

# Equations sheet for FYS1120

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## Electric fields

### Coulomb's law

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho}{r^2} \hat{\mathbf{r}} d\tau$$

### Dipoles

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \quad \boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

$$\boldsymbol{\mu} = IA \quad \mathbf{p} = q\mathbf{d}$$

$$U_B = -\boldsymbol{\mu} \cdot \mathbf{B} \quad U_E = -\mathbf{p} \cdot \mathbf{E}$$

## Potential, energy and work

$$W_{a \rightarrow b} = U_a - U_b$$

$$U = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{l}$$

$$\nabla V = -\mathbf{E}$$

Energy density in electromagnetic field

$$u = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$

Energy stored in solenoid and capacitor:

$$U_B = \frac{1}{2} LI^2, \quad U_E = \frac{1}{2} \frac{Q^2}{C}$$

## Maxwell's equations

### In general

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\oint_L \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left( I_c + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}}$$

### In matter

$$\nabla \cdot \mathbf{D} = \rho_f \quad \oint_S \mathbf{D} \cdot d\mathbf{A} = Q_{f \text{encl}}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \quad \oint_L \mathbf{H} \cdot d\mathbf{l} = I_{f \text{encl}} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{A}$$

### Definitions

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

### In linear media

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad \mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{M} = \chi_m \mathbf{H} \quad \mathbf{H} = \frac{1}{\mu} \mathbf{B}$$

### Lorentz force

$$\mathbf{F} = q \cdot \mathbf{v} \times \mathbf{B} + q \cdot \mathbf{E}$$

## Magnetism

Flux:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

Magnetic force on a conductor:

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B}$$

## Faraday's law and emf

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

All or parts of a closed loop moves in a  $\mathbf{B}$  field:

$$\mathcal{E} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

## Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \mathbf{r}}{r^2}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

## Self inductance & Mutual inductance

$$L = \frac{N\Phi_B}{i}, \quad \mathcal{E} = -L \frac{di}{dt}$$

$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

## Continuity of magnetic flux

$$\oint_S \mu_0 \mathbf{H} \cdot d\mathbf{A} = \mathbf{0}.$$

over a closed surface.

## Capacitor

$$C = \frac{Q}{V} = \epsilon_0 \epsilon_r \frac{A}{d}$$

Capacitors in series:

$$1/C_{eq} = 1/C_1 + 1/C_2 + \dots$$

Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + \dots$$

## Resistor

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

Resistors in series:

$$R_{eq} = R_1 + R_2 + \dots$$

Resistors in parallel:

$$1/R_{eq} = 1/R_1 + 1/R_2 + \dots$$

## Circuits

Current:

$$I = \frac{dQ}{dt} = n|q|v_d A$$

Effect:

$$P = VI.$$

Over ohmic resistance:

$$P = RI^2 = \frac{V^2}{R}.$$

## RC circuit

Charging capacitor in  $RC$ -circuit:

$$q = \mathcal{E}C \left[ 1 - e^{-t/(RC)} \right]$$

Discharging capacitor:

$$q = Q_0 e^{-t/(RC)}$$

## RL circuit

$$\mathcal{E} - L \frac{dI}{dt} - RI = 0$$

$$I = \frac{\mathcal{E}}{R} \left[ 1 - e^{-t(R/L)} \right]$$

Without emf:

$$-L \frac{dI}{dt} - RI = 0$$

$$I = I_m e^{-t(R/L)}$$

## LC circuit

$$\frac{q}{C} = -L \frac{dI}{dt} \quad (1)$$

$$\frac{d^2 q}{dt^2} + \frac{q}{LC} = 0 \quad (2)$$

## RCL

$$\frac{d^2 q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = 0 \quad (3)$$

$$q = e^{-t/\tau} (A \sin \omega t + B \cos \omega t)$$

$A$  and  $B$  are dependent on the initial conditions.

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \frac{1}{\tau^2}}$$

## Driven RCL

Complex current, voltage, impedance with inductive and capacitive reactance.

$$\hat{I} = \frac{V_m}{Z} e^{i\omega t - \phi} \quad \hat{V} = V_m e^{i\omega t}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

We have

$$L \frac{d^2 \hat{I}}{dt^2} + R \frac{d\hat{I}}{dt} + \frac{\hat{I}}{C} = i\omega V_m e^{i\omega t}$$

## Phase difference

$$\tan \phi = \frac{X_L - X_C}{R}$$

## Power of RCL

$$\bar{P} = RI_{rms}^2 = V_{rms} I_{rms} \cos \phi$$

## Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$X = X_L - X_C$$

$$\mathbf{Z} = R + jX = \frac{G - jB}{G^2 + B^2}$$

$$Z = \frac{V_m}{I_m}$$

## Reactance

Capacitive

$$X_C = -\frac{1}{\omega C}, \quad B = \omega C$$

Inductive

$$X_L = \omega L, \quad B = -\frac{1}{\omega L}$$

## Admittance

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB = \frac{R - jX}{R^2 + X^2}$$

## Transformers

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_1 I_1 = V_2 I_2$$

## Units

Henry:

$$H = \frac{\text{m}^2 \cdot \text{kg}}{\text{s}^2 \cdot \text{A}^2} = \frac{\text{J}}{\text{A}^2} = \frac{\text{Wb}}{\text{A}} = \frac{\text{V} \cdot \text{s}}{\text{A}} \quad (4)$$

$$= \frac{\text{J}/\text{C} \cdot \text{s}}{\text{C}/\text{s}} = \frac{\text{J} \cdot \text{s}^2}{\text{C}^2} = \frac{\text{m}^2 \cdot \text{kg}}{\text{C}^2} = \Omega \cdot \text{s} \quad (5)$$

Ampere:

$$A = \frac{C}{s}$$

Tesla:

$$T = \frac{\text{V} \cdot \text{s}}{\text{m}^2} = \frac{\text{N}}{\text{A} \cdot \text{m}} = \frac{\text{Wb}}{\text{m}^2} = \frac{\text{kg}}{\text{C} \cdot \text{s}} = \frac{\text{kg}}{\text{A} \cdot \text{s}^2}$$

## 1 Constants

Proton mass

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

Proton charge

$$q_p = 1e = 1.602 \times 10^{-19} \text{ C}$$

Electron mass

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

Electron charge

$$q_e = -1e = -1.602 \times 10^{-19} \text{ C}$$

Electrical permittivity in vacuum

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$$

Gravitational constant

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$$

Magnetic permeability

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ Tm/A}$$

Speed of light

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$