Gauss' Lov

Coulombs lov

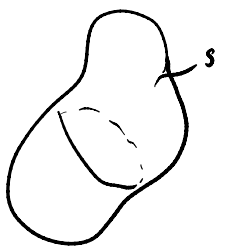
$$\vec{E}(\vec{r}) = \sum_i \frac{Q_i}{4\pi\epsilon_0}\frac{\hat{R}_i}{R_i^2}$$

$$\vec{E} = -\nabla V \qquad V(\vec{r}) = \sum \frac{Q_i}{4\pi\epsilon_0 R_i}$$

$$V = \int_r^{ref} \vec{E}\cdot d\vec{l}$$

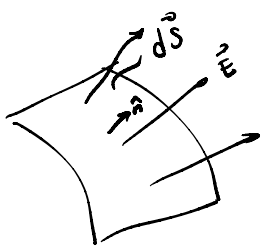
Gauss' lov

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{in}}{\epsilon_0}$$



FLUKS

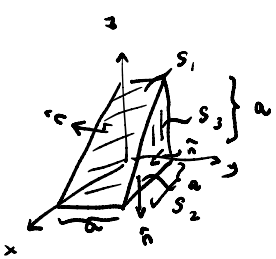
$$d\phi = \vec{E}\cdot d\vec{S}$$
$$dS\,\hat{n}$$



TEST

$$\vec{E} = E_0\hat{x}$$

ϕ_1 , ϕ_2 ϕ_3



$$S_2 : \int_{S_2} \vec{E}\cdot d\vec{S} = \vec{E}\cdot\int_{S_2}\overrightarrow{dS} \qquad \overrightarrow{S_2}$$

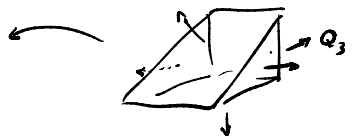
$$\vec{S}_2 = a^2(-\hat{z}) \qquad \phi_2 = \vec{E}\cdot\vec{S}_2 = 0$$

$$\vec{S}_3 = a^2\hat{x} \qquad \phi_3 = \vec{E}\cdot\vec{S}_3 = E_0\hat{x}\cdot a^2\hat{x} = E_0a^2$$

$\Phi_1 = \Phi_3$

$\Phi_1 + \Phi_2 + \Phi_3$

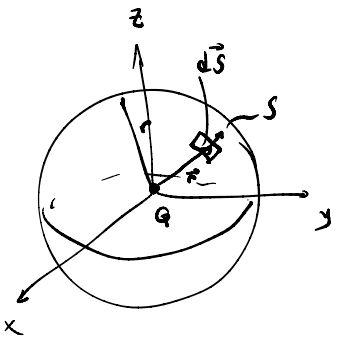
$= 0$



Gauss' law ~~←~~ Coulombs law

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}$$

$$\Phi = \oint_S \vec{E}\cdot d\vec{S}$$



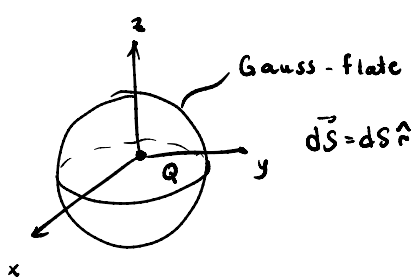
$$dS\,\hat{r}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\Phi = \oint_S \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\cdot dS\,\hat{r} = \frac{Q}{4\pi\epsilon_0 r^2}\overbrace{\oint_S dS}^{4\pi r^2} = \frac{Q}{\epsilon_0}$$

Gauss' law ⟶ Coulombs law

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_0}$$



$$\vec{E} = E_r(r,\phi,\theta)\,\hat{r}$$

$+ E_\theta(r,\phi,\theta)\,\hat{\theta} = 0$

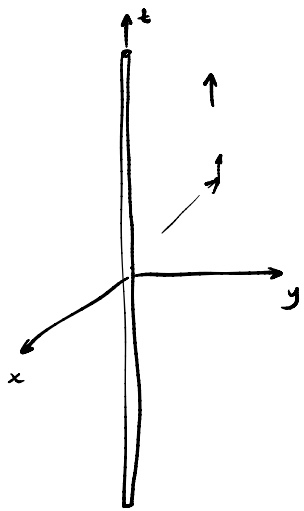
$+ E_\varphi(r,\phi,\theta)\,\hat{\varphi} = 0$

$$= E_r(r)\,\hat{r}$$

Gauss' law

$$\oint_S E_r(r)\,\hat{r}\cdot d\hat{S} = \oint_S E_r(r)\,\hat{r}\cdot\hat{r}\,dS = E_r(r)\overbrace{\oint_S dS}^{4\pi r^2} = \frac{Q}{\epsilon_0}$$

$$E_r(r) = \frac{Q}{4\pi r^2 \epsilon_0}$$



TEST

$\vec{E} = ?$

$E_r(r, \cancel{\varphi}, \cancel{z})\,\hat{r}$

$+ E_\varphi(r, \cancel{\varphi}, \cancel{z})\,\hat{\varphi}$

$+ E_z(r, \cancel{\varphi}, \cancel{z})\,\hat{z}$

$\vec{E} = E_r(r)\,\hat{r}$

METODE for å finne $\vec{E}(\vec{r})$

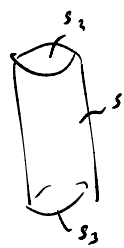
(1) Symmetri

$\vec{E}(\vec{r}) = E_r(r)\,\hat{r}$



(2) Velg Gauss-overflate slik at

$\vec{E} \cdot \hat{n}$ / konstant
\ eller 0



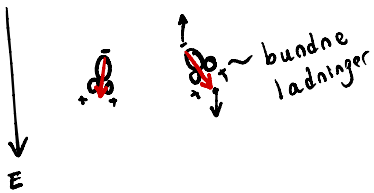
(3) Finn $\oint_S \vec{E} \cdot d\vec{S}$, husk S må være lukket!

(4) Flytter E utenfor integralet, løser for E

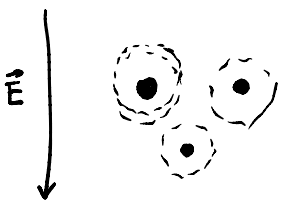
POLARISASJON

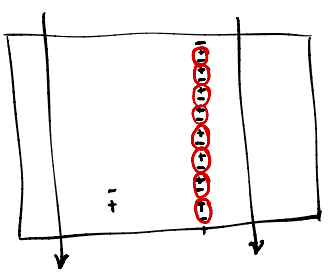
$\vec{E}$ - felt i dielektriske materialer

Vann



Fast stoff

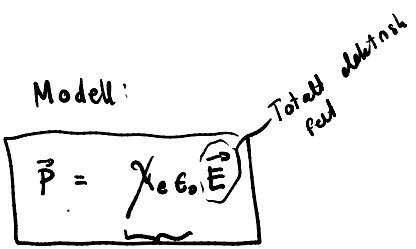


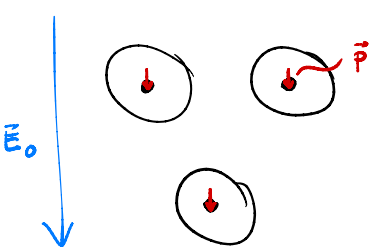




$$\vec{P}\, dv = \sum_{i\, dv} \vec{p}_i$$

Modell:

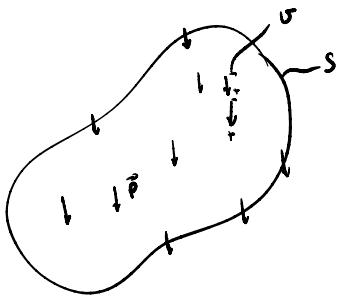




Lineært dielektriske

Bundne ladninger i et volum?

$$Q_b = -\oint_S \vec{P} \cdot d\vec{S}$$



Generalisert Gauss's lov

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{fritt}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (\vec{P} = \chi_e \epsilon_0 \vec{E}) = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E} = \underbrace{\epsilon_0 (1 + \chi_e)}_{\epsilon} \vec{E} \quad \Rightarrow \vec{E} = \frac{1}{\epsilon} \vec{D}$$

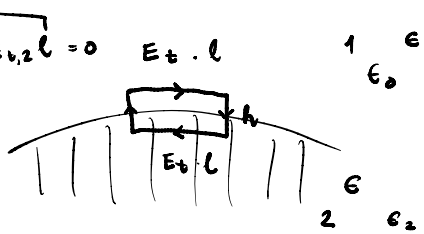
(ϵ_0 over $(1+\chi_e)$)

Grensebetingelser

$$\oint_C \vec{E} \cdot d\vec{l} = 0 \qquad E_{t,1} \cdot l + E_n \cdot h - E_{t,2} \cdot l = 0 \qquad E_t \cdot l$$

$$- E_n h$$

$$E_{t,1} = E_{t,2}$$



$$\oint \vec{D} \cdot d\vec{S} = Q_{fri} = \rho_{s,fri} \cdot S$$

$$D_{1,n} S - D_{n,2} S = \rho_{s,fri} \cdot S$$

$$D_{1,n} - D_{n,2} = \rho_{s,fri}$$

$$\epsilon_1 E_1 - \epsilon_2 E_2 = \rho_{s,fri}$$

