UNIVERSITY OF OSLO

Obligatory assignment 1: FYS2160, Thermodynamics and statistical physics, Fall 2019
Due: September 19. 2019
Hand in on Canvas

Requirement: You have to have made a good faith effort to all assignments in order to pass. All plots need to be discussed and formulas need to be explained. Three obligatory assignments have to be passed in order to take the final written exam.

1 First law of thermodynamics

Consider an ideal diatomic gas in a cylinder with a movable piston. Assume that the temperature is always such that the vibrational degrees of freedom are frozen, but the translational and rotational modes are active.

1.1 *Calculate the internal energy of the diatomic ideal gas with N molecules at a given temperature T. What is its heat capacity at constant volume?*

The diatomic ideal gas undergoes a process through the steps A, B and C shown in Fig. (1).

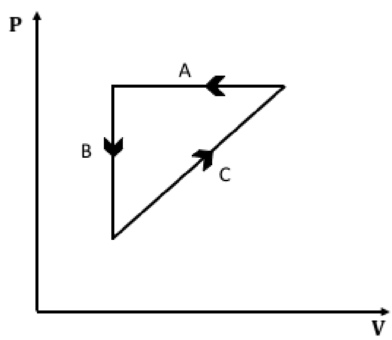

Figure 1: P-V diagram

1.2 *What is the change in the internal energy ΔU along each step and for the cyclic process?*

1.3 *Compute the work done W by the gas along each path.*

1.4 *Compute the heat exchange Q along each path.*

1.5 *Determine the sign of ΔU, W and Q for the whole cycle. Describe what the gas does in each of the steps, when does it absorb or emit heat, when it does work and what does this cycle of energy transformation accomplish.*

2 Multiplicity and Boltzmann's entropy in a paramagnet

Consider a paramagnetic system of N independent spins, where each spin can be in two possible states, $s_i = +1$ (up-spin) or $s_i = -1$ (down-spin) for $i = 1 \cdots N$ with equal probability. Thus for a given configuration, we have N_+ up-spins and N_- down-spins.

2.1 *How many microstates are there in a system of N-spins?*

2.2 *Find an expression of the total net spin, S, for the N-spins system.*

2.3 *Generate $M = 10^4$ microstates for a $N = 60$ spins randomly - assuming that all microstates are equally likely - and plot the spin S of the system for each of the microstate. Plot a histogram of the net spins using for example hist in matlab and compare it with the Gaussian distribution.*

2.4 *Show that the multiplicity of a macrostate $\Omega(N,S)$ with net spin S out of N spins given as*

$$\Omega(N,S) = \frac{N!}{(\frac{N+S}{2})!(\frac{N-S}{2})!} \tag{1}$$

2.5 *Using Stirling's approximation, show that the multiplicity function $\Omega(N,S)$ can be written as*

$$\Omega(N,S) = \Omega_{max} e^{-S^2/(2N)} \tag{2}$$

When is this formula valid? What is the maximum of the multiplicity $\Omega(N,S)$?

2.6 *Compare your analytical result with the histogram you generated of the microstates and comment on the results.*

2.7 *Using Boltzmann's formula $S_B = k \ln \Omega$, find the entropy as a function of N and the net spin S and plot it numerically for $N = 60$ spins.*