Interaction theory – Photons

Eirik Malinen
Introduction

- Photons
- Charged particles
- Neutrons

Interaction theory

Ionizing radiation

Atoms

Dosimetry

Ionizing radiation

Matter

Effects

Ionizing radiation

Molecules

Cells

Humans

Introduction

Radiation source

- Photons
- Charged particles
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H₂O

DNA

Objectives

- To understand primary effects of ionizing radiation

- How radiation doses are calculated and measured

- To appreciate applications of ionizing radiation
Contents FYSKJM4710

- Interactions between ionizing radiation and matter
- Radioactive and non-radioactive sources
- Calculations and measurement of absorbed doses (dosimetry)

Relevant issues

- X-ray and CT investigations
- Radiotherapy
- Positron emission tomography
- Radiation protection
- Radiation Biology
• X-ray contrast: only a matter of differences in density?

Basic theory of the atom
Bohr model

Hydrogen, H
Atomic number: 1
Mass number: 1
1 electron

Helium, He
Atomic number: 2
Mass number: 4
(2 protons + 2 neutrons)
2 electrons

Lithium, Li
Atomic number: 3
Mass number: 6
(3 protons + 3 neutrons)
3 electrons

Neon, Ne
Atomic number: 10
Mass number: 20
(10 protons + 10 neutrons)
10 electrons

Hydrogen
Ionization

- Liberation of electron from atom
- Requires energy transfer \( \sim 4-25 \text{ eV} \)
- A lethal whole-body dose of radiation (5 J/kg) results in a temperature increase of 0.001 °C

Ionizing radiation

**Directly ionizing radiation**: Fast charged particles, which deliver their energy to matter directly, through many small Coulomb-force interactions

**Indirectly ionizing radiation**: Photons (\( \gamma \) or X-rays) or neutrons, which transfer their energy to charged particles in the matter
Cross section 1

- Cross section $s$: "target area", effective target covering a certain area
- Proportional to the interaction strength between an incoming particle and the target particle
- Consider two discs, one target and one incoming:
  - $s$ is the total area: $\pi(r_1^2 + r_2^2)$

Cross section 2

- $N$ particles move towards an area $S$ with $n$ atoms
- Probability of interaction: $p = \frac{n\sigma}{S}$
- Number of interacting particles: $N_p = \frac{Nn\sigma}{S}$
Cross section 3

- Separate between *electronic* and *atomic* cross section

- The cross section depends on:
  - Type of target (nucleus, electron, ..)
  - Type of and energy of incoming particle
    (photon, electron...)

- Cross section calculated with quantum mechanics
  - here visualized in a classical window

Cross section 4

- *Differential cross section* with respect to scattering angle

\[
\frac{d\sigma}{d\Omega} = \frac{\text{number of particles scattered into } d\Omega}{\text{number of particles per unit area } d\Omega} \times \frac{1}{d\Omega}
\]
Photon interactions

- Photon represented by a plane wave \( \overline{A}_{in}(r,t) \sim e^{i(\vec{k} \cdot \vec{r} - \omega t)} \) in quantum mechanical calculations
- In principle, two different processes:
  
  - Absorption
  
  - Scattering

- Scattering: coherent (elastic) or incoherent (inelastic)

Coherent (Rayleigh) scattering

- Scattering without loss of energy: \( h\nu = h\nu' \)
- Photon is absorbed by atom, thereby emitted at a small deflection angle
- Depends on atomic structure and photon energy
- Atomic cross section:

\[
\sigma_r \propto \left( \frac{Z}{\hbar \nu} \right)^2
\]
Incoherent (Compton) scattering

- Scattering with loss of energy: $h\nu' < h\nu$
- Photon-electron scattering; electron may be assumed free (i.e. unbound)

![Diagram of Compton scattering]

- Thomson scattering: low energy limit, $h\nu \to 0$

Compton scattering – kinematics

- Conservation of energy and momentum:
  \[
  h\nu = h\nu' + T \\
  \frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + p \cos \varphi , \quad \frac{h\nu'}{c} \sin \theta = p \sin \varphi \\
  (pc)^2 = T^2 + 2Tm_c^2
  \]

\[
  h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_c^2} (1 - \cos \theta )} , \quad \cot \varphi = \left( 1 + \frac{h\nu}{m_c c^2} \right) \tan \left( \frac{\theta}{2} \right)
\]
Compton scattering – example

- An X-ray unit is to be installed, with the beam direction towards the ground. Employees in the floor above the unit are worried. Maximum X-ray energy is 250 keV. What is the maximum energy of the backscattered photons?

\[
\theta = 180^\circ \Rightarrow h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1 - \cos \theta)} = \frac{h\nu}{1 + \frac{2h\nu}{m_e c^2}}
\]

\[
h\nu = 250 \text{ keV} \Rightarrow h\nu' = \frac{250}{1 + \frac{2 \times 250}{511}} = \frac{126}{511} \text{ keV}
\]
Compton scattering – cross section 1

- Klein and Nishina derived the cross section for Compton scattering, assuming free electron

- Differential cross section:

\[
\left( \frac{d\sigma}{d\Omega} \right) = \frac{r_e^2}{2} \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right)
\]

\[d\Omega = \sin \theta d\theta d\phi\]

- \(r_e\) classical electron radius
- incoming photon along z-axis

Compton scattering – cross section 2

- Cylinder symmetry results in:

\[
\left( \frac{d\sigma}{d\theta} \right) = \pi r_e^2 \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right) \sin \theta
\]

- \(\sim\) probability of finding a scattered photon in the interval \([\theta, \theta + d\theta]\)

- Total electronic cross section:

\[
e\sigma = \int_0^\pi \pi r_e^2 \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right) \sin \theta d\theta
\]

- Atomic cross section: \(a\sigma = Z_e \sigma\)
Compton scattering – cross section

- Scattered photons are more forwardly directed with increasing photon energy:

![Graph showing Compton scattering cross section](image)

\[ \frac{d\sigma}{d(\hbar')} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d(\hbar')} = \frac{d\sigma}{d\Omega} \frac{2\pi \sin \theta}{d(\hbar')} \frac{d\theta}{d(\hbar')} \]

\[ \hbar' = \frac{\hbar}{1 + \frac{\hbar}{m_c^2} (1 - \cos \theta)} \]

\[ \Rightarrow \frac{d\sigma}{d(\hbar')} = \frac{\pi \hbar^2 m_c^2}{(\hbar')^2} \left[ \frac{\hbar'}{\hbar} + \frac{\hbar}{\hbar'} - 1 + \left( 1 - \frac{\hbar}{\hbar'} - 1 \right)^2 \right] \]
Compton scattering – cross section

- Correct atomic cross section:

The diagrams illustrate the variation of the Compton scattering cross section with photon energy for different materials.

1. For carbon (Z=6), the cross section shows a distinct peak at a certain photon energy, followed by a gradual decrease.
2. Copper (Z=29) and lead (Z=82) exhibit similar patterns, with the lead having a slightly different shape and peak position.

The diagrams also highlight the effect of electron binding energy on the cross section, indicating how the cross section changes with different binding energies.
Compton scattering – transferred energy 1

- The energy transferred to an electron in a Compton process:
  \[ T = h\nu - h\nu' \]
- The cross section for energy transfer:
  \[ \frac{d\sigma_{\nu}}{d\Omega} = \frac{d\sigma}{d\Omega} \frac{T}{h\nu} = \frac{d\sigma}{d\Omega} \frac{h\nu - h\nu'}{h\nu} \]
- Mean energy transferred:
  \[ \bar{T} = \frac{\int T \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega} = \frac{\int \frac{h\nu - h\nu'}{h\nu} \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega} = \frac{\sigma_{\nu}}{\sigma} \times h\nu \]

Compton scattering – transferred energy 2

- The fraction of incident energy transferred:

![Graph showing the fraction of incident energy retained by the photon versus photon energy (h\nu) in MeV. The graph shows a curve that increases from 0 to 1 as h\nu increases from 1 to 100.](image)
Photoelectric effect 1

- Photon is absorbed by atom/molecule; the result is an excitation or ionization

\[ \text{Photon} \rightarrow \text{Excitation or ionization} \]

- Atom may deexcite and emit characteristic radiation:

\[ \text{Atom} \rightarrow \text{Radiation} \]

Photoelectric effect 2

- In the kinematics, the binding energy of the ejected electron should be taken into account:

\[ T = \hbar \nu - E_b - T_a \approx \hbar \nu - E_b \]

- Assuming \( E_b = 0 \), the atomic cross section is:

\[ \frac{d\tau}{d\Omega} = 2\sqrt{2}e^2\alpha^4Z^5 \left(\frac{m_c^2}{\hbar\nu}\right)^{7/2} \sin^2 \theta \left(1 + 4 \left(\frac{2\hbar\nu}{m_c^2\cos \theta}\right) \right) \]

\( \alpha \): The fine-structure constant
\( \Omega \): Solid angle gives the direction of the ejected electron
Photoelectric effect 2

Characteristic radiation

- Energy of characteristic radiation depends on elektronic structure and transition probabilities
- "K- and L-shell" vacancies ↔ $\hbar\nu_K$ and $\hbar\nu_L$
- Isotropic emission
- Fraction of photoelectric interactions:
  $P_K [h\nu>(E_b)_K]$ and $P_L [(E_b)_L<h\nu<(E_b)_K]$
- Probability for emission: $Y_K$ og $Y_L$ (fluorescence yield)
- Energy emitted from the atom:
  $P_K Y_K h\nu_K + (1-P_K) P_L Y_L h\nu_L$
Auger effect

- Energy release by ejection of loosely bound electron
- Energy of emitted electron equal to deexcitation energy
- Low Z: Auger dominates
- High Z: characteristic radiation dominates

Photoelectric cross section

- General formula:
  \[ \tau \propto \frac{Z^n}{(\frac{hv}{m})^m}, \quad 4 < n < 5, \quad 1 < m < 3 \]
- Fraction of energy transferred to photoelectron:
  \[ \frac{T}{hv} = \frac{hv - E_b}{hv} \]
- However: don’t forget Auger electron(s)
- Cross section for energy transfer to photoelectron:
  \[ \tau_p = \tau \left( \frac{hv - P_k Y_k h_{k'} - (1 - P_k) P_L Y_L h_{L'}}{hv} \right) \]
Pair production 1

- Photon absorption in the nuclear electromagnetic field where an electron-positron pair is created

\[ h\nu = 2m_e c^2 + T^+ + T^- \]

- Triplet production: in the electromagnetic field of an electron

\[ T_0 \equiv 0 \text{ mom. } = 0 \]

Pair production 2

- Conservation of energy:

\[ h\nu = 2m_e c^2 + T^+ + T^- \]

- Average kinetic energy after absorption:

\[ \overline{T} = \frac{h\nu - 2m_e c^2}{2} \]

- Estimated electron/positron scattering angle:

\[ \overline{\theta} \approx \frac{m_e c^2}{\overline{T}} \]

- Total cross section:

\[ \kappa \approx \alpha_0^2 Z^2 \overline{P} \]
Discovery of pair production

- In the electromagnetic field from an electron, an electron-positron pair is created.
- Energy conservation:
  \[ h\nu = 2m_e^2 + T^+ + T_1^- + T_2^- \]
- Average kinetic energy:
  \[ \overline{T} = \frac{h\nu - 2m_e^2}{3} \]
- Primary electron is also given energy
- Threshold: \( 4m_0c^2 \)

Triplet production

- In the electromagnetic field from an electron, an electron-positron pair is created.
- Energy conservation:
  \[ h\nu = 2m_e^2 + T^+ + T_1^- + T_2^- \]
- Average kinetic energy:
  \[ \overline{T} = \frac{h\nu - 2m_e^2}{3} \]
- Primary electron is also given energy
- Threshold: \( 4m_0c^2 \)
Pair- and triplet production

- Pair production dominates:

Photonuclear reactions

- Photon (energy above a few MeV) excites a nucleus
- Proton or neutron is emitted
- \((\gamma, n)\) interactions may have consequences for radiation protection
- Example: Tungsten W \((\gamma, n)\)
Summary, interactions 1

Processes

- Compton
- Rayleigh
- Photoel. eff.
- Pair/triplet

Kinematics

\[ \Delta \nu' = \frac{T + (h \nu/m_e c)^2}{1 - \cos \theta} \]
\[ T = h \nu - h \nu' \]
\[ \cos \theta = \frac{1 + (h \nu/m_e c)^2}{2} \]

Atomic cross section

\[ \sigma = \frac{2 \pi}{\nu' \nu} \]
\[ \tau = \frac{Z^2}{\nu' \nu} \]

Mean energy transferred

\[ \bar{T} = \frac{h \nu}{\sigma} \]
\[ \tau = h \nu - P \Sigma Y \nu' - (1 - P) \Sigma Y \nu' \]

Photon-nuclear reactions

\[ \sigma = \frac{\sigma_0}{\cos \theta} \]
\[ P: \quad \sigma = 2m_e c^2 + 2T \]
\[ T: \quad \sigma = 2m_e c^2 + 3T \]

\[ \kappa = Z \pi (h \nu) \]
\[ \psi = Z \pi (h \nu) \]

Summary, interactions 2

Photon Energy \( h\nu \), in MeV

- Photoelectric effect dominant
- Pair production dominant

Compton effect dominant
### Attenuation coefficients 1

- $n_V$ atoms per volume = $\rho(N_A/A)$
- Number of atoms:
  \[n = n_V V = n_V \Sigma dx\]
- Interaction probability:
  \[p = n \sigma / \Sigma = n_V \sigma dx\]
- Probability per unit length:
  \[\mu = p / dx = n_V \sigma = \rho(N_A/A)\sigma\]
  $\mu$: linear attenuation coefficient

### Attenuation coefficients 2

- $N_A$: Avogadro’s constant; $6.022 \times 10^{23}$ mole\(^{-1}\)
- $A$: number of grams per mole
- $N_A/A$: number of atoms per gram
- $N_AZ/A$: number of electrons per gram
- Number of atoms per volume: $r(N_A/A)$
- Etc.
Attenuation coefficients 3

- Total mass attenuation coefficient:
  \[
  \frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_r}{\rho}
  \]

- Coefficient for energy transfer:
  \[
  \frac{\mu_{ir}}{\rho} = \frac{\mu}{\rho} \frac{T}{h \nu}
  \]

- Braggs rule for mixture of atoms:
  \[
  \left( \frac{\mu}{\rho} \right)_{\text{mix}} = \sum_{i=1}^{n} f_i \left( \frac{\mu}{\rho} \right)_i, \quad f_i = \frac{m_i}{\sum_{i=1}^{n} m_i}
  \]
Attenuation coefficients

X-ray images at different energies

Conventional X-rays (120 kV)  Linear accelerator (5 MV)
Attenuation 1

- Beam with $N$ photons impinge absorber with thickness $dx$:

\[ \text{Beam} \xrightarrow{dx} \text{Absorber} \xrightarrow{N \cdot dN} \]

- Probability for interaction: $\mu dx$

- Number of photons interacting: $N\mu dx$

\[ dN = N \mu dx \quad \Rightarrow \quad \int \frac{dN}{N} = \int \mu dx \]

\[ \Rightarrow N = N_0 e^{-\mu x} \]

Note that $\mu$ is the interaction probability per unit length – not the absorption probability

Attenuation 2

- Note that $\mu$ is the interaction probability per unit length – not the absorption probability

- $e^{-\mu x}$ corresponds to a narrow beam measurement geometry:
Attenuation 3

- 'Probability' for photon not interacting: $e^{-\mu \cdot x}$
- Normalized probability

$$p_{ni} = Ce^{-\mu x}, \int_0^\infty p_{ni} \, dx = 1, \Rightarrow p_{ni} = \mu e^{-\mu x}$$

- Mean free path:

$$\langle x \rangle = \int_0^\infty x p_{ni} \, dx = \int_0^\infty x \mu e^{-\mu x} \, dx = \frac{1}{\mu}$$

Attenuation 4
Attenuation 4

- 'Probability' for photon not interacting: \( \sim e^{-\mu x} \)
- Normalized probability
  \[
  p_n = Ce^{-\mu x}, \quad \int_0^\infty p_n dx = 1, \quad \Rightarrow \quad p_n = \mu e^{-\mu x}
  \]
- Mean free path:
  \[
  \langle x \rangle = \int_0^\infty x p_n dx = \int_0^\infty x \mu e^{-\mu x} dx = \frac{1}{\mu}
  \]
- A distance of 3 MFP reduces the beam intensity to 5%

Attenuation - example

- 2 MeV photons
  
  \[
  \begin{align*}
  \text{Pb:} & \quad \mu = 0,516 \text{ cm}^{-1} \\
  \text{H}_2\text{O:} & \quad \mu = 0,049 \text{ cm}^{-1} \\
  e^{-\mu_{\text{H}_2\text{O}} x_{\text{H}_2\text{O}}} & = e^{-\mu_{\text{Pb}} x_{\text{Pb}}} \\
  \Rightarrow \quad \frac{x_{\text{H}_2\text{O}}}{x_{\text{Pb}}} & = \frac{\mu_{\text{Pb}}}{\mu_{\text{H}_2\text{O}}}
  \end{align*}
  \]
- 10 times as much water necessary
Broad beam attenuation

- Broad-beam geometry: every scattered or secondary uncharged particle strikes the detector

\[ e^{\mu x} \]: number of primary photons at a given depth

- What about the scattered photons?

Scattered photons

- Monte Carlo simulations
Primary and scattered photons, 100 keV

Primary and scattered photons, 1 MeV