Two myths about special relativity

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Misconceptions about special relativity theory are common and pernicious. I address two such misconceptions: the low-speed behavior of the Lorentz transformation and the meaning of the phrase, "the constancy of the speed of light," as Einstein used it in 1905.
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I. INTRODUCTION

In the course of a century, the special theory of relativity has given rise to a variety of misconceptions. I address two significant instances.

II. LOW-SPEED BEHAVIOR OF THE LORENTZ TRANSFORMATION

Q. Does the Lorentz transformation reduce to the Galilean transformation when the ratio v/c is small?

A. No.

At least two routes are available to substantiate the negative response. I will first give a direct, algebraic route and then a more sophisticated indirect route.

Consider the usual pair of inertial reference frames, the primed frame moving with speed v along the x axis of the unprimed frame. To avoid any spurious dependence on the origins of coordinate systems, consider a pair of physical events. The Lorentz transformation for the time interval between the events takes the form

$$\Delta t' = \frac{1}{\sqrt{1 - v^2/c^2}}\left(\Delta t - \frac{v}{c^2}\Delta x\right). \tag{1}$$

Let the ratio v/c be as small as desired (but nonzero). Then it is always possible to find an event pair for which Δx is large enough that the term with Δx dominates over the term with Δt. This behavior is entirely different from what the Galilean transformation, $\Delta t' = \Delta t$, asserts.

For a sophisticated justification, note that the composition (the successive use) of two Lorentz transformations is equivalent to another Lorentz transformation. This equivalence is the group property of the Lorentz transformation. Moreover, the Lorentz transformation is differentiable with respect to v/c, and the derivative is nonzero at $v/c=0$. Consequently, any Lorentz transformation with finite speed can be constructed by iterating a Lorentz transformation with a small (and ultimately infinitesimal) ratio v/c.

If the Lorentz transformation for infinitesimal v/c were to reduce to the Galilean transformation, then the iterative process could never generate a finite Lorentz transformation that is radically different from the Galilean transformation. But the finite transformations are indeed radically different, and so—however subtly—the infinitesimal Lorentz transformation must differ significantly from the Galilean transformation.

I could stop here, but allow me a brief amplification. In the present context, the infinitesimal Lorentz transformation for time is given by Eq. (1) after the square root has been set equal to 1. The infinitesimal Lorentz transformation for the spatial interval (along the frames' relative velocity) is given by the Galilean transformation: $\Delta x' = \Delta x - (v/c)c\Delta t$. In short, the infinitesimal Lorentz transformation is the full Lorentz transformation after truncation to first order in v/c. In a masterful but little known paper, Wendell Furry established the infinitesimal Lorentz transformation by simple, direct reasoning and then iterated it by matrix methods for an arbitrary direction of the frames' relative velocity.[1] More about the mathematics of the comparison at small v/c is provided in an appendix.

To make the point of this section most succinctly: no matter how small (but nonzero) the ratio v/c, simultaneity remains a relative notion.

III. CONSTANCY OF THE SPEED OF LIGHT

Q. Does the phrase, "the constancy of the speed of light," have the same meaning today that it had when Einstein used it in 1905?

A. No.

Today, the primary meaning of the phrase is that, given a specific burst of light, the burst's speed is measured to have the same numerical value in all inertial frames. That is, the speed is constant with respect to changes in the reference frame in which it is observed.

A secondary meaning also exists: in any given frame, bursts of light from sources with different velocities all have the same speed. That is, the speed of light is constant with respect to changes in the source's velocity.

When the typical contemporary textbook uses the phrase, "the constancy of the speed of light," it intends that both the primary and the secondary meaning apply.

In the years immediately preceding 1905 and in Einstein's seminal paper, the phrase, "the constancy of the speed of light," meant *only* that the speed of light is independent of the source's velocity.

For the usage prior to 1905, I turn to Einstein's collected papers. In a note, the editors quote Wilhelm Wien as using the phrase in 1904 to mean strictly that the propagation of radiation is independent of the source's motion.[2]

As for Einstein's usage, the best evidence for my assertion comes from a line-by-line scrutiny of Einstein's paper.[3] To derive the Lorentz transformation, Einstein *used* only the principle that the speed of light is independent of the state of motion of the emitting (or reflecting) body and the relativity principle (the laws of physics are the same in all inertial frames).

Ancillary evidence comes in several places. First, Einstein asserted the constancy only for the "stationary" coordinate system.[4] Second, Einstein wrote, "Now we have to *prove* [my italics] that, measured in the moving system, every light ray propagates with the speed V [we would write c] if it does so, as we have assumed, in the stationary system… ."[5]

Third, at an intermediate stage in his derivation of the Lorentz transformation, Einstein notes "that (as required by the principle of the constancy of the speed of light *in conjunction with* [my italics] the principle of relativity) light propagates also with speed V when measured in the moving system."[6] In short, constancy with respect to change in the source's velocity (asserted for the stationary system) *plus* the principle of relativity imply constancy with respect to change in the reference frame.[7,8]

Other writers have made some or all of the foregoing points about what Einstein actually postulated and how he went about his derivation of the Lorentz transformation. Among them are the authors cited in Ref. 9. Some especially pertinent quotations and editorial comments in Einstein's collected papers are listed in Ref. 10.

Banesh Hoffmann, who worked with Einstein in the 1930s, put the matter neatly in his book, *Relativity and Its Roots*:

> The second of the two principles in Einstein's paper said that the motion of light is not affected by the motion of the source of light. Nothing, it would seem, could be more orthodox and obvious. For if a source of light generates light waves in the ether, once the waves are launched they are no longer linked to their source; they are on their own, moving at the rate set by the elastic properties of the ether….
>
> If it was so obvious, though, why did he need to state it as a principle? Because, having taken from the idea of light waves in the ether the one aspect that he needed, he declared early in his paper, to quote his own words, that "the introduction of a 'luminiferous ether' will prove to be superfluous."
>
> We see in all this the working of an extraordinary intuition. The beautiful thing about Einstein's cunningly chosen pair of principles is that each by itself seems harmless, yet the two together form an explosive mixture destined to rock the very foundations of science.[11]

I suggest that textbook writers have something to learn from Hoffmann's description. To take as a postulate that the speed of light is constant relative to changes in reference frame is to assume an apparent absurdity. It goes against common sense. No wonder, thinks a student, that we can derive other absurdities, such as time dilation and length contraction, from the premises. Far better to start much closer to where Einstein started and to *derive* the logical consequence that the speed of any given light pulse has the same value in all inertial frames. The derivation can be accomplished with one simple diagram.[12,13]

IV. NEXT STEPS

The special theory of relativity is not a particularly difficult topic. Clarity and accuracy enable one to teach the subject successfully to first year college students, both those in the sciences and those in other disciplines. But the twin characteristics—clarity and accuracy—are essential to the enterprise. Attention to history and to the referred literature will help.

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APPENDIX

This appendix extends the discussion about limits that was given in Sec. II. First note that, as $v/c \to 0$, both the Lorentz and the Galilean transformation reduce to the identity transformation. There is, however, no significant comparative information in that fact. To see clearly why that is so, let me introduce an analogous, easily visualized situation.

Consider a hard cover book. We can rotate the book by an angle θ around an axis perpendicular to the front cover and passing through the center of mass; call that a FC rotation. Also, we can rotate the book about an axis parallel to the book's spine and passing through the center of mass; call that a BS rotation. When $\theta = 20°$, the FC and BS rotations are obviously different. As θ goes to zero, both the FC and the BS rotation reduce to the identity rotation, that is, to no effect at all.

Does this passage imply that a FC rotation reduces to a BS rotation when θ is small? Not at all. Specify that $\theta = 20°/n$ and that the integer $n \gg 1$. For any finite n, the FC and BS rotations continue to differ significantly. To prove this assertion, we need only iterate the tiny rotations n times and recover the rotations by $\theta = 20°$, which are manifestly different. In short, iteration amplifies a difference and reveals it.

To return to the Lorentz and Galilean transformations, recall that a paragraph near the end of Sec. II presented the Lorentz transformation to first order in v/c. First order determines the generators of the Lie group and hence, ultimately, the high-speed behavior. For the spatial transformation, the Lorentz and Galilean transformations agree to first order; for the temporal transformation, they differ in first order. Iteration of the first-order transformations will amplify the difference, leading ultimately to an equation like Eq. (1) for the Lorentz transformation. We need only write $v/c = v_0/nc$, where v_0 is a constant, iterate n times, and then let n go to infinity. (Some remarks about this process were made in Ref. 1.)

In short, the difference in the temporal transformations in first order in v/c implies that the Lorentz transformation fails to reduce to the Galilean transformation. Note that this line of reasoning holds for all nonzero values of Δx and Δt, which may be specified either prior to iteration or subsequently. The conclusion is independent of order.

Now let me return to my first line of reasoning, the argument based directly on Eq. (1). Suppose a student objects to the argument, saying, "You stacked the deck in your favor by prescribing the sequence, pick the relative speed and then choose the event pair. Can you still carry through a proof if first you pick the event pair and then let me pick the relative speed?"

I believe that the alternative sequence is not what a mathematician would understand the phrase, "the Lorentz transformation reduces to the Galilean transformation," to mean. At a specified level of accuracy, a single speed should suffice for all conceivable event pairs. But I can offer a "proof" under the inverted sequence nonetheless.

Specify that "reduces" means that the ratio $\Delta t'_{\mathrm{LT}}/\Delta t'_{\mathrm{GT}}$ differs from 1 by less than one part in a thousand, say. The subscripts LT and GT indicate that the time interval is computed with the Lorentz and Galilean transformation, respectively. I use a ratio rather than a difference because a ratio is independent of the units of time, seconds versus microseconds or centuries, for example.

Now I choose an event pair for which $\Delta t=0$, but $\Delta x=5$ m (or any nonzero value). Then $\Delta t'_{\mathrm{LT}}$ is nonzero at every nonzero relative speed, but $\Delta t'_{\mathrm{GT}}$ remains zero always. The ratio of transformed times is infinite for all nonzero values of the relative speed. The skeptic cannot meet the prescribed accuracy bound.

Moreover, we can put the situation more dramatically. According to the Galilean transformation, the events are always simultaneous in the new reference frame. According to the Lorentz transformation, the events are never simultaneous.

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[1]W. H. Furry, "Lorentz transformation and the Thomas precession," Am. J. Phys. **23**(8), 517–525 (1955). For the standard context in my Sec. II, one can form linear combinations $\Delta x' \pm c\Delta t'$ and iterate them readily to obtain the finite Lorentz transformation. For the iteration, replace v/c in the infinitesimal transformation by r/n, where r is a finite constant; iterate n times; and then let n go to infinity. The finite Lorentz transformation emerges in terms of the parameter r, sometimes called the *rapidity*. The rapidity r satisfies the relationship $v/c=\tanh r$, where v here is the finite relative speed of the two frames.

[2]*The Collected Papers of Albert Einstein*, edited by John Stachel, David C. Cassidy, Jürgen Renn, and Robert Schulmann (Princeton U.P., Princeton, NJ, 1989), Vol. 2, p. 307, note 5.

[3]Reference 2, pp. 275–306.

[4]Reference 2, p. 280.

[5]Reference 2, p. 285.

[6]Reference 2, p. 284.

[7]H. M. Schwartz, "Einstein's first paper on relativity," Am. J. Phys. **45**(1), 18–25 (1977). The author interprets the passage differently. In my opinion, Schwartz missed the insight into Einstein's reasoning that Banesh Hoffmann expressed in the remarks that I quote from Ref. 11.

[8]For an indication of how the logic runs, see Refs. 12 and 13. For the context closest to Einstein's context, consult Ref. 13, take Alice's frame to correspond to Einstein's "stationary" frame, and let the relative motion of Bob and Alice have an arbitrary algebraic value. Alice uses the "principle of the constancy of the speed of light" in Einstein's 1905 sense. (The text on the lower half of p. 190 uses the modern meaning of the same phrase.).

[9]W. Pauli, *Theory of Relativity* (Pergamon, New York, 1958), p. 5. (The German original was published in 1921.) Philipp Frank, *Einstein: His Life and Times* (Knopf, New York, NY, 1947), pp. 54–55; Robert B. Williamson, "Logical economy in Einstein's 'On the Electrodynamics of Moving Bodies,' " Stud. Hist. Philos. Sci. **8**, 49–60 (1977); Roberto Torretti, *Relativity and Geometry* (Pergamon, Elmsford, NY, 1983), pp. 54–61; Harvey R. Brown and Adolfo Maia, Jr., "Light-speed constancy versus light-speed invariance in the derivation of relativistic kinematics," Br. J. Philos. Sci. **44**, 381–407 (1993); Robert Rynasiewicz, "The construction of the special theory: Some queries and considerations," in *Einstein: The Formative Years, 1879–1909*, edited by Don Howard and John Stachel (Birkhäuser, Boston, 2000) pp. 163, 167–169; Alberto A. Martinez, "Kinematic subtleties in Einstein's first derivation of the Lorentz transformations," Am. J. Phys. **72**(6), 790–798 (2004).

[10]Reference 2, pp. 257 and 263–5.

[11]Banesh Hoffmann, *Relativity and Its Roots* (Freeman, New York, 1983), p. 92.

[12]Reference 11, pp. 92–94.

[13]Ralph Baierlein, *Newton to Einstein: The Trail of Light* (Cambridge U.P., New York, 1992), pp. 185 and 188–191. Moreover, Chapter 8 presents direct experimental evidence that the speed of light is independent of the source's velocity.