Oppgave 1

a) Grunntilstand har lavest energi, $n_1 = n_2 = n_3 = 1$.

$$\underline{E_0} = E_{111} = \frac{\hbar^2\pi^2}{2m}\left(\left(\frac{1}{L}\right)^2 + \left(\frac{1}{L}\right)^2 + \left(\frac{1}{L/2}\right)^2\right)$$

$$= \frac{\hbar^2\pi^2}{2m}\cdot\frac{6}{L^2} = \underline{\frac{3\hbar^2\pi^2}{mL^2}}$$

$$\underline{E_0} = \frac{3\hbar^2c^2\pi^2}{mc^2L^2} = \frac{3\cdot(197{,}3\text{ nm eV})^2\pi^2}{0{,}511\text{ MeV}\cdot(2\text{nm})^2} = \underline{0{,}56\text{ eV}}$$

b) Første eksiterte nivået har nest lavest energi.

Får det ved $\underline{(n_1, n_2, n_3) = (2,1,1)}$ eller $\underline{(1,2,1)}$.

$$\underline{E_1} = E_{211} = \frac{\hbar^2\pi^2}{2m}\left(\left(\frac{2}{L}\right)^2 + \left(\frac{1}{L}\right)^2 + \left(\frac{1}{L/2}\right)^2\right) = \frac{\hbar^2\pi^2}{2m}\left(\frac{4}{L^2} + \frac{1}{L^2} + \frac{4}{L^2}\right)$$

$$= \frac{9\hbar^2\pi^2}{2mL^2} = \underline{\frac{3}{2}E_0}$$

Et degenerert energinivå har flere tilstander med samme energi. Antall slike tilstander er degenerasjonsgraden. E_1 har degenerasjonsgrad 2.

c) Paulis eksklusjonsprinsipp sier at to fermioner ikke kan befinne seg i samme tilstand. Mer generelt er bølgefunksjonen antisymmetrisk ved ombytte av de to fermionene: $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$. $\left(\psi(\vec{r}_1,\vec{r}_2)\chi(m_{s_1}, m_{s_2}) = -\psi(\vec{r}_2,\vec{r}_1)\chi(m_{s_2}, m_{s_1})\right)$

Et elektron har spin-$\frac{1}{2}$, dvs. $m_s = \pm\frac{1}{2}$. Da kan vi ha kun to elektroner per tilstand. E_0 har en tilstand (1,1,1) og derfor plass til to elektroner, E_1 har to tilstander og derfor plass til fire elektroner

$$\underline{E} = 2E_0 + 4E_1 = 2E_0 + 4\cdot\frac{3}{2}E_0 = \underline{8E_0}$$

d) For bosoner kan alle partiklene være i samme tilstand slik at energien er $\underline{E = 6E_0}$.

Med spin-$\frac{3}{2}$ kan vi ha $m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ slik at vi kan ha fire "elektroner" (fermioner) i grunntilstanden og vi får

$$\underline{E} = 4E_0 + 2\cdot E_1 = 4E_0 + 2\cdot\frac{3}{2}E_0 = \underline{7E_0}$$

Oppgave 2

a) Normering av bølgefunksjon krever

$$\int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = 1$$

$$\left(|e^{ix}|^2 = e^{ix} e^{-ix} = 1 \right)$$

Da er

$$\int_{-\infty}^{\infty} |f(x)|^2 |e^{\frac{i}{\hbar}p_0 x}|^2 dx = \underline{\int_{-\infty}^{\infty} |f(x)|^2 dx = 1} \qquad \left(2\int_0^{\infty} |f(x)|^2 dx = 1 \right)$$

b) Forventningverdien er

$$\langle x \rangle_0 = \int_{-\infty}^{\infty} \Psi^*(x,0)\, x\, \Psi(x,0)\, dx = \int_{-\infty}^{\infty} f^*(x) \cancel{e^{-\frac{i}{\hbar}p_0 x}}\, x\, f(x) \cancel{e^{\frac{i}{\hbar}p_0 x}}\, dx$$

$$= \int_{-\infty}^{\infty} x |f(x)|^2 dx = 0$$ pga. $x|f(x)|^2$ er antisymmetrisk.

c) $\underline{\langle p \rangle} = \int_{-\infty}^{\infty} \Psi^*(x,0)\, \hat{p}\, \Psi(x,0)\, dx = \int_{-\infty}^{\infty} \Psi^*(x,0) \left(-i\hbar \frac{\partial}{\partial x}\right) \Psi(x,0)\, dx$

$$= \int_{-\infty}^{\infty} f^*(x)\, e^{-\frac{i}{\hbar} p_0 x} \left(-i\hbar \frac{\partial}{\partial x}\right) f(x)\, e^{\frac{i}{\hbar} p_0 x}\, dx$$

$$= -i\hbar \int_{-\infty}^{\infty} f^*(x) \cancel{e^{-\frac{i}{\hbar} p_0 x}} \left(\frac{\partial f(x)}{\partial x} \cancel{e^{\frac{i}{\hbar} p_0 x}} + f(x) \frac{i}{\hbar} p_0 \cancel{e^{\frac{i}{\hbar} p_0 x}} \right) dx$$

$$= \underbrace{-i\hbar \int_{-\infty}^{\infty} f^*(x) \frac{\partial f(x)}{\partial x}\, dx}_{0 \text{ pga. anti-sym.}} - i\hbar \frac{i}{\hbar} p_0 \underbrace{\int_{-\infty}^{\infty} |f(x)|^2 dx}_{1}$$

$$\underline{= p_0}$$

$f(x) = x^2$

$\frac{df}{dx} = 2x$

d) $\Psi(x,0)$ er en energiegentilstand dersom $\underline{\hat{H}\, \Psi(x,0) = E\, \Psi(x,0)}$
TUSL

der E er en konstant (energi).

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m\omega^2 x^2$$

tidligere

$$\hat{H}\Psi = -\frac{\hbar^2}{2m} \left(\cancel{\frac{m^2\omega^2 x^2}{\hbar^2}} - \frac{2 i m\omega p_0 x}{\hbar^2} - \frac{m\omega}{\hbar} - \frac{p_0^2}{\hbar^2} \right) \Psi(x,0) + \cancel{\frac{1}{2} m\omega^2 x^2\, \Psi(x,0)} \neq E\, \Psi(x,0)$$

Svaret er nei.

c) Forventningsverdien for energien er (ved $t=0$)

$$\underline{\langle E \rangle} = \int_{-\infty}^{\infty} \Psi^*(x,0)\, \hat{H}\, \Psi(x,0)\, dx$$

$$= \int_{-\infty}^{\infty} \Psi^*(x,0) \left(-\frac{\hbar^2}{2m} \left(-\frac{2im\omega p_0 x}{\hbar^2} - \frac{m\omega}{\hbar} - \frac{p_0^2}{\hbar^2} \right) \right) \Psi(x,0)\, dx$$

$$= +\frac{\hbar^2}{2m} \left[\underbrace{\int_{-\infty}^{\infty} \Psi^*(x,0) \frac{2im\omega p_0 x}{\hbar^2} \Psi(x,0)\, dx}_{0 \text{ antisymmetrisk}} + \int_{-\infty}^{\infty} \Psi^* \underline{\left(\frac{m\omega}{\hbar} + \frac{p_0^2}{\hbar^2} \right)} \Psi\, dx \right]$$

$$= \frac{\hbar^2}{2m} \left(\frac{m\omega}{\hbar} + \frac{p_0^2}{\hbar^2} \right) = \underline{\frac{1}{2}\hbar\omega + \frac{p_0^2}{2m}}$$

$$\int_{-\infty}^{\infty} \Psi^* \hat{H} \hat{H} \Psi\, dx = \int_{-\infty}^{\infty} \Psi^* \hat{H} \Psi'\, dx$$

$$\overset{\text{herm.}}{=} \int_{-\infty}^{\infty} (\hat{H}\Psi)^* \Psi'\, dx = \int_{-\infty}^{\infty} \hat{H}\Psi^* \hat{H}\Psi\, dx$$

d) $$\frac{d}{dt}\langle E \rangle = \underline{\frac{d}{dt} \int_{-\infty}^{\infty} \Psi^*(x,t)\, \hat{H}\, \Psi(x,t)\, dx}$$

$$= \int_{-\infty}^{\infty} \left(\frac{\partial \Psi^*}{\partial t} \hat{H} \Psi + \Psi^* \hat{H} \frac{\partial \Psi}{\partial t} \right) dx$$

$$= \int_{-\infty}^{\infty} \underline{\frac{i}{\hbar} \hat{H}\Psi^* \hat{H}\Psi} - \underline{\frac{i}{\hbar} \Psi^* \hat{H} \hat{H} \Psi}\, dx$$

$$= 0$$

$$\frac{\partial \hat{H}}{\partial t} = 0$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi \quad \text{TASL}$$

$$\frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} \hat{H}\Psi, \quad \frac{\partial \Psi^*}{\partial t} = \frac{i}{\hbar} \hat{H}\Psi^*$$