Project 11.1: Trapping atoms

In this project you will use a simple model for how we trap and cool atoms. Today, it is possible to cool atoms to about 10^{-9}K, which corresponds to about $-273.15°$. Typically, you want to both trap the atom and cool it, so that you can study it in detail. The motivation comes from quantum mechanics, because some quite exotic phenomena can only be systematically studied at low temperatures. For example, experiments on Bose-Einstein condensates are based on similar techniques, and quantum computers may be based on cold atoms as its basic tools.

Here we study a magneto-optical trap (MOT) (See figure 11.30). Detailed calculations of the interactions in the trap are beyond the scope of this course, since that requires detailed quantum-mechanical calculations. However, we will introduce a model that can be justified by quantum-mechanical calculations and that captures the main features of the process.

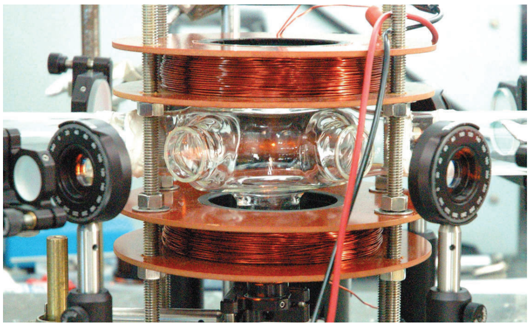

Figure 11.30: *Illustration of a MOT. The atoms are "collected" in the center of the glass container. A laser enters from the sides, while a magnetic field is generated by the coils.*

We address a one-dimensional system. An atom moves along the x-axis with a kinetic energy $K = \frac{1}{2}mv^2$. In the range $-x_0 < x < x_0$ the atom enters the trap, and is affected by a magnetic field. The interaction with the magnetic field gives rise to a poential $U(x)$, which we model as:

$$U(x) = \begin{cases} U_0 & |x| \geq x_0 \\ U_0 \frac{|x|}{x_0} & |x| < x_0 \end{cases} . \tag{11.199}$$

Notice that this is all we need to know about the interaction between the atom and the magnetic field.

(a) Make a sketch of $U(x)$. Discuss the motion of the atom for representative values of the total energy E of the atom. Find equilibrium points and discuss their stability.

(b) Find the force $F(x)$ acting on the atom from the magnetic field. Is this force conservative?

(c) If the atom of mass m has the velocity $v_0 = \sqrt{4U_0/m}$ at $x = 0$, find the velocity at $x = x_0/2$ and $x = 2x_0$.

(d) If the atom of mass m has the velocity $v_0 = -\sqrt{4U_0/m}$ at $x = 0$, find the velocity at $x = -x_0/2$ and $x = -2x_0$.

Let us also assume that the atom is charged and subject to a constant electrostatic force, F_0 acting in the positive x-direction.

(e) If the atom has the kinetic energy $K = 0$ at $x = 0$, how large must F_0 be in order for the atom to escape? And if the kinetic energy is $K = U_0/2$ at $x = 0$, how large must F_0 then be in order for the atom to escape?

In the following, let us assume that the atom is only affected by the magnetic field. In addition, while the atom is in the trap, we send photons with a particular wavelength at the atom. For example, for Li-atoms, a wavelength of 671nm is used. The force on the atom due to a continuous adsorption (and emission) of photons, can be written as

$$F = -\alpha v , \tag{11.200}$$

where v is the velocity of the atom, and α is a constant. This force also only acts in the range $-x_0 < x < x_0$.

(f) Is the force F conservative? (Provide an argument for your answer).

The equations of motion for the atom may be non-dimensionalized, but we will not address the details of this. You can in the following use the non-dimensional values $U_0 = 150$, $m = 23$, $x_0 = 2$, and $\alpha = 39.48$, and describe the motion using non-dimensional positions, times, and velocities.

The equations of motion for the atom are difficult to solve analytically, but can be addressed using numerical methods. In the following exercises, use your experience with numerical solutions of the equation of motion to solve the problems.

(g) Find an expression for the acceleration of the atom. What are the initial conditions for the motion?

(h) Write a program to find the position, $x(t)$, as a function of time for the atom given the expression for the acceleration and the initial conditions from above.

(i) Find the motion, $x(t)$, of an atom with velocity $v_0 = 10.0$ at $x = -5$. Describe what happens.

(j) Find the motion, $x(t)$, of an atom with velocity $v_0 = 8.0$ at $x = -5$. Describe what happens.

(k) (Optional) Find the maximal initial velocity v_0 the atom may have and still be trapped.