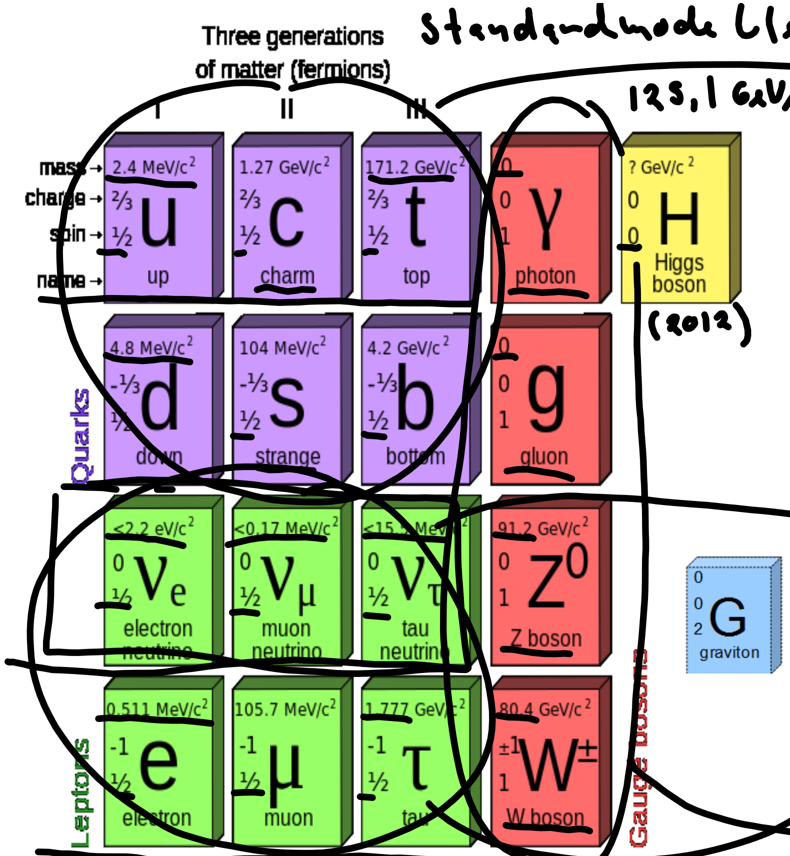
Standardmodellen (SM)



125,1 GeV/c²

(2012)

kvarker

materiepartikler spin-$\frac{1}{2}$

leptoner

Bygger opp

protoner p (uud)

nøytron n (ddu)

atomer p, n og e^-

nøytrinoer

$n \to p^+ e^- \bar{\nu}_e$

kraftbærende partikler spin-1

×2 anti-partikler med motsatt ladning

(untatt $\gamma, g, Z^0, W^\pm, H$) f.eks. positron e^+, anti-up kvark $\bar{u}$

Universets synlige byggesteiner: $\underbrace{p, n, e^-, \nu_e, \nu_\mu, \nu_\tau, \gamma,}_{4\%}$ $\pi^0(\bar{u}u+\bar{d}d), \pi^-(\bar{u}d), \pi^+(u\bar{d})$

Usynlige greier: mørk materie 20%, mørk energi 76%

Henfall

F. eks. $H \to b\bar{b}$.

Kan ikke beskrive henfall med vanlige Hamiltonoperatorer.

Vi tar utgangspunkt i SL for en partikkel i et potensial V

$$\hat{H}_0 \Psi = i\hbar \frac{\partial}{\partial t} \Psi \quad , \quad \hat{H}_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

La nå $\hat{H} = \hat{H}_0 - i\frac{\Gamma}{2}$ hvor $\Gamma \in \mathbb{R}$ (Γ kalles vidde/bredde)

Ikke en Hermitisk operator.

Vi finner de stasjonære tilstandene ved separasjon av variable

$$\Psi(x,t) = \psi(x)\phi(t)$$

Da er $\hat{H}\Psi = i\hbar\frac{\partial}{\partial t}\Psi \Rightarrow \hat{H}_0\Psi = \left(i\hbar\frac{\partial}{\partial t} + i\frac{\Gamma}{2}\right)\Psi$

$$\Rightarrow \phi(t)\hat{H}_0\psi(x) = \psi(x)\left(i\hbar\frac{\partial}{\partial t}\phi(t) + i\frac{\Gamma}{2}\phi(t)\right) \quad \Big| \cdot \frac{1}{\phi(t)\psi(x)}$$

$$\Rightarrow \underline{\frac{1}{\psi(x)}\hat{H}_0\psi(x)} = \frac{1}{\phi(t)}\left(i\hbar\frac{\partial}{\partial t}\phi(t) + i\frac{\Gamma}{2}\phi(t)\right) \underline{= E_0}$$

$$\Rightarrow \hat{H}_0\psi = E_0\psi \quad \text{og} \quad i\hbar\frac{\partial}{\partial t}\phi + i\frac{\Gamma}{2}\phi = E_0\phi$$

Vi kan skrive

$$\frac{d\varphi(t)}{dt} = -\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)\varphi(t)$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{d\varphi(t)}{\varphi(t)} = -\int \frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)dt$$

$$\ln\varphi(t) = -\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t + C'$$

$$|e^{ix}|^2 = 1 \quad x \in \mathbb{R}$$

$$\varphi(t) = e^{-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t} = e^{-\frac{i}{\hbar}E_0 t}\, e^{-\frac{\Gamma}{2\hbar}t}$$

Sannsynligheten for å finne partikkelen (i det hele tatt)

$$P(t) = \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty} |\psi(x)|^2 |\varphi(t)|^2 dx = \int_{-\infty}^{\infty} |\psi(x)|^2 e^{-\frac{\Gamma}{\hbar}t} dx = e^{-\frac{\Gamma}{\hbar}t} \underbrace{\int_{-\infty}^{\infty} |\psi(x)|^2 dx}_{1} = e^{-\frac{\Gamma}{\hbar}t}$$

Vi har fått eksponentielt henfall med levetid

$$\tau = \frac{\hbar}{\Gamma}, \quad P(t) = e^{-t/\tau}$$

Rottmann (Fouriertrans.)

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ipx} g(p)\, dp$$

$$f(x) = e^{-\beta x} e^{i\lambda x}, \quad g(p) = \frac{i}{\lambda + p + i\beta}$$

$$x = t, \quad p = \frac{E}{\hbar}, \quad \beta = \frac{\Gamma}{2\hbar}, \quad \lambda = -\frac{E_0}{\hbar}$$

Hvorfor heter Γ vidde?

Vi fant $\varphi(t) = e^{-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t}$. Dette kan skrives som

$$\varphi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{i}{E - E_0 + i\Gamma/2}\, e^{-\frac{i}{\hbar}Et}\, dE$$

Tolken $\psi(E) = \sqrt{\frac{\Gamma}{2\pi}} \frac{1}{E - E_0 + i\frac{\Gamma}{2}}$ som en sannsynlighet for å finne partikkelen med energien E.

$$\left(\phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{\frac{1}{E - E_0 + i\frac{\Gamma}{2}}}_{\psi(E)} e^{-\frac{i}{\hbar}Et} \, dE \right)$$

Dvs. $|\psi(E)|^2$ er sannsynligheten.

$$\left(\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underline{\Psi(x,0)} \, e^{-ikx} \, dx \right)$$

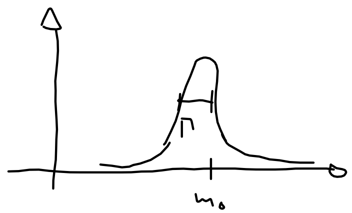
$$|\psi(E)|^2 = \frac{\Gamma}{2\pi} \frac{1}{(E-E_0)^2 + (\Gamma/2)^2}$$

I hvilesystemet til partikkelen $E = mc^2$ og da blir $\left(\frac{dE}{dm} = c^2 \right)$

$$\underline{|\psi(m)|^2} = |\psi(E = mc^2)|^2 \left|\frac{dE}{dm}\right| = c^2 \frac{\Gamma}{2\pi} \frac{1}{(mc^2 - m_0c^2)^2 + (\Gamma/2)^2}$$

$$= \frac{\Gamma}{2\pi c^2} \frac{1}{(m-m_0)^2 + \left(\frac{\Gamma}{2c^2}\right)^2}$$

Breit-Wigner fordelingen (ikke-relativistiske)



$E_0 = m_0 c^2$

$m_Z = 91{,}1876 \pm 0{,}0021 \; GeV/c^2$

$\Gamma/c^2 = 2{,}4952 \pm 0{,}0023 \; GeV/c^2$