UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: FYS1120 – Electromagnetism
Day of exam: 6. December 2012
Exam hours: 14:30 – 18:30
This examination paper consists of: 2 pages
Appendices: Collection of equations (3 pages)
Permitted materials: The attached collection of equations and calculator
Angell (eller Øgrim) og Lian: Fysiske størrelser og enheter
Rottman: Matematisk formelsamling

Make sure that your copy of this examination paper is complete before answering.

Problem 1

a) Light bulb L1 has twice the resistance of light bulb L2 (we assume the resistance to be constant – i.e. independent of the current through the light bulbs). We then connect the light bulbs to a battery. Which of the bulbs will emit the strongest light if they are connected in series? Explain why.

De er koblet i serie, altså er strømmen den samme gjennom pærene. Siden $P = VI = RI^2$, så vil effekten være proporsjonal med resistansen, og pæra med høyest resistans vil lyse sterkest.

b) What if they are connected in parallel – which one will then emit the strongest light? Explain why.

Her vil spenningen over pærene være den samme (for begge pærene).
Siden $P = VI = V^2 / R$, så vil effekten være omvendt proporsjonal med resistansen, og pæra med lavest resistans vil lyse sterkest.

Problem 2

Two wires are suspended 15 m over the ground with a distance of 3 m between them. Each of the wires carries a current of 10 A, but in opposite direction.

a) Find the magnitude and direction of the mutual magnetic force per 100 m of wire.

$$\vec{F} = I\vec{l} \times \vec{B} = I_2 lB = I_2 l \frac{\mu_0 I_1}{2\pi R} \quad \Rightarrow \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi R}$$

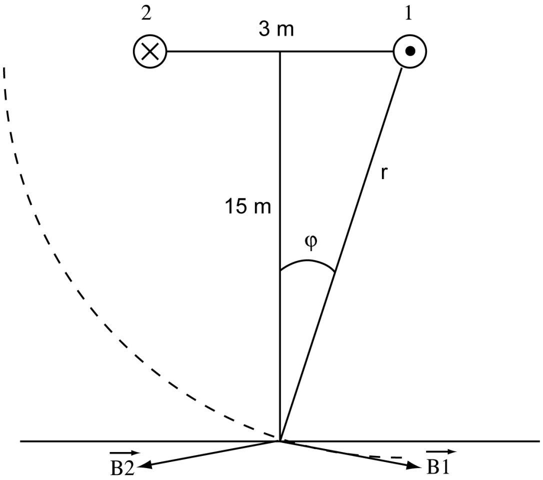
der I_1 og I_2 er strømmene i de to ledningene og R=3m er avstanden mellom ledningene.

Innsatt gir dette F = $\underline{6{,}7 \cdot 10^{-4}\ \text{N}}$ (frastøtende kraft).

b) Write down Ampere's law and explain all the symbols in the equation. Then use this law to calculate the magnitude of the magnetic field at a point on the ground at the midpoint between the wires.

Se boka for Amperes lov …

Vi velger en Ampère-vei som følger en feltlinje og som også går gjennom det aktuelle punktet på bakken (se figuren hvor deler av Ampère-veien er stiplet). Gjør vi dette for ledning nummer 1 (se figuren), så får vi regnet ut $\vec{B1}$. Horisontalkomponenten av $\vec{B1}$ vil kanselleres av horisontalkomponenten av $\vec{B2}$, slik at det totale feltet blir vertikalt nedover på figuren og lik to ganger vertikalkomponenten av $\vec{B1}$.



Ampèreveiens radius: $r = \sqrt{15^2 + 1{,}5^2} = \underline{15{,}07}\ \text{m}$

Ampèreveiens omkrets: $l = 2\pi r = \underline{94{,}7}\ \text{m}$

Vinkel φ = $\arctan\frac{1{,}5}{15} = \underline{5{,}71°}$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \cdot I_{innenfor} \Rightarrow B \cdot l = \mu_0 \cdot I_{innenfor}$$

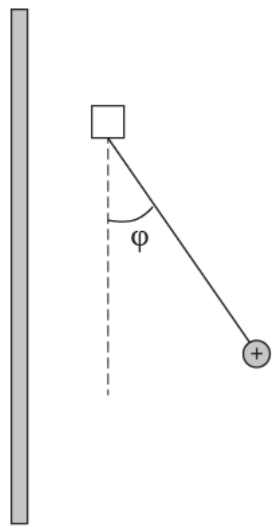
$$B1 = \frac{4\pi \cdot 10^{-7} \cdot 10}{94{,}7} = \underline{1{,}33 \cdot 10^{-7} [T]}$$

Total $B = 2 \cdot B1 \cdot \sin 5{,}71° = \underline{\underline{2{,}65 \cdot 10^{-8} [T]}}$ vertikalt nedover.

Problem 3

A small sphere having a mass of 1.0 gram carries a charge of 100 μC. It is suspended in a thin wire (without mass) close to a large, thin, vertical sheet that carries a uniform charge density of 2.5 nC/m^2.

a) Write down Gauss' law and explain all the symbols in the equation. Then use this law to calculate the electric field outside the sheet.



For Gauss' lov, se boka …

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} \Rightarrow 2EA = \frac{qA}{\varepsilon_0} \Rightarrow E = \frac{q}{2\varepsilon_0} = \underline{\underline{141.2 \tfrac{V}{m}}}$$

hvor q er ladningstetthet.

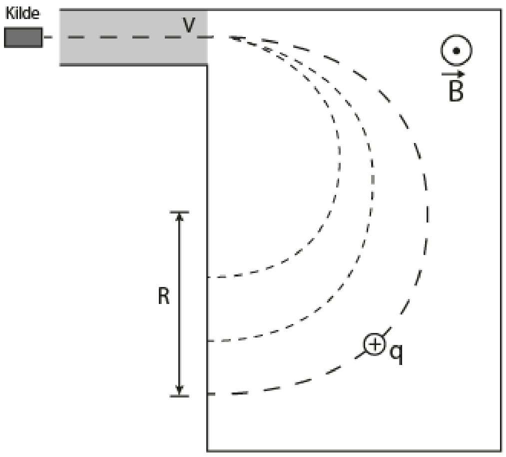
b) Find the angle φ of the wire, as shown in the figure to the right.

Horisontalt: $F = E \cdot Q = 141.2 \cdot 10^{-4} = \underline{0.0141 N}$

Vertikalt: $\text{F} = \text{m} \cdot \text{g} = 10^{-3} \cdot 9.8 = \underline{0.0098 N}$

$$Atg\frac{0.0141}{0.0098} = \underline{\underline{55.20°}}$$

Problem 4



A mass spectrograph is used to measure the masses of ions, or to separate ions of different masses. In the left figure, ions of mass *m* and charge *q* are accelerated through a potential difference *V* (grey area in the figure). The ions have no velocity before they enter into this potential difference.

They then enter into a uniform magnetic field *B* that is perpendicular to their velocity, and they are deflected in a semicircular path of radius *R*. A detector measures where the ions complete the semicircle and *R* is calculated from this.

The set-up is shown in the figure to the left.

a) Show that the equation for calculating the mass of the ions from measurements of *B, V, R* and *q*, is:

$$m = \frac{q\,B^2\,R^2}{2V}$$

$$V = \frac{E}{q} = \frac{\frac{1}{2}mv^2}{q} = \frac{mv^2}{2q} \Rightarrow v = \sqrt{\frac{2Vq}{m}}$$

$$F = qvB = m\frac{v^2}{R} \Rightarrow m = \frac{qBR}{v} = \frac{qBR}{\sqrt{\frac{2Vq}{m}}}$$, som utregnet gir svaret over

b) What potential difference *V* is needed so that single ionized ^{12}C (mass = 2.00×10^{-26} kg and positive charge = 1.602×10^{-19} C) will have *R* = 50.0 cm in a 0.150 T magnetic field?

Bare å sette inn i likningen over (løst mhp V): V = **22.53 kV**

c) Suppose the beam consists of a mixture of ^{12}C and ^{14}C ions. Calculate the distance between these two isotopes at the detector, if *V* and *B* have the same values as in part b). ^{14}C has a mass of 2.34×10^{-26} kg and the same charge as the ^{12}C ions.

Vi ser av likningen at: $\frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{2,34}{2,00}} = 1,08 \Rightarrow R_1 = 50cm \cdot 1,08 = 54,1cm$

Forskjell i diameter blir da 2·4,1 cm = **8,2 cm**

d) We now leave the mass spectrograph and consider a general case: If a charged particle moves in a straight line through space, does that mean that there is no magnetic field present? Please explain.

Nei, det betyr det ikke. Partikkelen kan bevege seg parallelt eller antiparallelt med magnetfeltet, eller det kan være et elektrisk felt som akkurat oppveier kraften fra magnetfeltet.

Problem 5

A resistor of 100 kΩ is connected in series with a capacitor of 10 nF.

a) What is the impedance *Z* of this series circuit at a frequency of 200 Hz? Find both the modulus (magnitude) and phase angle.

$$\vec{Z} = R + jX = 10^5 - j\frac{1}{2 \cdot \pi \cdot 200 \cdot 10^{-8}} = 10^5 - j79{,}6 \cdot 10^3$$

$$|Z| = \sqrt{R^2 + X^2} = 128 \cdot 10^3 = \underline{\underline{128\ \text{k}\Omega}}$$

$$\varphi = atg\frac{-79{,}6 \cdot 10^3}{10^5} = \underline{\underline{-38{,}5\ \text{deg}}}$$

b) A parallel connection (circuit) of a resistor and a capacitor has the same impedance at 200 Hz (modulus and phase) as the series connection in part a). Calculate the resistance *R* and capacitance *C* of this parallel connection.

$$G = Y \cdot \cos(-\varphi) = \frac{1}{Z}\cos(-\varphi) = \frac{1}{128 \cdot 10^3}\cos(38{,}5) = 6{,}11 \cdot 10^{-6} \Rightarrow R = \frac{1}{6{,}11 \cdot 10^{-6}} = \underline{\underline{163{,}6\ [\text{k}\Omega]}}$$

$$B = Y \cdot \sin(-\varphi) = \frac{1}{Z}\sin(-\varphi) = \frac{1}{128 \cdot 10^3}\sin(38{,}5) = 4{,}86 \cdot 10^{-6} \Rightarrow$$

$$C = \frac{B}{2\pi f} = \frac{4{,}86 \cdot 10^{-6}}{2 \cdot 3{,}14 \cdot 200} = 3{,}87 \cdot 10^{-9} = \underline{\underline{3{,}87\ [\text{nF}]}}$$

c) A coil is then connected in parallel with the two components in part b). Draw a circuit diagram that shows these three components. What inductance *L* must the coil have in order to make the phase angle of the impedance of this parallel circuit be zero degrees at 200 Hz?

$$L = \frac{1}{2\pi f B} = \frac{1}{2 \cdot 3{,}14 \cdot 200 \cdot 4{,}86 \cdot 10^{-6}} = \underline{\underline{164\ [\text{H}]}}$$