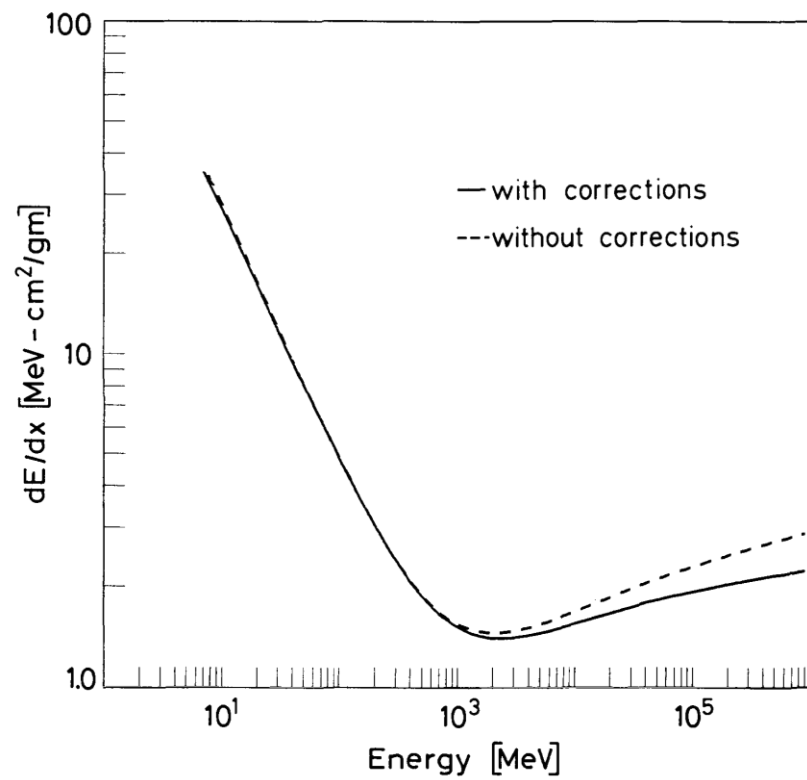
h : Planck constant

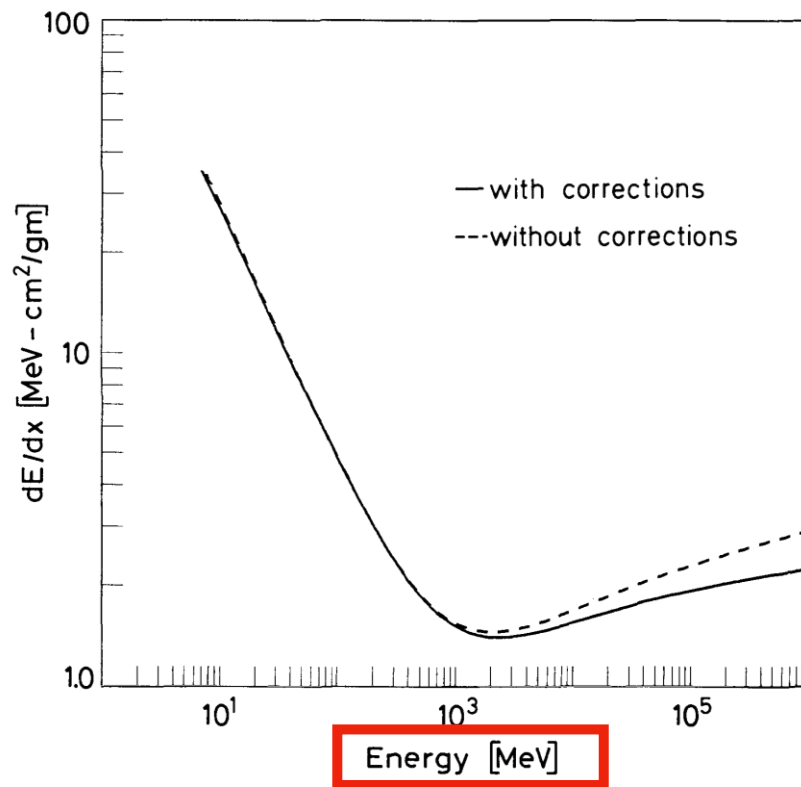
What happens at low and high particle energies?

Shell and density effects, respectively, need to be taken into account. We will see these effects soon.

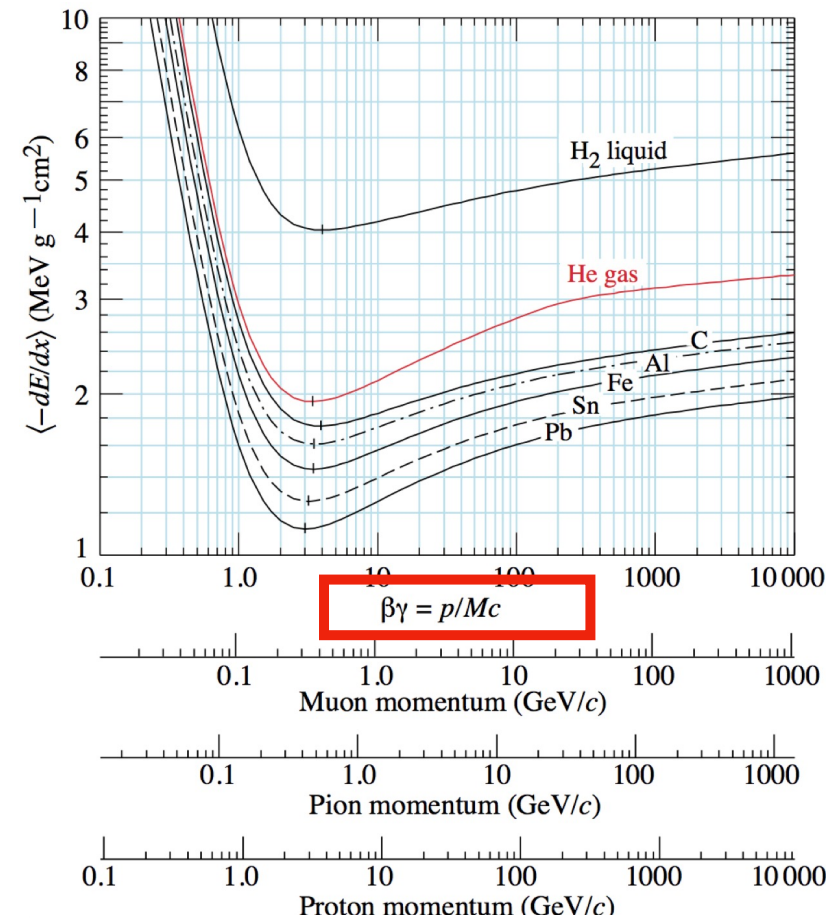
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Nuclear physicists show the x-axis in “Energy”.



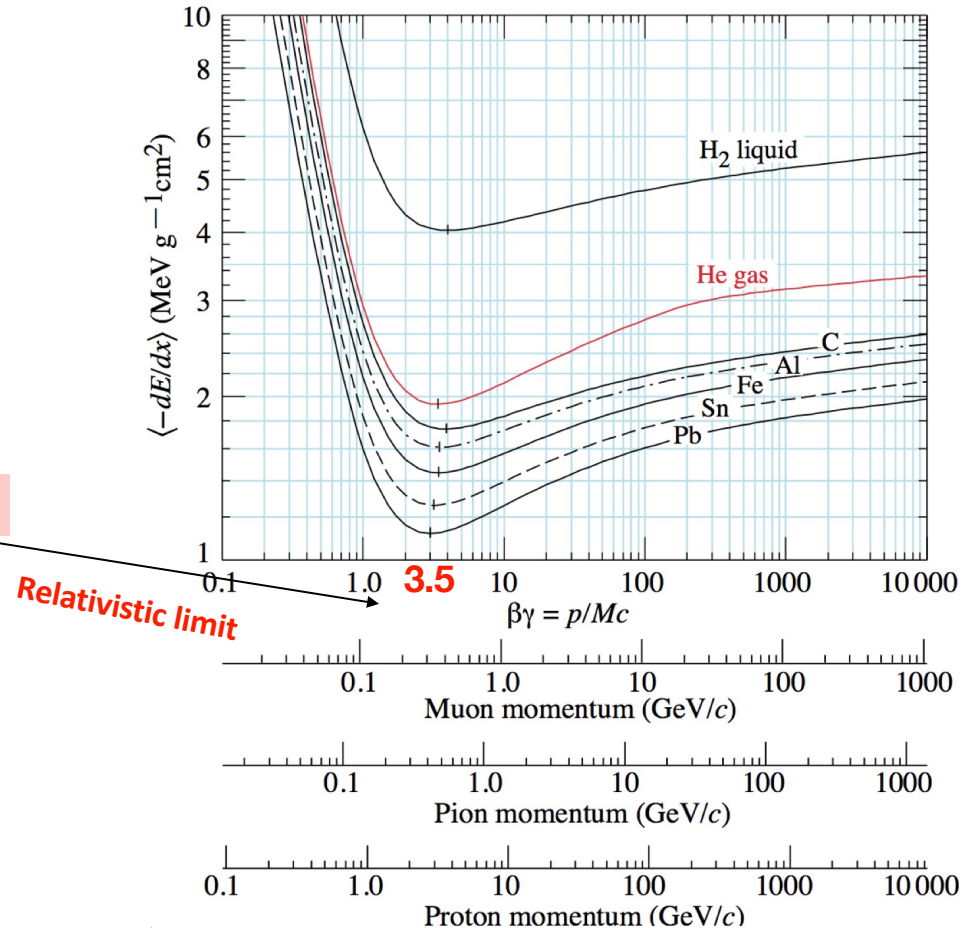
Particle physicists show the x-axis in “ $\beta\gamma$ or momentum”.

Calculate $\beta\gamma$ for different β values of the particle. What do you get?

$$\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

v: Particle velocity

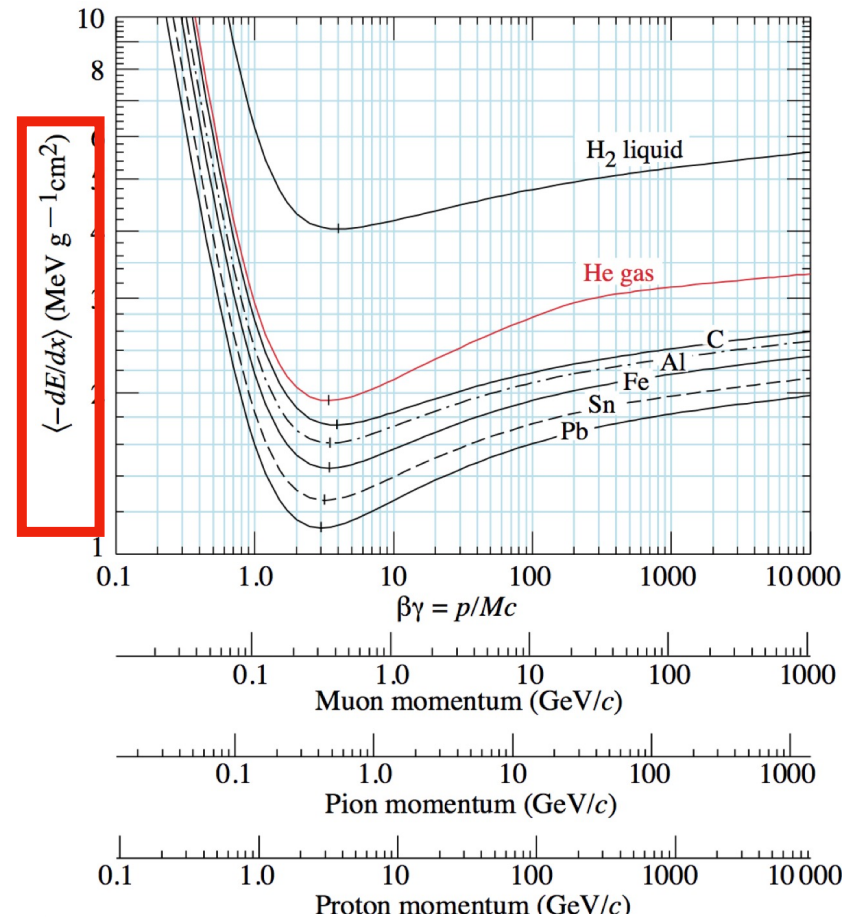
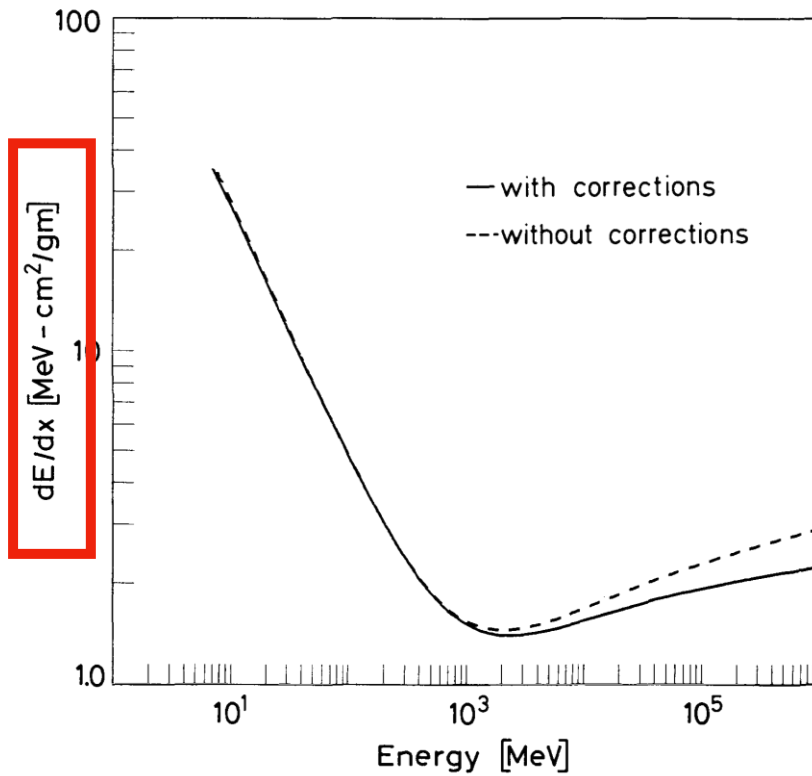
v	β	γ	$\beta\gamma$
10 % c	0,1	1,005	0,101
50 % c	0,5	1,400	0,980
96 % c	0,96	3,571	3,429
99 % c	0,99	7,089	7,018
99,5 % c	0,995	10,013	9,962
99,9 % c	0,999	22,366	22,344
99,99 % c	0,9999	70,712	70,705
99,99999 % c	0,99999999	7071,068	7071,068
100 % c	1	infinite	infinite



$$-\frac{1}{\rho} \frac{dE}{dx}$$

Mass stopping power

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$



What about the y-axis?

Mass Thickness: Why use it instead of distance?

Definition

Mass thickness: $d\varepsilon = \rho dx$

dx : geometric thickness, length (cm)

ρ : density (g/cm^3)

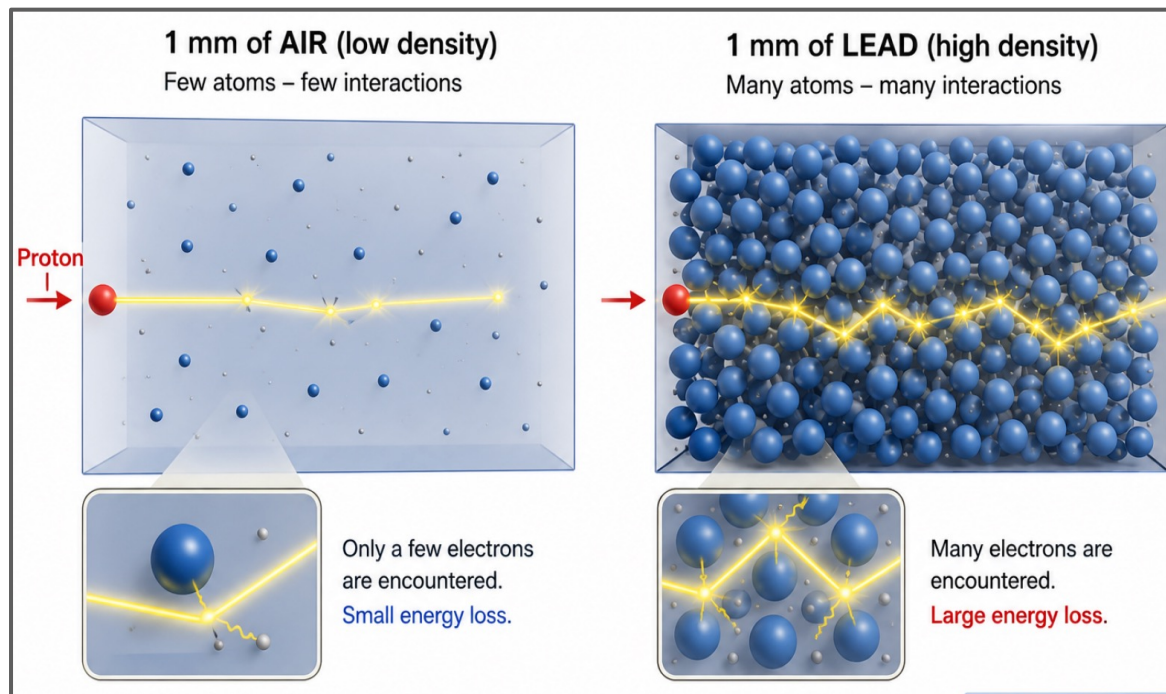
$d\varepsilon$: mass per unit area (g/cm^2 or mg/cm^2)

Key idea

Charged particles lose energy by interacting with electrons, not by travelling a certain distance.

Therefore, they 'see' the amount of material (number of electrons), not the physical thickness.

Same geometric thickness (1 mm)



Mass Stopping Power: Why is it useful?

Allows direct comparison of different materials.

- Removes the effect of density.
- Since $Z/A \approx 1/2$ for most elements, it is nearly material independent.
- $I(Z)$ is in the logarithmic term and thus mass stopping power varies very little with $I(Z)$.

$$-\frac{dE}{d\epsilon} = -\frac{1}{\rho} \frac{dE}{dx} = z^2 \frac{Z}{A} f(B, I)$$

A 10 MeV proton loses approximately the same amount of energy in

- 100 mg/cm² of aluminium
- 100 mg/cm² of copper.

Although the foil thicknesses are different:

- Aluminium: 0.37 mm
- Copper: 0.11 mm

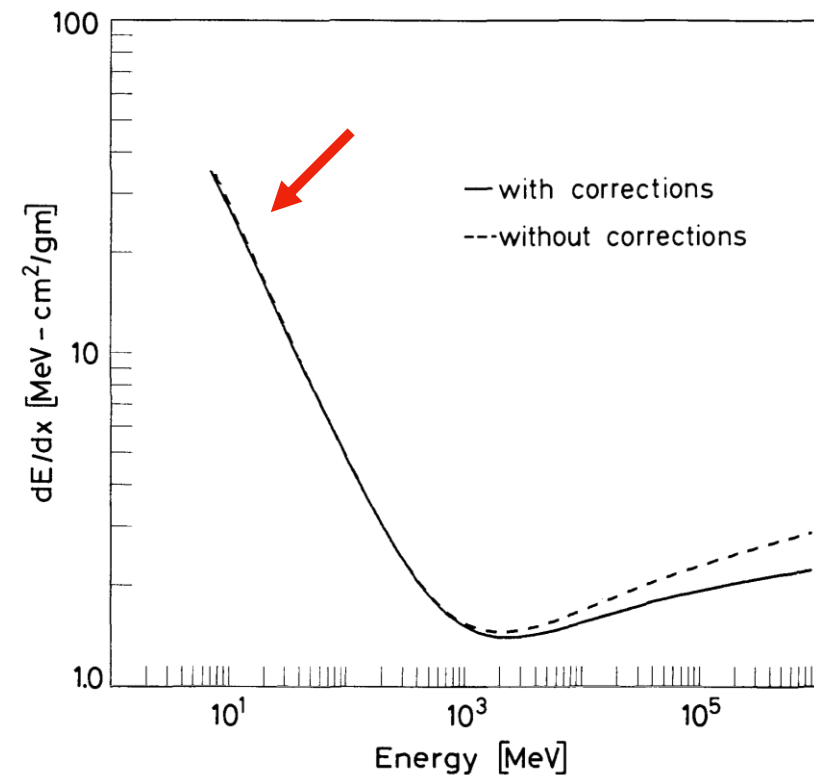
Equal mass thickness $\Rightarrow$ approximately equal energy loss.

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \left(\frac{C}{Z} \right) \right]$$

Shell correction

Shell Correction (at low heavy-particle energies)

The shell correction accounts for effects which arise when the velocity of the incident particle is comparable or smaller than the orbital velocity of the bound electrons. At such energies, the assumption that the electron is stationary with respect to the incident particle is no longer valid and the Bethe-Bloch formula breaks down. The correction is generally small as can be seen in the figure above.



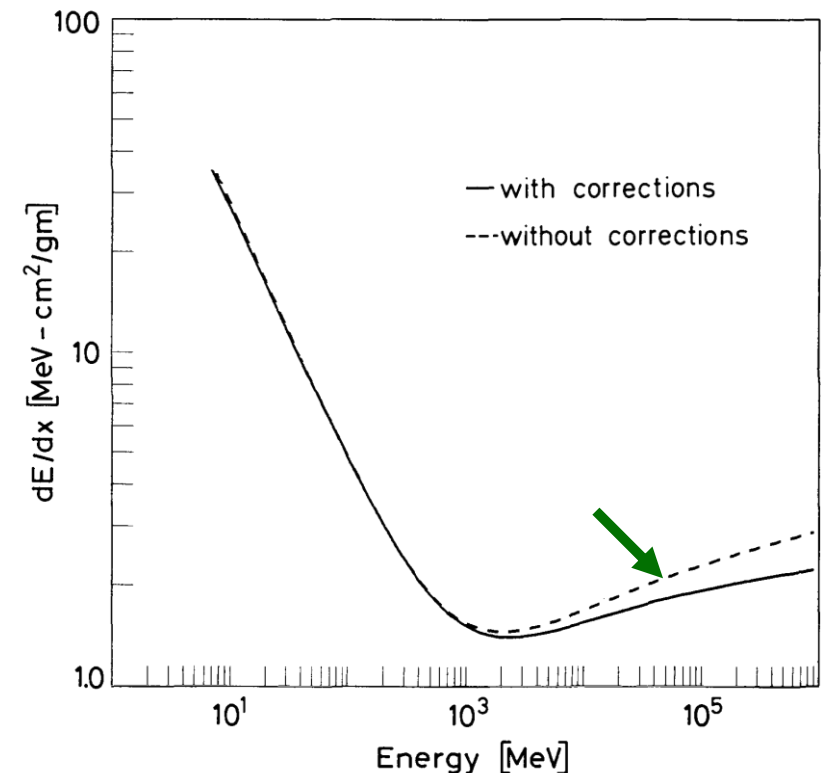
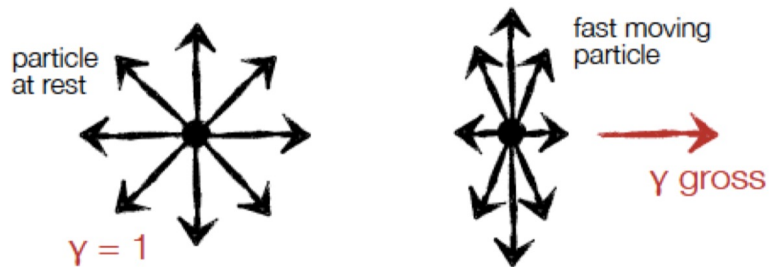
$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 \overset{\text{density correction}}{\delta} - 2 \frac{C}{Z} \right]$$

Density Correction (at high heavy-particle energies, $\gamma \gg 1$)

Density Correction ($\gamma \gg 1$)

- Fast particle polarizes the medium.
- Polarized electrons partially screen the particle's electric field.
- Distant electrons contribute less to ionization.
- Result: stopping power is reduced.

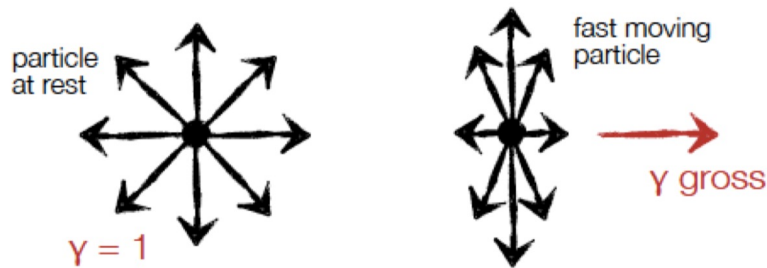
Density correction (δ) accounts for this effect.



Long text for reading later:

- A fast-moving charged particle creates an electric field that interacts with the electrons in the absorber.
- At very high energies, the particle's electric field polarizes the surrounding medium.
- The polarized electrons, produce an opposing electric field that partially screens (shields) the particle's field.
- As a result, distant electrons contribute less to ionization and excitation, reducing the energy loss.
- This reduction is accounted for by the density correction term, δ , in the Bethe–Bloch equation.

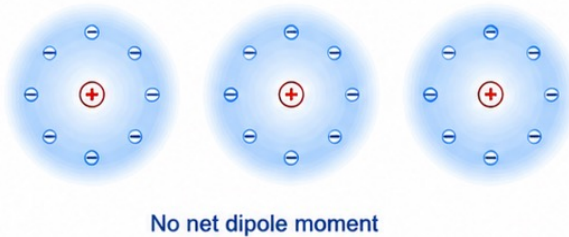
Consequence: Without the density correction, the stopping power would continue to increase at high energies. The density effect suppresses this increase, giving the lower solid curve in the figure.



At very high energies, the electric field of the charged particle polarizes the medium. This polarization partially screens the particle's field, so distant electrons no longer contribute effectively to ionization. Consequently, the stopping power is reduced. This is why the Bethe–Bloch equation includes the density correction term, δ , which becomes important at high energies.

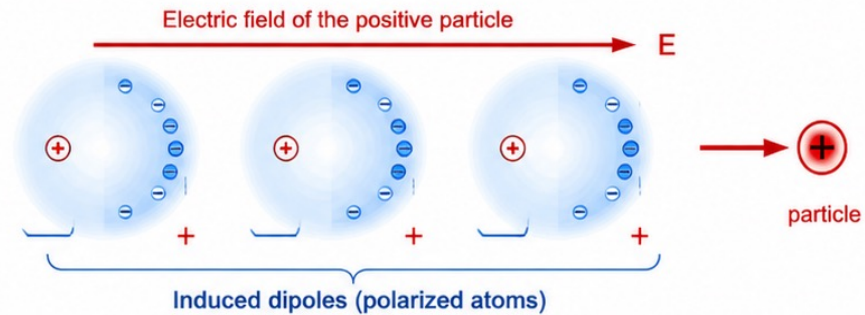
1. Before the particle arrives

Atoms are neutral. The electron clouds are symmetric around the nuclei.



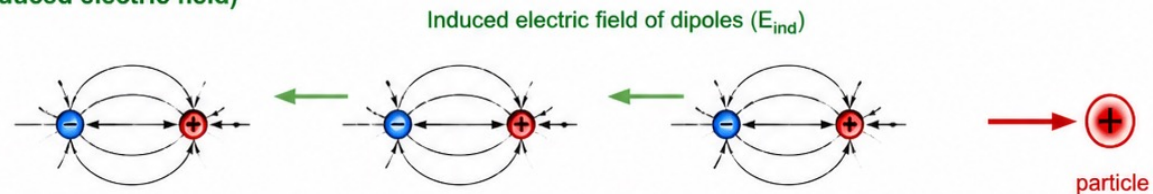
2. A positive charged particle approaches

The electric field of the incoming positive charge pulls the electrons slightly toward it and pushes the nuclei slightly away (nuclei hardly move).



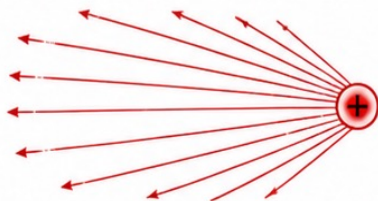
3. Creation of dipole field (induced electric field)

Each polarized atom (dipole) creates its own electric field. On the right side of a dipole (near the particle) the field points to the left (opposite to the particle field).

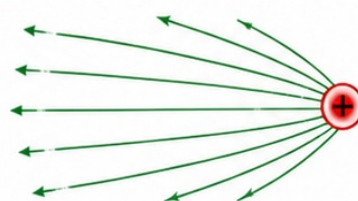


4. Superposition: particle field + induced field → net (reduced) field

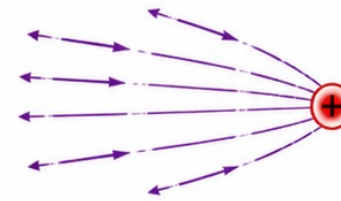
Electric field of the positive particle (E) points away from the particle.



Induced electric field of polarized atoms (E_{ind}) points toward the particle (opposite direction).



Net electric field ($E_{total} = E + E_{ind}$) is weakened (partial cancellation).



At large distances from the track, the net field is much smaller. Therefore distant electrons are much less disturbed and receive less energy.



Physical picture: The positive particle **polarizes** the medium. The induced dipoles create an electric field **opposite** to the particle's field. At large distances the fields partially cancel, so distant electrons feel a **weaker force** and contribute less to the energy loss. This is the essence of the **density effect**.

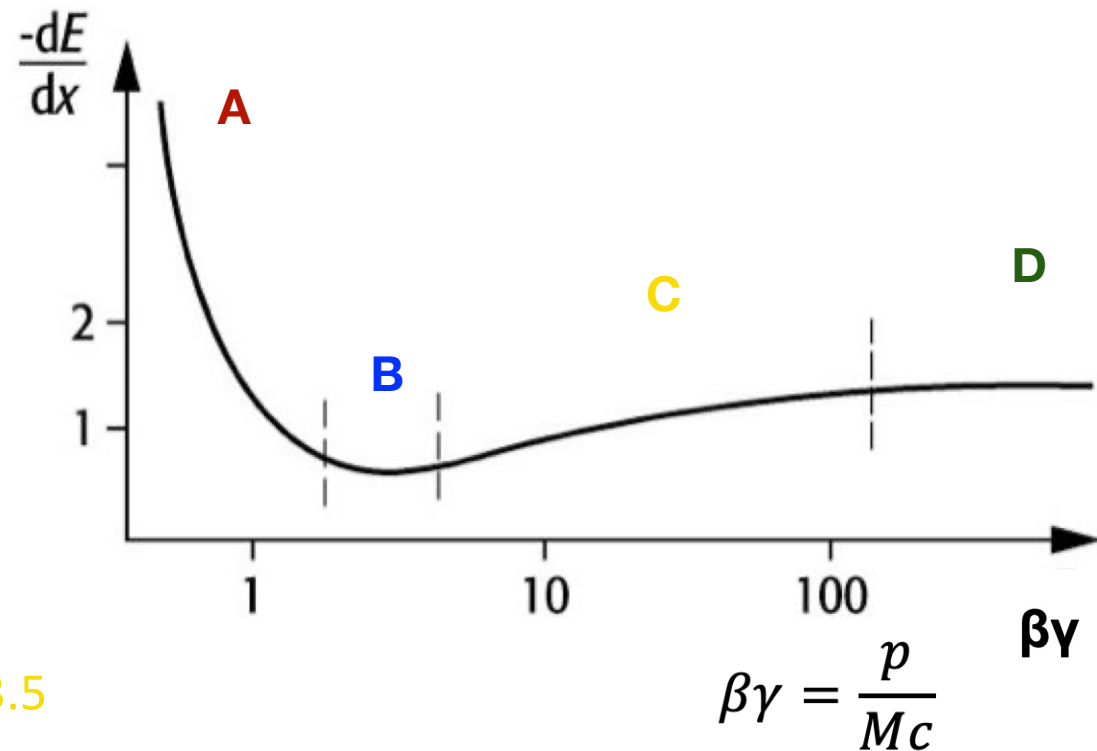
$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

A: Low velocity regime: $\beta\gamma < 3.5$

$$\frac{1}{\beta^2}$$

B: Minimum ionization loss: $\beta\gamma \approx 3.5$

$$\beta \sim 1 \Rightarrow \beta\gamma \approx 3.5$$



C: Relativistic (Logarithmic) rise: $\beta\gamma > 3.5$

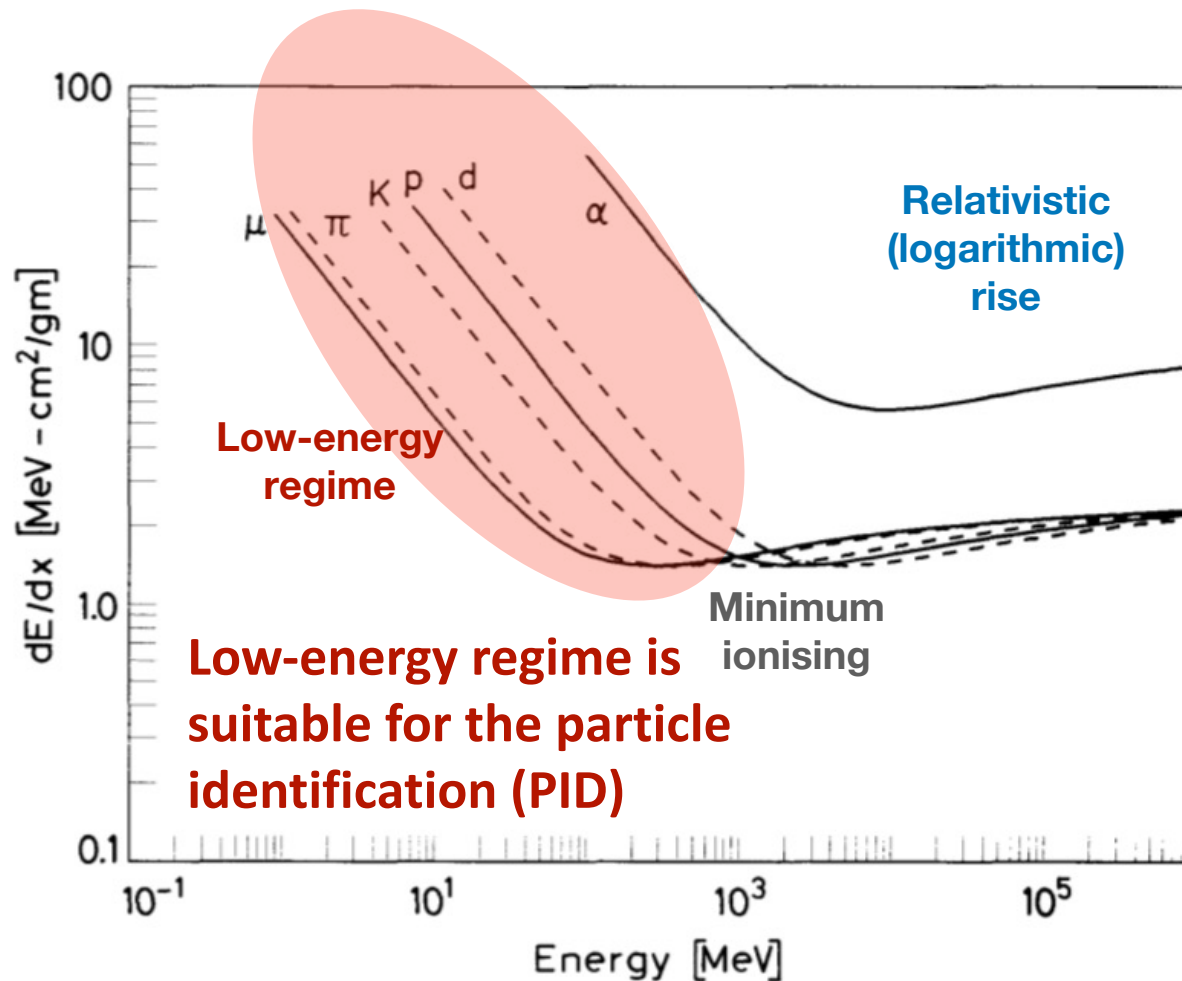
$$\ln \gamma^2 \beta^2$$

D: Saturation region/Density effect: $\beta\gamma \gg 3.5$

Today

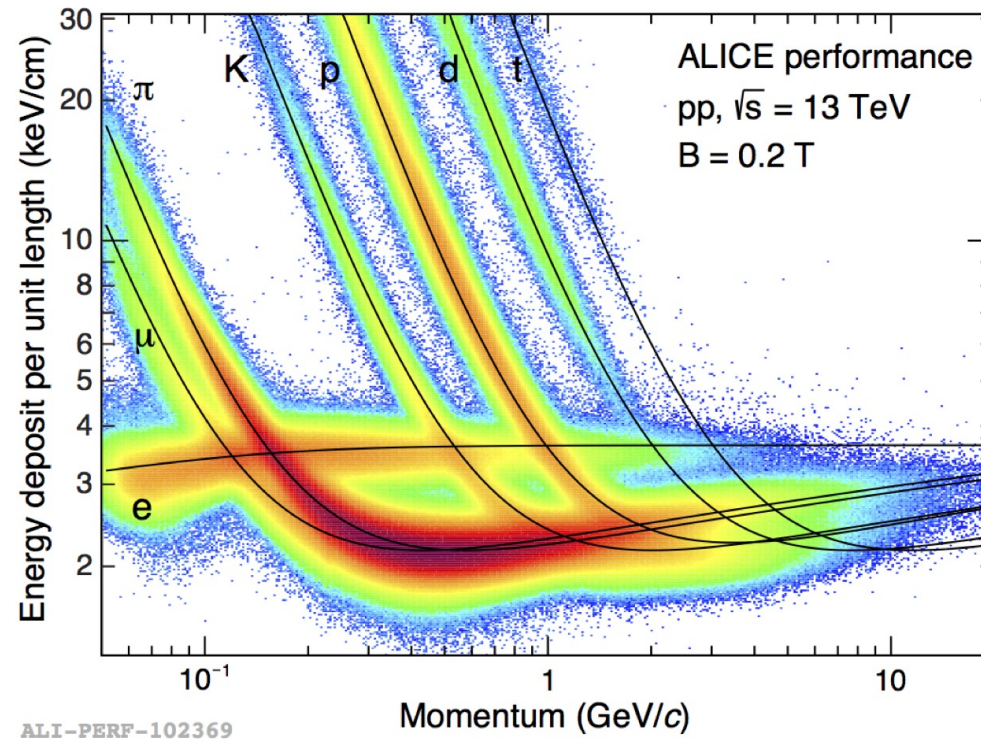
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 - Applications in NP, HEP, MP

Particle identification in HEP



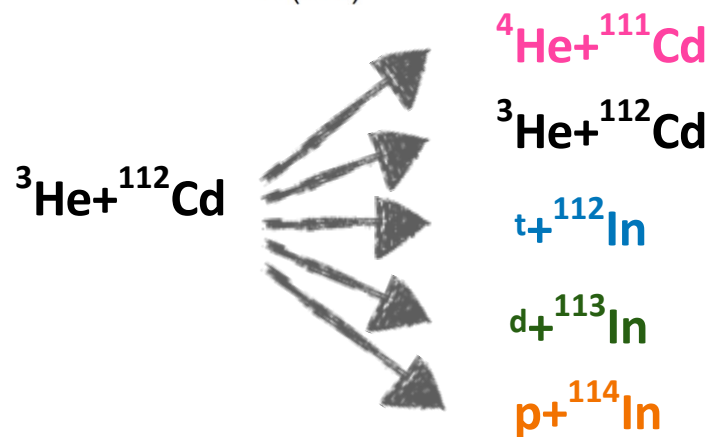
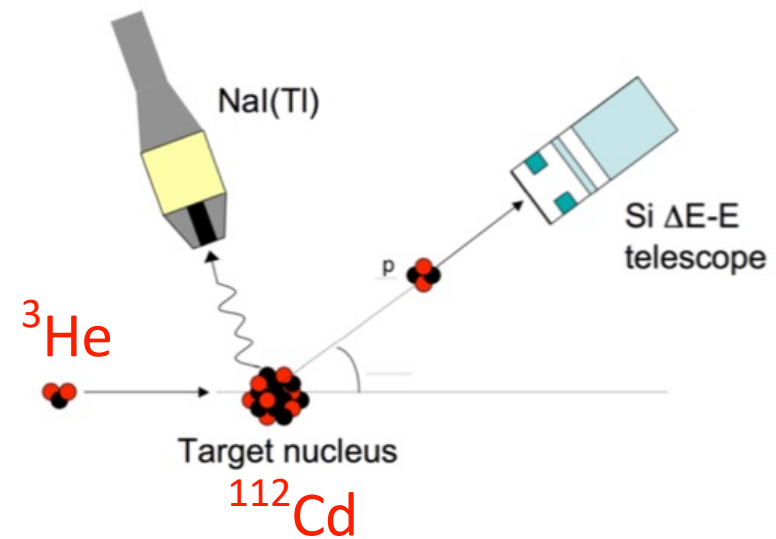
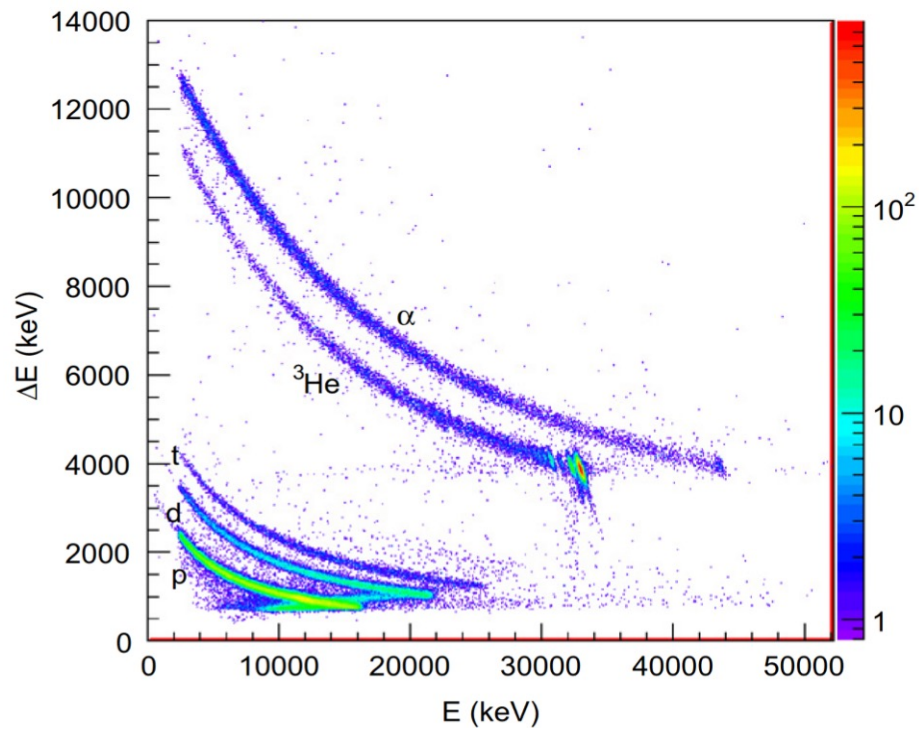
- The **minimum value of dE/dx** is almost the same for all particles of the same charge.
- Relativistic rise is also the same for particles with the same charge.

Particle identification in HEP



ALICE TPC (Time Projection Chamber)

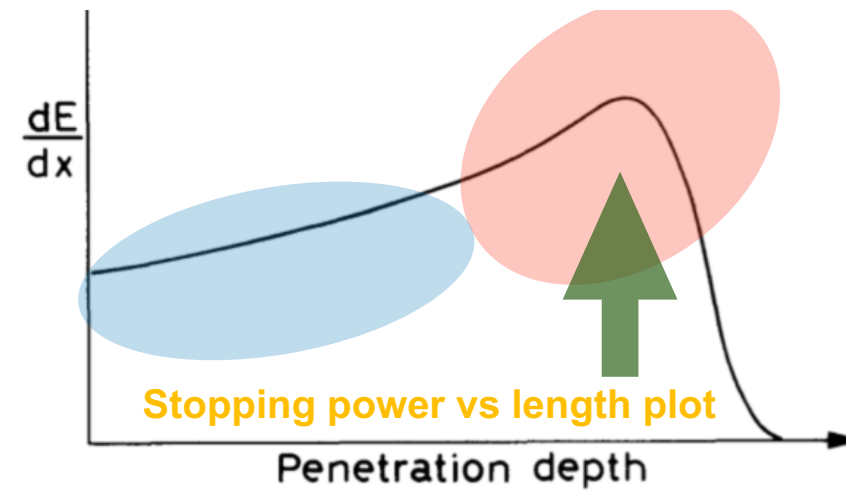
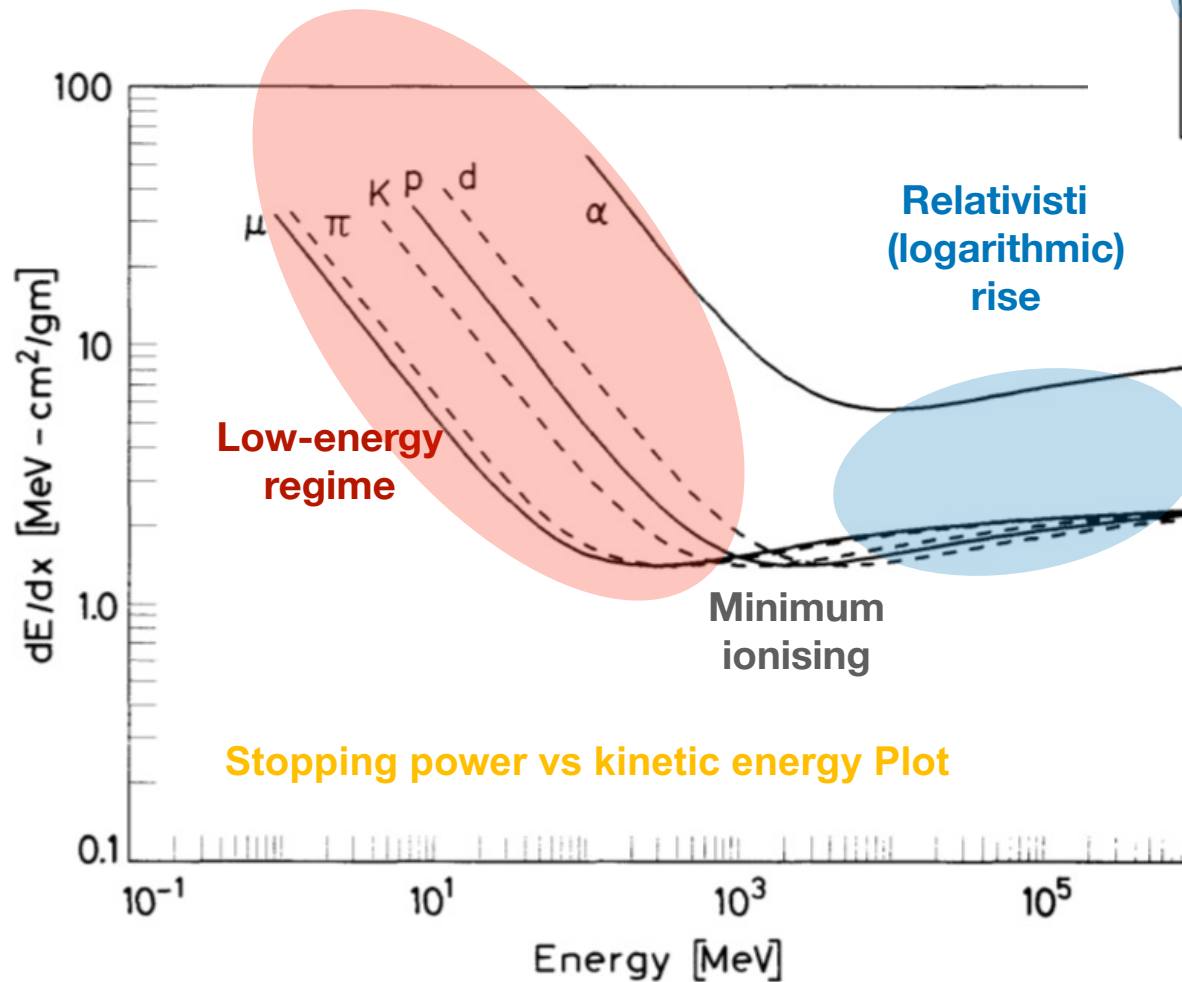
Particle identification and selection of reaction channels in NP



Entrance channel

Exit channels

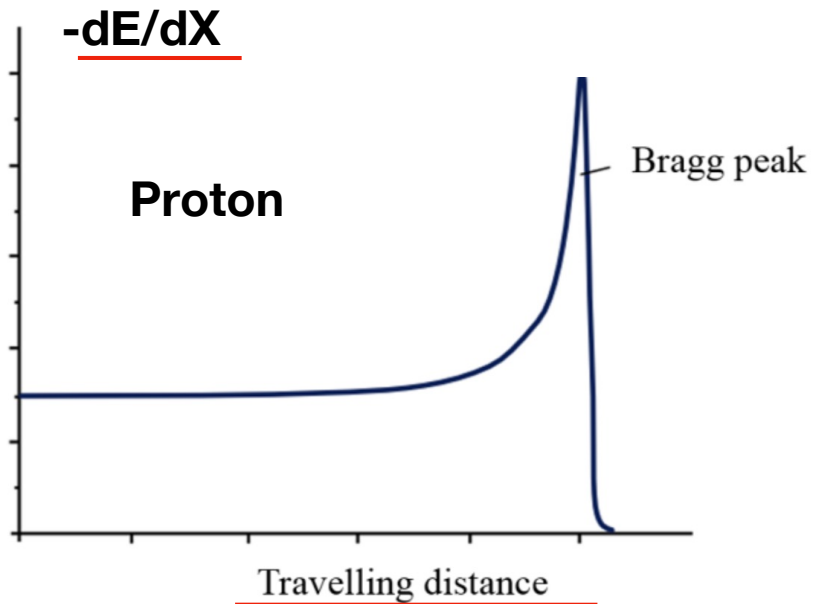
Stopping power in Nuclear Medicine: radiation therapy



Bragg Curve & Bragg peak

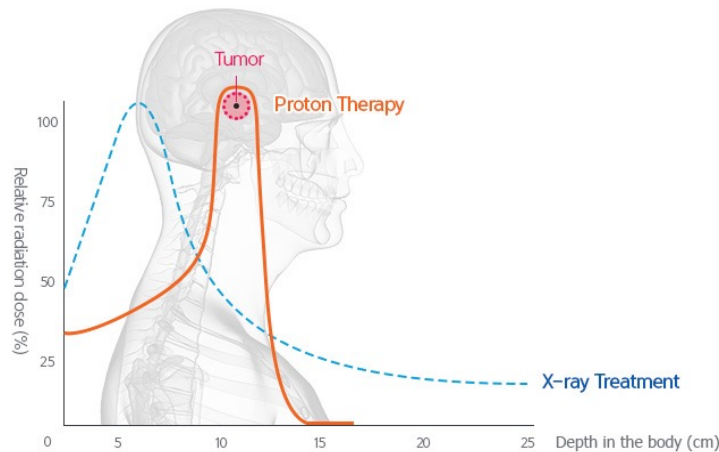
Length = distance of transverse=
penetration depth = traveling
distance

Bragg curve



Energy loss increases as the particle slows down.

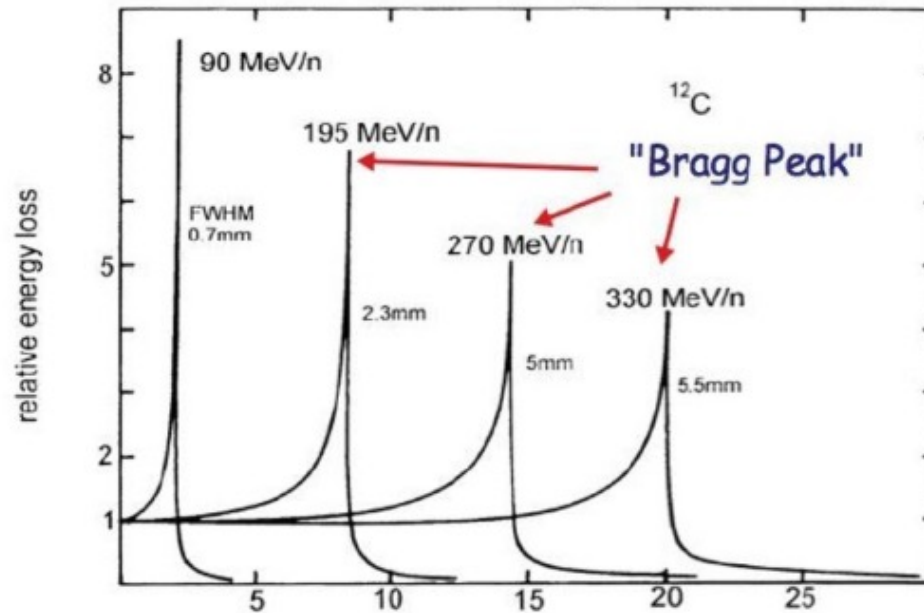
- Maximum energy deposition occurs just before the particle stops (Bragg peak).
- After stopping, no further dose is deposited.



This allows the tumour to receive the highest dose while sparing healthy tissue.

Bragg curve

Ionisation profile for ^{12}C ions in water



Carbon ions have a sharper Bragg peak than protons.

Why isn't the Bragg peak infinitely sharp in general but seems to be sharper for Carbon?

Energy straggling:

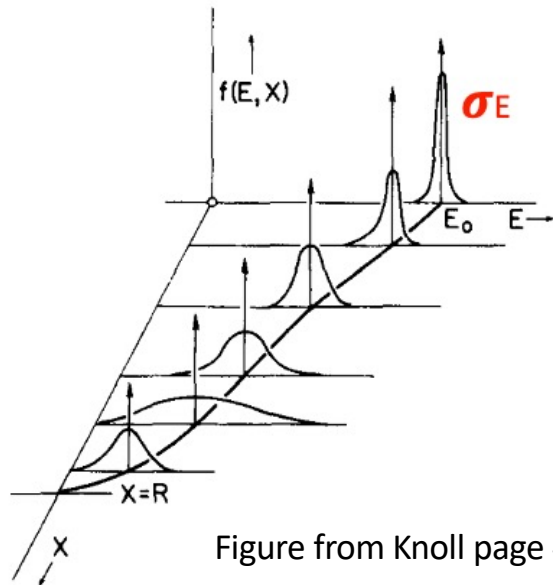


Figure from Knoll page # 33

Why isn't the Bragg peak infinitely sharp?

- Every collision is random.
- Identical particles lose slightly different amounts of energy.

→ Energy straggling
Energy distribution broadens.

→ Range straggling
Range: *length before the particles lose all of their energy*
Particles stop at slightly different depths.

Result:

- Bragg peak becomes broader.
- Heavier ions exhibit less straggling. See the expression below

Range straggling:

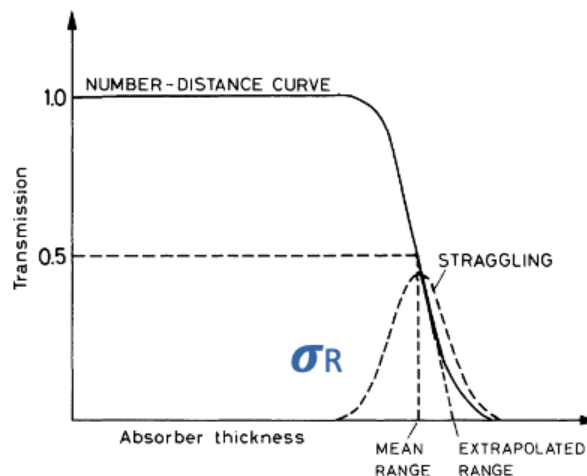


Figure from Leo

$$\sigma_r = \frac{0.012 \cdot r^{0.951}}{\sqrt{A}}$$

r: Range

A: Atomic mass number

A=12 for Carbon while A=1 for proton

σ_r is smaller for Carbon

Today

- Heavy charged particles

Why BB formula?

- Name some heavy charged particles
- Energy loss mechanism
- Stopping power
- Bethe-Bloch formula
- Bethe-Bloch plot
- Applications in NP, HEP, MP

- Electrons

- Energy loss mechanism
- Bremsstrahlung radiation
- Critical energy

Today

- Heavy charged particles

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- Electrons

- Energy loss mechanism
- Bremsstrahlung radiation
- Critical energy

Why BB formula?

It predicts energy loss (stopping power), $-dE/dx$

Many applications:

nuclear physics,
particle physics,
medical physics (proton and ion therapy),
radiation protection,
detector design, thickness
particle identification ($\Delta E-E$ technique)
Monte Carlo simulations such as Geant4.

1- Energy loss by collisions when passing through matter



Bethe-Bloch formula for electrons

$$-\frac{dE}{dx} = 2 \pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2 \frac{C}{Z} \right]$$



where τ is the kinetic energy of particle in units of $m_e c^2$,

$$F(\tau) = 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2r+1) \ln 2}{(\tau+1)^2} \quad \text{for } e^-$$

$$F(\tau) = 2 \ln 2 - \frac{\beta^2}{12} \left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3} \right) \quad \text{for } e^+$$

2- Energy loss by Bremsstrahlung (Radiative loss)

- Charged particle entering electric field emits **Bremsstrahlung radiation**.
- “Breaking” or “deceleration” radiation.
- It is most dominant for electrons than heavy charged particles
- Electrons are deflected by the charge of the nucleus and photons are emitted
- Sudden loss in electron(kinetic) energy causes the creation of photons (energy conservation).

- **Electrons:** energy loss by bremsstrahlung dominates energy loss by ionisation:

$$\left(\frac{dE}{dx}\right)_{\text{bremsstrahlung}}^{\text{electrons}} \gg \left(\frac{dE}{dx}\right)_{\text{ionisation}}^{\text{electrons}}$$

- **Heavier particles:** energy loss by bremsstrahlung is much less important for muons that are 200 times heavier than electrons, and less important still for everything else. Hence energy loss by ionisation dominates bremsstrahlung:

$$\left(\frac{dE}{dx}\right)_{\text{bremsstrahlung}}^{\text{heavier particles}} \ll \left(\frac{dE}{dx}\right)_{\text{ionisation}}^{\text{heavier particles}}$$

Collision loss Radiative loss

$$\frac{dE}{dx} = \left(\frac{dE}{dx} \right)_c + \left(\frac{dE}{dx} \right)_r$$

$$-\left(\frac{dE}{dx} \right)_r = \frac{\overset{\text{★}}{N} \overset{\text{★}}{E} \overset{\text{★}}{Z}(Z+1)e^4}{137 \underset{\text{★}}{m_0^2 c^4}} \left(4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3} \right)$$

Greater E:

More energetic electrons lose energy via bremsstrahlung more

Z²:

Absorbing materials with higher Z cause more bremsstrahlung radiation on electrons

m₀c² (mass)

Heavier mass, less chance of radiative loss. Think about muons, next lightest charged particle. (40000 times less energy loss than electrons!)

Critical Energy: E_c

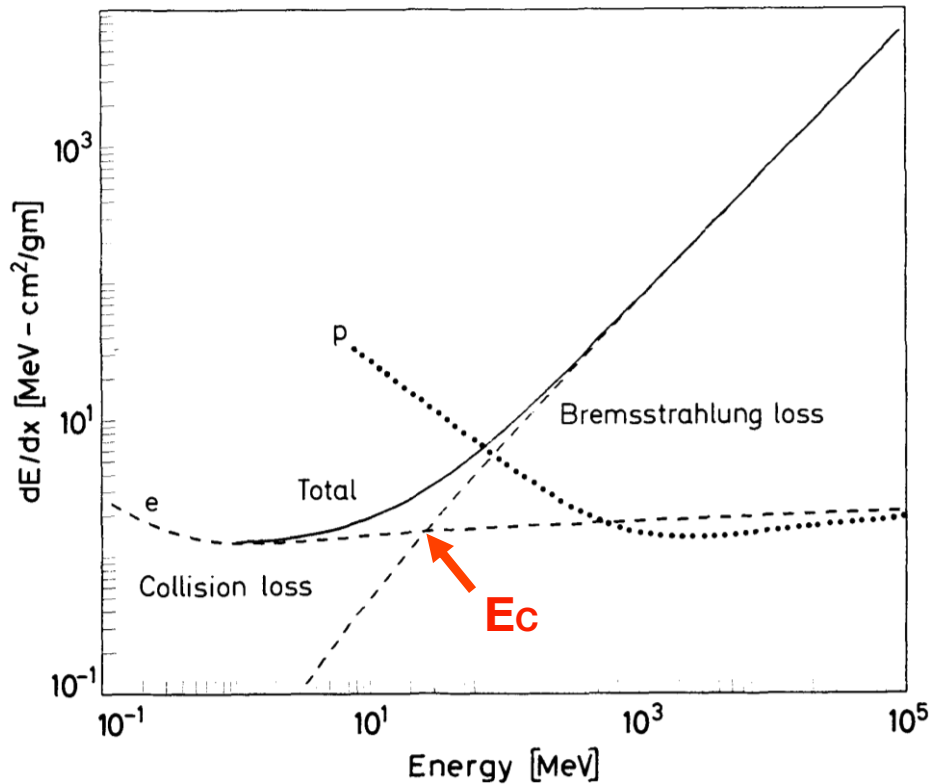


Fig. 2.10. Radiation loss vs. collision loss for electrons in copper. For comparison, the dE/dx for protons is also shown

The energy loss by radiation depends strongly on the absorbing material. For each material, we can define a *critical* energy, E_c , at which the radiation loss equals the collision loss.

$$\left(\frac{dE}{dx}\right)_{\text{ionisation}} = \left(\frac{dE}{dx}\right)_{\text{bremsstrahlung}}$$

(for electrons)

Table 2.2. Critical energies of some materials

Material	Critical energy [MeV]
Pb	9.51
Al	51.0
Fe	27.4
Cu	24.8
Air (STP)	102
Lucite	100
Polystyrene	109
NaI	17.4
Anthracene	105
H ₂ O	92

$$E_c \simeq \frac{800 \text{ MeV}}{Z + 1.2} ,$$

$$-\left(\frac{dE}{dx}\right)_r = \frac{\star NEZ(Z+1)e^4}{137m_0^2c^4} \left(4 \ln \frac{2E}{m_0c^2} - \frac{4}{3}\right)$$

Z²:
Absorbing materials with higher Z
cause more bremsstrahlung radiation
on electrons

Radiation length is even more frequently used than E_c . We will see it in the interaction of high-energetic photons in the next lecture!

Backscattering of low-energy electrons

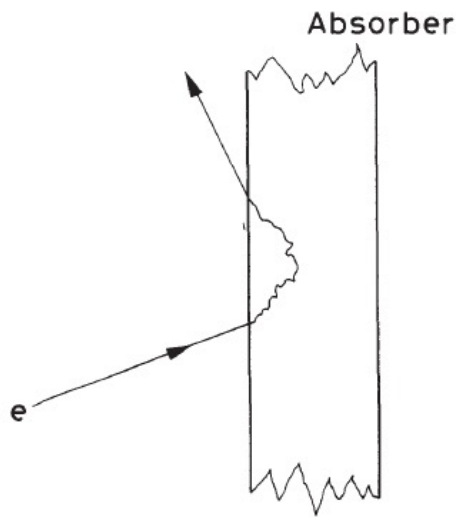


Fig. 2.16. Backscattering of electrons due to large angle multiple scatterings

- electrons undergo to large angle deflections by scattering from nuclei because of their small masses.
- Because the scattering probability is very high, electrons often undergo multiple scattering events. As a result, some are deflected so strongly that they reverse their direction and are backscattered out of the absorber.
- The effect is particularly strong for low energy electrons and increases with the atomic number Z of the material.
- Fraction of backscattering = backscattering coefficient

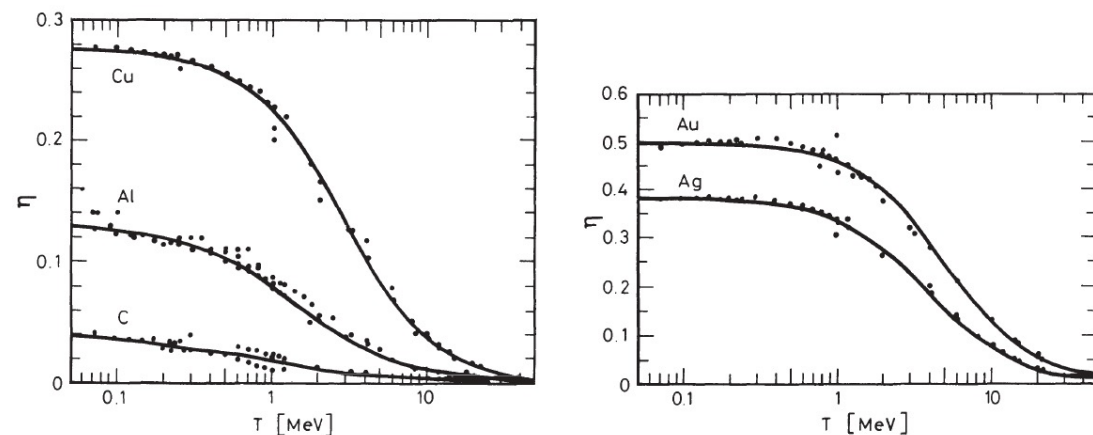
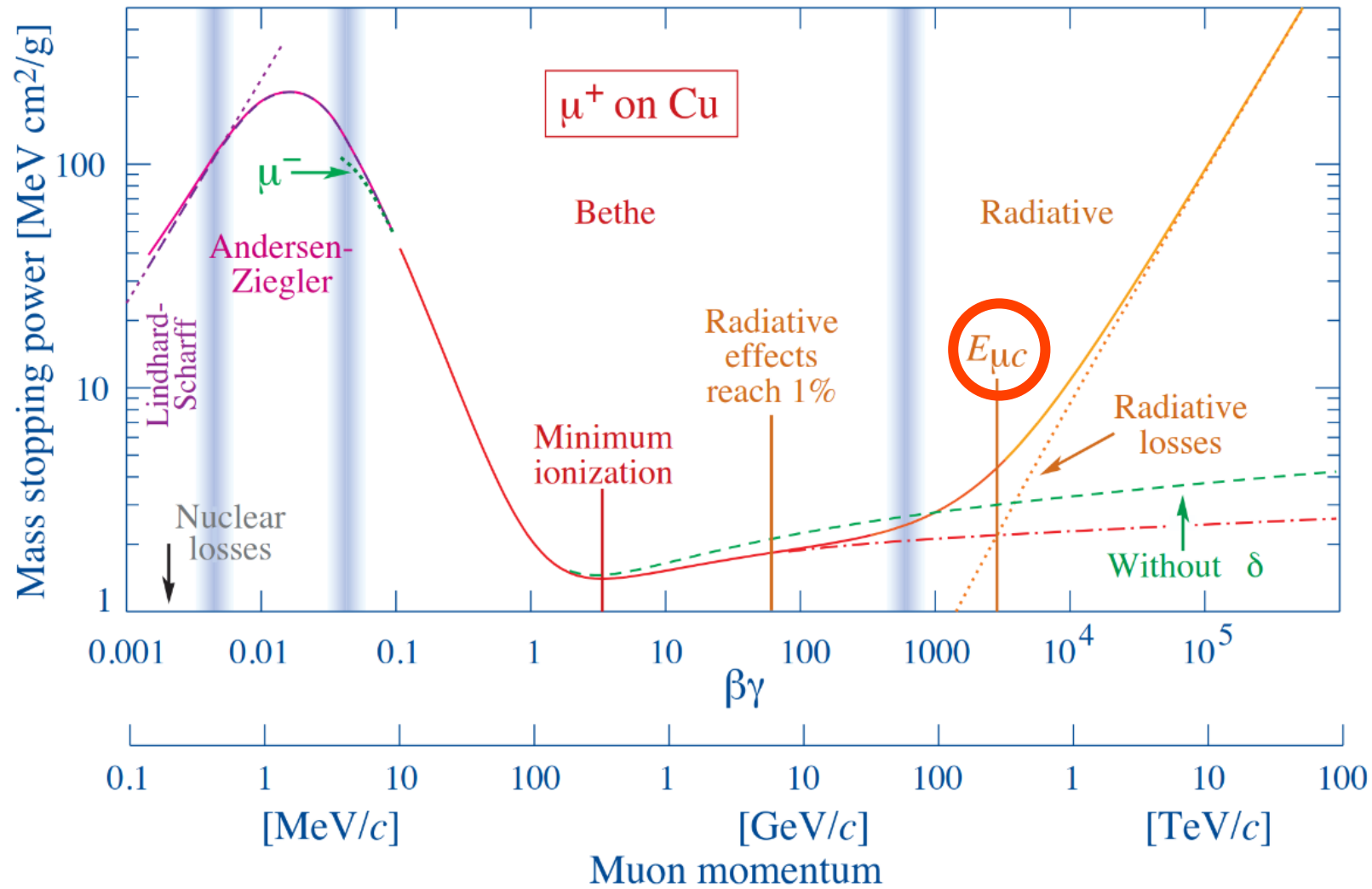
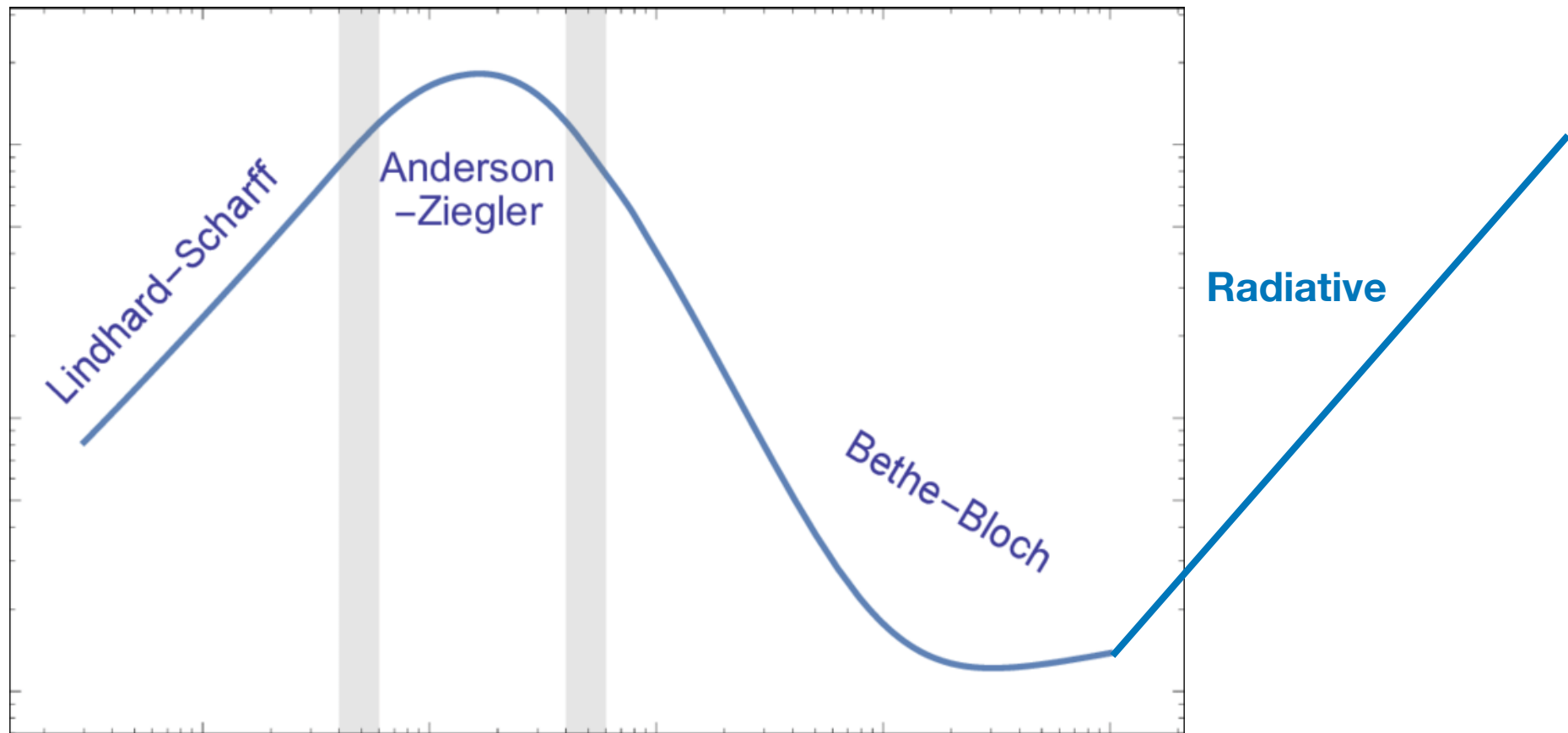


Fig. 2.17. Some measured electron backscattering coefficients for various materials. The electrons are perpendicularly incident on the surface of the sample (from Tabata et al. [2.24])

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{2m_e c^2 \gamma^2 \beta^2 W_{\max}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$





The Bethe-Bloch and Lindhard-Scharff regions are well predicted by theory. The Anderson-Ziegler region is an interpolation from data, and the gray bands denote the interface between these regions.

https://www.researchgate.net/figure/The-average-ionization-loss-of-a-charged-particle-moving-through-solid-copper-as-a_fig1_333861988

Questions?

- How do heavy charged particles lose energy?
- Explain the terms of the Bethe-Bloch equation. What processes are important at different particle momenta?
- What corrections are important at what energies?
- Can we use the Bethe-Bloch formula for particle identification? Can we use this technique in both Nuclear Physics and High-energy Physics? Examples?
- What is a Bragg peak? Which domains do use it? Can you give an example?
- What makes electrons different compared to heavy charged particles with respect to their interactions with material?
- Explain bremsstrahlung. When is it a dominant process?