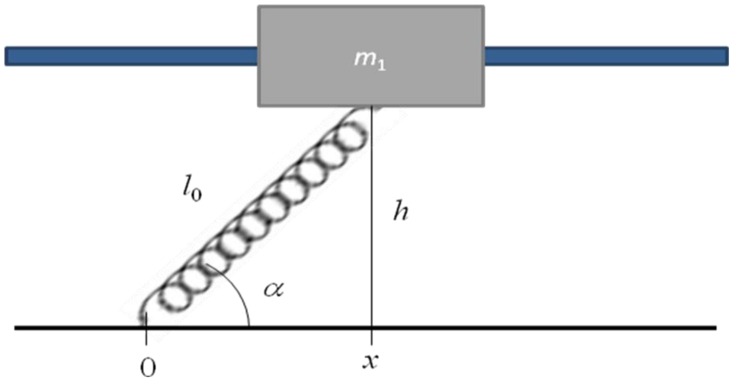
FYS-MEK 1110 – 2013 – Oblig 5 – Midtveis-hjemmeeksamen

A metal cylinder of mass m = 5 kg with a coaxial hole is sliding on a metal rod. A spring with spring constant k = 500 N/m is attached to the cylinder. The other end of the spring is attached on the table at a vertical distance of h = 0.3 m below the cylinder. The spring has an equilibrium length of l_0 = 0.5 m.



a) What is the horizontal position x_0 of the cylinder when the spring is at its natural equilibrium length of l_0 = 0.5 m ? Define the attachment point of the spring on the table as position x = 0.

b) What is the length l of the spring when the cylinder is at any given position x ?

c) Draw a free-body diagram of the cylinder.

d) Show that the horizontal force which the spring exerts on the cylinder is:

$$F_x = -kx\left(1 - \frac{l_0}{\sqrt{x^2+h^2}}\right)$$

e) Plot the horizontal spring force as a function of position of the cylinder between x = –0.75 m and x = +0.75 m and explain what you expect for the motion of the cylinder.

f) For the moment we assume that there is neither friction nor air resistance. You pull the cylinder to the position x = 0.6 m and let it go without giving it any initial velocity. Write a program to solve the equation of motion numerically and plot the position and velocity of the cylinder for the first 10 seconds of its motion. Describe and interpret the motion of the cylinder.

g) Run your program again, but this time choose a starting position of x = 0.65 m. Plot the position and velocity as a function of time for the first 10 seconds of motion. Describe the motion of the cylinder and use energy arguments to explain.

h) Show that the vertical component of the spring force acting on the cylinder is:

$$F_y = -kh\left(1 - \frac{l_0}{\sqrt{x^2+h^2}}\right)$$

i) What is the normal force from the rod acting on the cylinder when the spring is at its natural equilibrium length ?

j) What is the normal force *N* from the rod acting on the cylinder when the cylinder is at a position *x* ?

k) Show that the normal force becomes zero for:

$$x = \pm\sqrt{\left(\frac{l_0}{1+\frac{mg}{kh}}\right)^2 - h^2}$$

l) We will now consider a more realistic model and include friction. Due to polished surfaces and the use of a lubricant the (dynamic) friction coefficient is only $\mu = 0.05$. Modify your program to include friction. Start at the position $x = 0.75$ m and plot again position and velocity as a function of time. Explain the motion.

m) Make an additional figure where you plot the kinetic energy versus the position for the first 10 seconds of the motion. Explain the results.

n) Find the work that is needed to bring the cylinder from its equilibrium position (that you have found in part a) to the position x = 0.75 m numerically. You can use the function “trapz” or write your own program for the numerical integration. Pay attention to vectorized operations.

o) Write a program to find the potential energy for the spring force as a function of the position of the cylinder. Plot the result and compare with the kinetic energy from part m). You can use the function “cumtrapz” or program your own function for the numerical integration.

p) Where are the equilibrium points of the system? Characterize them as stable or unstable.