Equations sheet for FYS1120

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Electric fields

Coulomb's law

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r^2}\hat{\mathbf{r}}\,d\tau$$

Dipoles

$$\tau = \boldsymbol{\mu}\times\mathbf{B} \qquad \tau = \mathbf{p}\times\mathbf{E}$$

$$\boldsymbol{\mu} = I\mathbf{A} \qquad \mathbf{p} = q\mathbf{d}$$

$$U_B = -\boldsymbol{\mu}\cdot\mathbf{B} \qquad U_E = -\mathbf{p}\cdot\mathbf{E}$$

Potential, energy and work

$$W_{a\to b} = U_a - U_b$$

$$U = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot\mathbf{dl}$$

$$\nabla V = -\mathbf{E}$$

Energy density in electromagnetic field

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

Energy stored in solenoid and capacitor:

$$U_B = \frac{1}{2}LI^2, \quad U_E = \frac{1}{2}\frac{Q^2}{C}$$

Maxwell's equations

In general

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \oint_S \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \oint_S \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot \mathrm{dl} = -\frac{d\Phi_B}{dt}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}$$

$$\oint_L \mathbf{B}\cdot \mathrm{dl} = \mu_0\left(I_c + \epsilon_0\frac{d\Phi_E}{dt}\right)_{encl}$$

In matter

$$\nabla\cdot\mathbf{D} = \rho_f \qquad \oint_S \mathbf{D}\cdot \mathrm{d}\mathbf{A} = Q_{f_{enc}}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \oint_S \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot \mathrm{dl} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{A}$$

$$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H}\cdot \mathrm{dl} = I_{f_{enc}} + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{A}$$

Definitions

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

In linear media

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E} \quad \mathbf{D} = \epsilon\mathbf{E}$$
$$\mathbf{M} = \chi_m\mathbf{H} \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

Lorentz force

$$\mathbf{F} = q\cdot\mathbf{v}\times\mathbf{B} + q\cdot\mathbf{E}$$

Magnetism

Flux:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

Magnetic force on a conductor:

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B}$$

Faraday's law and emf

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

All or parts of a closed loop moves in a **B** field:

$$\mathcal{E} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \mathbf{r}}{r^2}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

Self inductance & Mutual inductance

$$L = \frac{N\Phi_B}{i}, \quad \mathcal{E} = -L\frac{di}{dt}$$

$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

$$\mathcal{E}_2 = -M\frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt}$$

Continuity of magnetic flux

$$\oint_S \mu_0 \mathbf{H} \cdot d\mathbf{A} = \mathbf{0}\,.$$

over a closed surface.

Capacitor

$$C = \frac{Q}{V} = \epsilon_0 \epsilon_r \frac{A}{d}$$

Capacitors in series:

$$1/C_{eq} = 1/C_1 + 1/C_2 + ...$$

Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + ...$$

Resistor

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

Resistors in series:

$$R_{eq} = R_1 + R_2 + ...$$

Resistors in parallel:

$$1/R_{eq} = 1/R_1 + 1/R_2 + ...$$

Circuits

Current:

$$I = \frac{dQ}{dt} = n|q|v_d A$$

Effect:

$$P = VI\,.$$

Over ohmic resistance:

$$P = RI^2 = \frac{V^2}{R}\,.$$

RC circuit

Charging capacitor in RC-circuit:

$$q = \mathcal{E}C\left[1 - e^{(-t/(RC))}\right]$$

Decharging capacitor:

$$q = Q_0 e^{(-t/(RC))}$$

RL circuit

$$\mathcal{E} - L\frac{dI}{dt} - RI = 0$$

$$I = \frac{\mathcal{E}}{R}\left[1 - e^{-t(R/L)}\right]$$

Without emf:

$$-L\frac{dI}{dt} - RI = 0$$

$$I = I_m e^{-t(R/L)}$$

LC circuit

$$\frac{q}{C} = L\frac{dI}{dt} \qquad (1)$$

$$\frac{d^2q}{dt^2} - \frac{q}{LC} = 0 \qquad (2)$$

$$\rightarrow q = Q_m \cos(\omega_0 + \phi) \qquad (3)$$

RCL

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \tag{4}$$

$$q = Q_m e^{-t/\tau}\cos(\omega t + \phi)$$

Q_m and ϕ are dependent on the initial conditions.

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \frac{1}{\tau^2}}$$

Driven RCL

Complex current, voltage, impedance with inductive and capacitive reactance.

$$\hat{I} = \frac{V_m}{Z}e^{i\omega t - \phi} \quad \hat{V} = V_m e^{i\omega t}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{V_m}{\sqrt{2}}$$

We have

$$L\frac{d^2\hat{I}}{dt^2} + R\frac{d\hat{I}}{dt} + \frac{\hat{I}}{C} = i\omega V_m e^{i\omega t}$$

Phase difference

$$\tan\phi = \frac{X_L - X_C}{R}$$

Power of RCL

$$\bar{P} = RI_{rms}^2 = V_{rms}I_{rms}\cos\phi$$

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$X = X_L - X_C$$

$$\mathbf{Z} = R + jX = \frac{G - jB}{G^2 + B^2}$$

$$Z = \frac{V_m}{I_m}$$

Reactance

Capacitive

$$X_C = \frac{1}{\omega C}, \quad B = \omega C$$

Inductive

$$X_L = \omega L, \quad B = -\frac{1}{\omega L}$$

Admittance

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB = \frac{R - jX}{R^2 + X^2}$$

Transformers

$$\frac{V2}{V_1} = \frac{N_2}{N_1}$$

$$V_1 I_1 = V_2 I_2$$

Units

Henry:

$$\mathrm{H} = \frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^2 \cdot A^2}} = \frac{\mathrm{J}}{\mathrm{A^2}} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V \cdot s}}{\mathrm{A}} \tag{5}$$

$$= \frac{\mathrm{J/C \cdot s}}{\mathrm{C/s}} = \frac{\mathrm{J \cdot s^2}}{\mathrm{C^2}} = \frac{\mathrm{m^2 \cdot kg}}{\mathrm{C^2}} = \Omega \cdot \mathrm{s} \tag{6}$$

Ampere:

$$\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}}$$

Tesla:

$$\mathrm{T} = \frac{\mathrm{V \cdot s}}{\mathrm{m^2}} = \frac{\mathrm{N}}{\mathrm{A \cdot m}} = \frac{\mathrm{Wb}}{\mathrm{m^2}} = \frac{\mathrm{kg}}{\mathrm{C \cdot s}} = \frac{\mathrm{kg}}{\mathrm{A \cdot s^2}}$$

1 Constants

Proton mass

$$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$$

Proton charge

$$q_p = 1e = 1.602 \times 10^{-19}\,\mathrm{C}$$

Electron mass

$$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$$

Electron charge

$$q_e = -1e = -1.602 \times 10^{-19}\,\mathrm{C}$$

Electrical permittivity in vacuum

$$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$$

Gravitational constant

$$G = 6.67 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$$

Magnetic permeability

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7}\,\mathrm{Tm/A}$$

Speed of light

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$