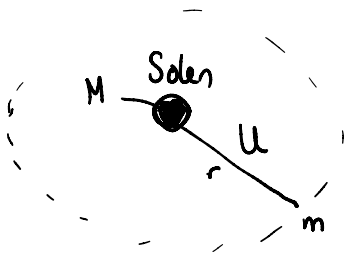
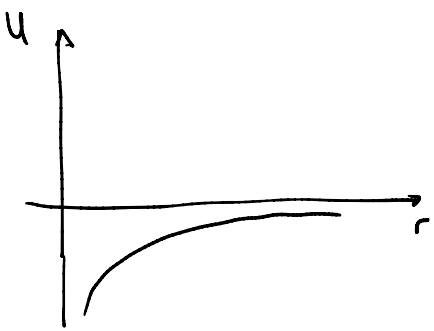
ELEKTRISK POTENSIAL

$$U(r) = -G\frac{mM}{r}$$

$$= -Gm\frac{M}{r}$$

$$= -m\frac{GM}{r}$$

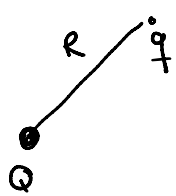




← Coulombs lov

$$\vec{F} = \frac{qQ}{4\pi\epsilon_0}\frac{\hat{R}}{R^2} = \vec{E}(\vec{r})$$

$$= q\,\vec{E}(\vec{r})$$



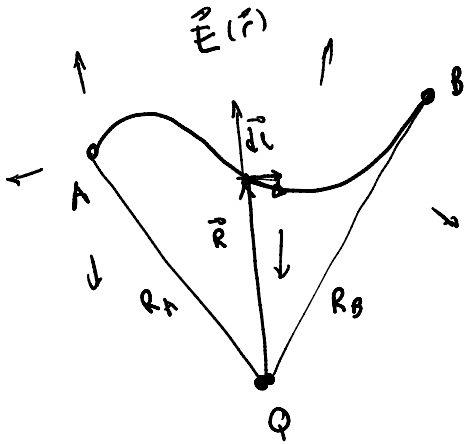
Elektriske felt er konservative!

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{l}$$

$$= q \int_A^B \vec{E} \cdot d\vec{l}$$

$$= q \int_A^B \frac{Q}{4\pi\varepsilon_0} \frac{dl}{R^2}$$

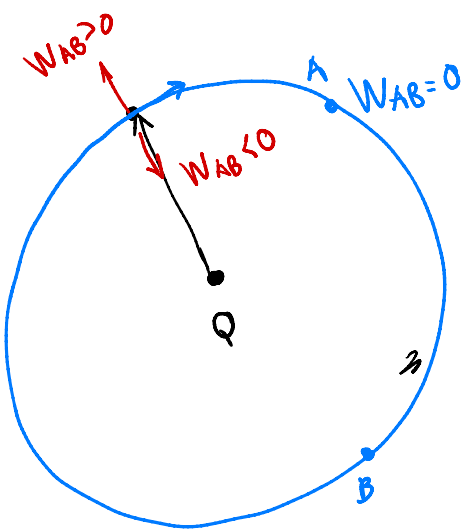
W_{AB}



TEST

$W_{AB} = 0$
> 0
< 0

qQ



POTENSIAL FOR ELEKTRISKE KREFTER

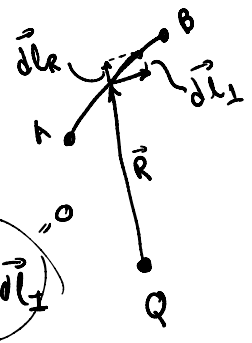
$$U_A = \int_A^{\text{ref}} \vec{F} \cdot d\vec{l} = q \int_A^{\text{ref}} \vec{E} \cdot d\vec{l} = q V_A$$

$\vec{F} = q\vec{E}$

V_A

B = ref

A



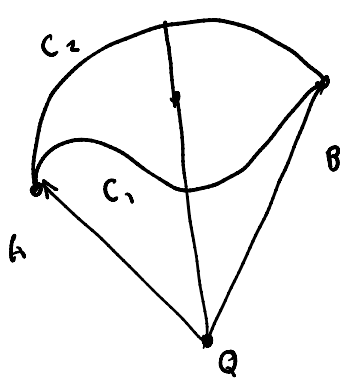
$$d\vec{l} = d\vec{l}_R + d\vec{l}_\perp$$

$$\vec{E} \cdot d\vec{l} = \vec{E} \cdot d\vec{l}_R + \underbrace{\vec{E} \cdot d\vec{l}_\perp}_{=0}$$

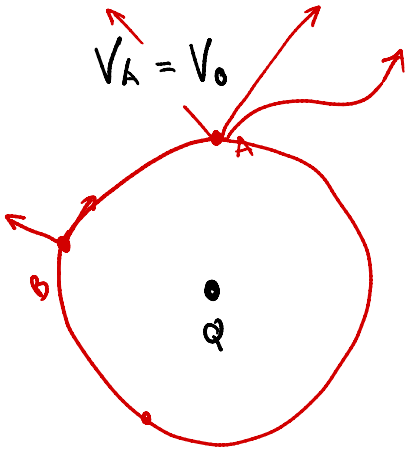
Det elektriske feltet er konservativt!

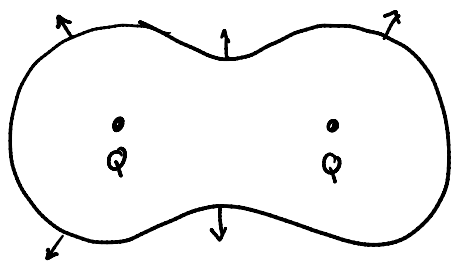
$$V_{AB} = \int_A^B \vec{E} \cdot d\vec{l} \quad \text{er uavhengig av veien}$$

$$V_{C_1} = \int_{A,C_1}^B \vec{E} \cdot d\vec{l} = \int_{A,C_2}^B \vec{E} \cdot d\vec{l} = \int_A^B \frac{Q}{4\pi\varepsilon_0} \frac{dR}{R^2}$$



TEST $V_A = \int_A^{ref} \vec{E} \cdot d\vec{l} = V(\vec{r})$





Egenskaper til det elektriske potensialet

$$V_A = \int_A^{ref} \vec{E} \cdot d\vec{l}$$

a) $V(\vec{r})$ kun av punkt, ikke av banen

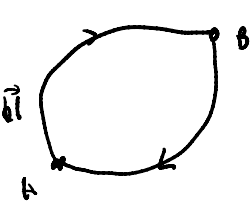
b) $$\vec{E} = \sum_{i=1}^{N} \vec{E}_i \Rightarrow V_A = \int_A^{ref} \vec{E} \cdot d\vec{l} = \int_A^{ref} \sum_i \vec{E}_i \cdot d\vec{l}$$

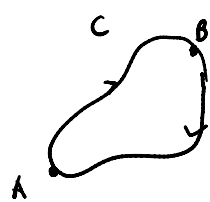
$$= \sum_i \int_A^{ref} \vec{E}_i \cdot d\vec{l} = \sum V_i(\vec{r})$$

c) $V_{AB} + V_{BA} = 0$

$$\oint_C \vec{E} \cdot d\vec{l} = \int_A^B \vec{E} \cdot d\vec{l} + \int_B^{\hat{A}} \vec{E} \cdot d\vec{l}$$

$$= 0$$

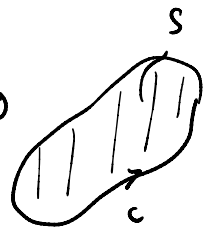




$$\oint_C \vec{E}\cdot d\vec{l} = 0 \qquad \text{(Kirchoffs spenningslov)}$$

d) $$\overbrace{\oint_C \vec{E}\cdot d\vec{l} = \int_S (\nabla \times \vec{E})\cdot d\vec{S}}^{\text{stokes teorem}} = 0$$

$$\nabla \times \vec{E} = 0$$



e)



Newton

$$\vec{F} = -G\frac{mM}{R^2}\hat{R}$$

$$U = -G\frac{mM}{R}$$

Coulomb

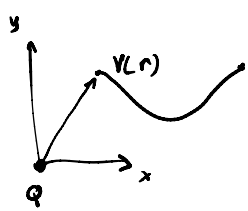
$$\vec{F} = +\frac{qQ}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}$$

$$U = \frac{qQ}{4\pi\epsilon_0}\frac{1}{R} = qV$$

Potensiale for en ladning

$$V(\vec{r}) = \int_{\vec{r}}^{\infty} \vec{E}\cdot d\vec{l} = \int_{\vec{r}}^{\infty} \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\cdot d\vec{l}$$

$$= \int_r^{\infty} \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_r^{\infty}$$



$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

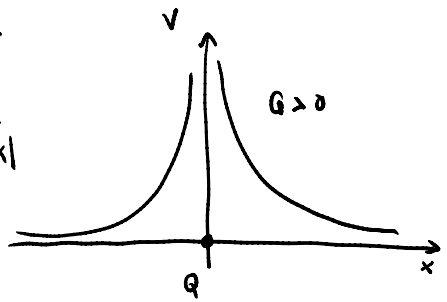
$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}$$

$$V(r) = \frac{Q}{4\pi\epsilon_0 R}$$

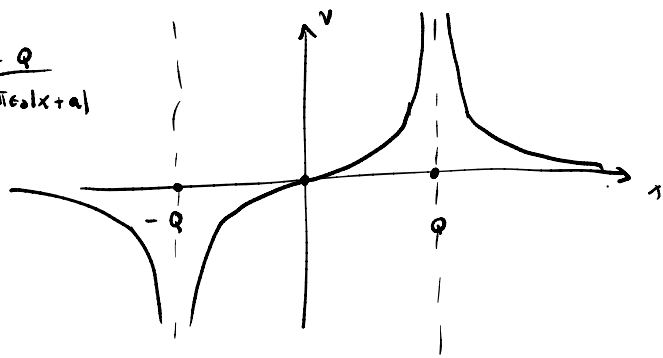
TEST

$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

$$= \frac{Q}{4\pi\epsilon_0 |x|}$$

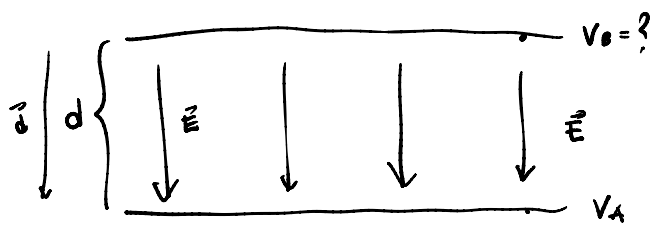


$$V(r) = \frac{Q}{4\pi\epsilon_0 |x-a|} + \frac{-Q}{4\pi\epsilon_0 |x+a|}$$



TEST

Finn V_B



$$V_B = \int_B^{ref} \vec{E} \cdot d\vec{l}$$

$$= \int_B^A \vec{E} \cdot d\vec{l} + \underbrace{\int_{ref}^A \vec{E} \cdot d\vec{l}}_{V_A} = \vec{E} \cdot \underbrace{\int_A^B d\vec{l}}_{\vec{d}} + V_A = \vec{E} \cdot \vec{d} + V_A$$

$$Ed + V_A$$

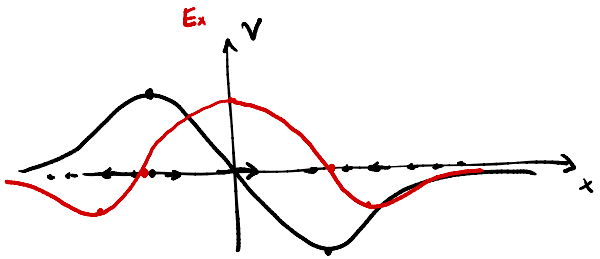
FELT OG POTENSIAL

$$\vec{E}(\vec{r}) \longrightarrow V(\vec{r}) = \int_{\vec{r}}^{\text{ref}} \vec{E} \cdot d\vec{l}$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{-\nabla U}{q} = \frac{-\nabla qV}{q} = -\nabla V$$

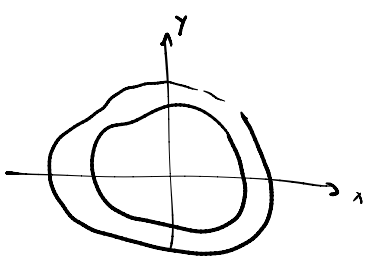
$$V(\vec{r}) \longrightarrow \vec{E} = -\nabla V$$

TEST



$$\vec{E} = -\nabla V = \left(\underbrace{-\frac{\partial V}{\partial x}}_{E_x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

$\vec{E} = -\nabla V$ er normal på ekvipotensialoverflater



Metode

(A) $V(\vec{r}) = \sum V_i$

$= \sum \frac{Q_i}{4\pi\epsilon_0 R_i}$

$= \int_v \frac{\rho(\vec{r}')}{4\pi\epsilon_0 R} dv'$

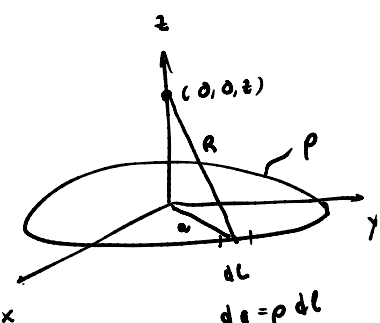


(B) Kjenner $V(\vec{r}) = \int_{\vec{r}}^{ref} \vec{E} \cdot d\vec{l}$

↓ ↓ ↓ ↓

EKSEMPEL

$V(0,0,z) = \int_c \frac{\rho\, dl}{4\pi\epsilon_0 R}$



$R^2 = a^2 + z^2$

$= \int_c \frac{\rho\, dl}{4\pi\epsilon_0\sqrt{a^2+z^2}}$

$dq = \rho\, dl$

$V = \frac{\rho}{4\pi\epsilon_0\sqrt{a^2+z^2}} \underbrace{\int dl}_{2\pi a} = \frac{\overbrace{\rho 2\pi a}^{Q}}{4\pi\epsilon_0\sqrt{a^2+z^2}} = \frac{Q}{4\pi\epsilon_0\sqrt{a^2+z^2}}$

$\vec{E} = -\nabla V = \Rightarrow E_z = -\frac{\partial V}{\partial z}$

$V(\vec{r}) = \frac{Q_i}{4\pi\epsilon_0 R_i}$