№ 13.1 (#31.36)

Given	SI
$V_0 = 120\,V$	
$L = 0.28\,H$	
$C = 4\,\mu F$	$4 \cdot 10^{-6}\,F$
$I_0 = 1.7\,A$	
$\omega_0 - ?$	
$R - ?$	
$V_L, V_C, V_R - ?$	

a) Impedance of RLC-circuit is defined as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

There is resonance in the circuit when impedance is minimal (we get maximal current).
So, Z is minimal when reactances are equal

$$X_L = X_C$$

Where inductive reactance is

$$X_L = \omega L$$

and capacitive reactance is

$$X_C = \frac{1}{\omega C}.$$

We obtain the following equation for resonance frequency:

$$\omega_0 L = \frac{1}{\omega_0 C}$$

that gives

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{0.28H \cdot 4 \cdot 10^{-6}F}} \approx 945\ s^{-1}.$$

b) Current in RLC-circuit is defined by the impedance:

$$I_0 = \frac{V_0}{Z}, \text{ where } Z = Z(\omega).$$

At resonance we have:

$$Z_0 = Z(\omega_0) = \sqrt{R^2 + \left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2} = \sqrt{R^2 + \left(\sqrt{\tfrac{L}{C}} - \sqrt{\tfrac{L}{C}}\right)^2} = R$$

Therefore active resistance is

$$R = Z_0 = \frac{V_0}{I_0} \Rightarrow \quad R = \frac{120\,V}{1.7\,A} \approx 70.6\ \Omega$$

c) Voltage over coil and ~~condon~~ capacitor depends on their reactances (for resistor we can use usual resistance):

$$I_c = \frac{V_c}{X_c} \quad ; \quad I_L = \frac{V_L}{X_L} \quad ; \quad I_R = \frac{V_R}{R}$$

The element are in series in the LRC-circuit, so

$$I_L = I_c = I_R = I_0 .$$

The reactance are equal at resonance:

$$X_L^0 = X_c^0 = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} \quad \text{or} \quad X_L = X_c = \sqrt{\frac{0.28\,H}{4\cdot 10^{-6}F}} \approx 264.6\,\Omega \approx$$

$$\approx 265\,\Omega .$$

That gives the voltage:

$$\underline{V_c = I_0 X_c^0 = I_0\sqrt{\frac{L}{C}}}$$

$$\Rightarrow \underline{V_L = V_c} = 1.7\,A \cdot 265\,\Omega \approx \underline{450\ V}$$

$$\underline{V_L = I_0 X_L^0 = I_0\sqrt{\frac{L}{C}}}$$

We can see that at resonance voltage amplitude over capacitor and coil is bigger than voltage of source !

Voltage of the resistor is

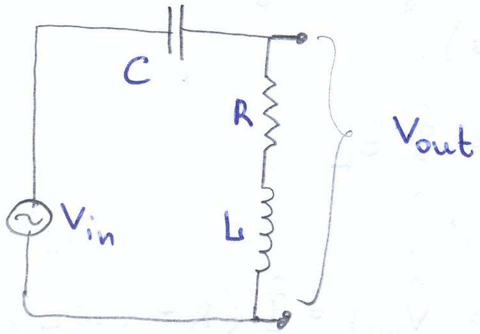
$$\underline{V_R = I_0 \cdot R} \Rightarrow \quad \underline{\underline{V_R}} = 1.7\ A \cdot 70.6\,\Omega \approx \underline{120\ V}$$

Note: Voltages over the capacitor and coil are shifted with opposite phase, so their total contribution is zero at resonance.

№ 13.2 (#31.49)

R, L, C V_{in}, ω
$\frac{V_{out}}{V_{in}}$ - ?

The sketch of the problem



a) In order to define Vout we can use complex reactance of the elements in the circuit:

$$\hat{z}_R = R \quad - \text{ real}$$

$$\hat{z}_L = i\omega L = iX_L; \qquad \hat{z}_c = \frac{X_c}{i} = -iX_c = \frac{1}{i\omega C}$$

The elements R, L and C are in series, so the total impedance

$$\hat{z} = \hat{z}_c + \hat{z}_R + \hat{z}_L$$

$$\Rightarrow \hat{z} = R + i(X_L - X_c)$$

Therefore the impedance (its real value, that corresponds to relation of current and voltage amplitudes):

$$z = \text{Re}.\hat{z} = \sqrt{\hat{z}\hat{z}^*} = \sqrt{(R + i(X_L - X_c))(R - i(X_L - X_c))}$$

$$\Rightarrow z = \sqrt{R^2 + (X_L - X_c)^2}$$

So ~~the~~ magnitude of the current through the circuit is

$$I = \frac{V_{in}}{z} \Rightarrow I = \frac{V_{in}}{\sqrt{R^2 + (X_L - X_c)^2}}$$

Or in complex form (when we keep phase):

$$\hat{I} = \frac{\hat{V}_{in}}{\hat{z}} \Rightarrow \hat{I} = \frac{\hat{V}_{in}}{R + i(X_L - X_c)}$$

Voltage across R and L - elements is

$$\hat{V}_{out} = \hat{V}_R + \hat{V}_c,$$

where $\hat{V}_R$, $\hat{V}_L$ are complex voltages across ~~to~~ the resistor and solenoid.
(Note: we should use complex voltage in general case, because we have to include phase ~~shift~~).

$$\hat{V}_R = \hat{I}\,\hat{Z}_R \qquad \text{and} \qquad \hat{V}_L = \hat{I}\,\hat{Z}_L .$$

Therefore:

$$\hat{V}_{out} = \hat{I}\,(\hat{Z}_R + \hat{Z}_L) \quad \Rightarrow \quad \hat{V}_{out} = \hat{I}\,(R + i\overbrace{\omega L}^{X_L})$$

or for the magnitudes:

$$V_{out} = I \cdot \sqrt{R^2 + X_L^2}$$

Finally we obtain that

$$\hat{V}_{out} = \hat{V}_{in}\,\frac{\hat{Z}_R + \hat{Z}_L}{\hat{Z}} = \hat{V}_{in}\,\frac{\hat{Z}_R + \hat{Z}_L}{\hat{Z}_R + \hat{Z}_L + \hat{Z}_C} = \frac{\hat{V}_{in}}{1 + \dfrac{\hat{Z}_C}{\hat{Z}_R + \hat{Z}_L}}$$

or in explicit form:

$$\hat{V}_{out} = \frac{\hat{V}_{in}}{1 - \dfrac{iX_C}{R + iX_L}} = \frac{\hat{V}_{in}}{1 - i\,\dfrac{1/\omega C}{R + i\omega L}} .$$

For magnitude we have:

$$V_{out} = V_{in}\,\frac{|\hat{Z}_R + \hat{Z}_L|}{Z} = V_{in}\,\frac{\sqrt{R^2 + X_L^2}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The relation is

$$\boxed{\frac{V_{out}}{V_{in}} = \sqrt{\frac{R^2 + X_L^2}{R^2 + (X_L - X_C)^2}} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}\;;$$

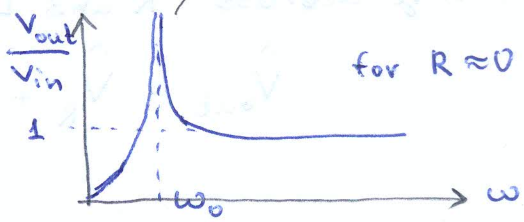
or by introducing

$$\omega_0 = \frac{1}{\sqrt{CL}} \quad \Rightarrow \quad \boxed{\frac{V_{out}}{V_{in}} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + (\omega L)^2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2}}}$$

height depends on R.

In case when R is smal:

$$\left.\frac{V_{out}}{V_{in}}\right|_{R \approx 0} = \frac{1}{\left|1 - \frac{\omega_0^2}{\omega^2}\right|} = \frac{\omega^2}{|\omega^2 - \omega_0^2|}$$



b) For low frequency : $\omega \to 0$:

$$\left.\frac{V_{out}}{V_{in}}\right|_{\omega \to 0} = \lim_{\omega \to 0} \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \frac{\cancel{\omega^2} L^2}{\omega^4}(\omega^2 - \omega_0^2)^2}} =$$

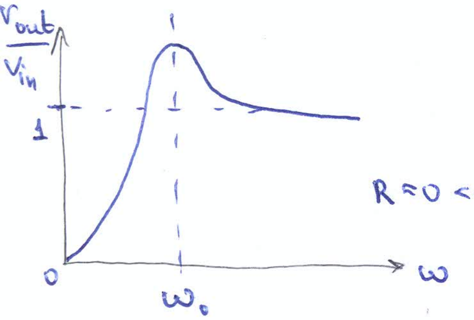
$$= \lim_{\omega \to 0} \omega \sqrt{\frac{R^2 + \underbrace{\omega^2 L^2}_{\to 0}}{\underbrace{R^2 \omega^2}_{0} + L^2(\underbrace{\omega^2}_{0} - \omega_0^2)^2}} = \lim_{\omega \to 0} \omega \cdot \sqrt{\frac{R^2}{L^2 \omega_0^4}} = 0$$

c) For high frequency: $\omega \to \infty$ $(\omega >> \omega_0)$:

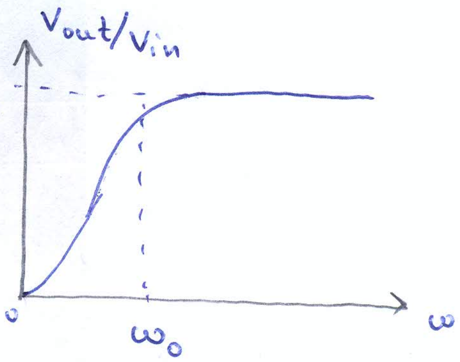
$$\left.\frac{V_{out}}{V_{in}}\right|_{\omega \to \infty} = \lim_{\omega \to \infty} \sqrt{\frac{\underbrace{R^2/\omega^2}_{\to 0} + L^2}{\underbrace{R^2/\omega^2}_{0} + L^2\left(1 - \underbrace{\frac{\omega_0^2}{\omega^2}}_{0}\right)^2}} = \sqrt{\frac{L^2}{L^2}} = 1.$$

Therefore we have high-pass filter with corner frequenc is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$R \approx 0 << \omega L$



$R \sim \omega L$

SI | № 13.3 (#31.59)

Given	SI
$\varphi = 54^\circ$	$\frac{54^\circ}{180^\circ}\pi = 0.3\pi$
$X_c = 350\,\Omega$	
$R = 180\,\Omega$	
$\bar{P} = 140\ W$	
$X_L - ?$	
$I_{rms} - ?$	
$V_{rms} - ?$	

a) Phase angle of RLC-circuit is given by

$$\tan\varphi_0 = \frac{X_L - X_c}{R}$$

That gives

$$X_L = X_c + \tan\varphi_0 \cdot R$$

where φ_0 is phase angle when voltage leads current. But in our case current leads the voltage (the voltage lags the current), so φ_0 is negative:

$$\varphi_0 = -\varphi \quad \Rightarrow \quad \tan\varphi_0 = -\tan\varphi$$

Therefore:

$$\boxed{X_L = X_c - R\cdot\tan\varphi} \qquad \text{and} \qquad X_L < X_c$$

or numerical value is

$$X_L = 350\,\Omega - 180\,\Omega\cdot\underbrace{\tan 54^\circ}_{\approx 1.376} \approx 102.3\,\Omega$$

b) The mean effect is given by:

$$\bar{P} = I_{rms}^2 \cdot R$$

(because ~~only~~ resistor only consumes energy).

So

$$\boxed{I_{rms} = \sqrt{\frac{\bar{P}}{R}}} \qquad \text{or} \qquad I_{rms} = \sqrt{\frac{140\ W}{180\,\Omega}} \approx 0.882\ A$$

c) On the other hand the mean effect is

$$\bar{P} = V_{rms}\cdot I_{rms}\cdot\cos\varphi,$$

where $\cos\varphi$ is power factor of the circuit.

That gives

$$V_{rms} = \frac{\bar{P}}{I_{rms}\cdot\cos\varphi} \qquad \text{or} \qquad \boxed{V_{rms} = \frac{1}{\cos\varphi}\sqrt{\bar{P}R}}$$

Or

$$V_{rms} = \frac{1}{\underbrace{\cos 54^\circ}_{\approx 0.588}}\sqrt{140\ W\cdot 180\,\Omega} \approx 270\ V$$

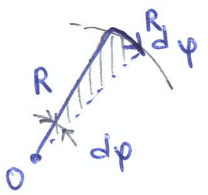
№ 13.4 $\quad d\mathcal{E} = (\vec{v}\times\vec{B})\,d\vec{l} \quad F_M = q(\vec{v}\times\vec{B}) \quad F_E = q\vec{E}$

R, ω, B
$\mathcal{E}$ - ?

a) In order to define emf we can use Faraday's law

$$\mathcal{E} = -\frac{d\varphi}{dt}$$

by considering imaginary loop duaring rotation of the disk



In time dt area of the loop changes on

$$dA = \frac{1}{2} R \cdot R\,d\varphi \quad \text{- area of triangle}$$

Therefore the magnitude of emf is

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left[\, d\Phi = B\cdot dA \,\right] = B\frac{dA}{dt} \quad \text{- for } B = \text{const.}$$

so

$$\underline{\underline{\mathcal{E}}} = B\cdot\frac{dA}{dt} = \frac{1}{2}BR^2\cdot\underbrace{\frac{d\varphi}{dt}}_{=\omega} = \underline{\underline{\frac{1}{2}BR^2\omega}}$$

b) The same result can be obtained by considering magnetic force:

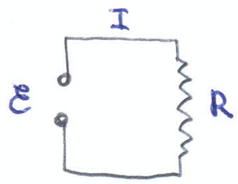
$$d\mathcal{E} = (\vec{v}\times\vec{B})\,d\vec{l} \quad \text{- for each segment}$$

that gives finally $(v = \omega r)$

$$\underline{\underline{\mathcal{E}}} = B\omega\int_0^R r\,dr = \underline{\underline{\frac{1}{2}B\omega R^2}}$$

, see #11.3

c) Due to energy conservatio law we must provide the same energy that is consumed by the resistor.



Due to Ohm's law:

$$I = \frac{\mathcal{E}}{R_\Omega}$$

So the dissipated energy in the resistor per secont is

$$P_\mathcal{E} = I^2R_\Omega = \frac{\mathcal{E}^2}{R_\Omega}$$

Therefore we must provide the mecanical power is

$$\underline{\underline{P_M}} = P_\mathcal{E} = \frac{\mathcal{E}^2}{R_\Omega} = \underline{\underline{\frac{1}{4}\frac{B^2\omega^2R^4}{R_\Omega}}}$$