

UiO • University of Oslo

Electron gas

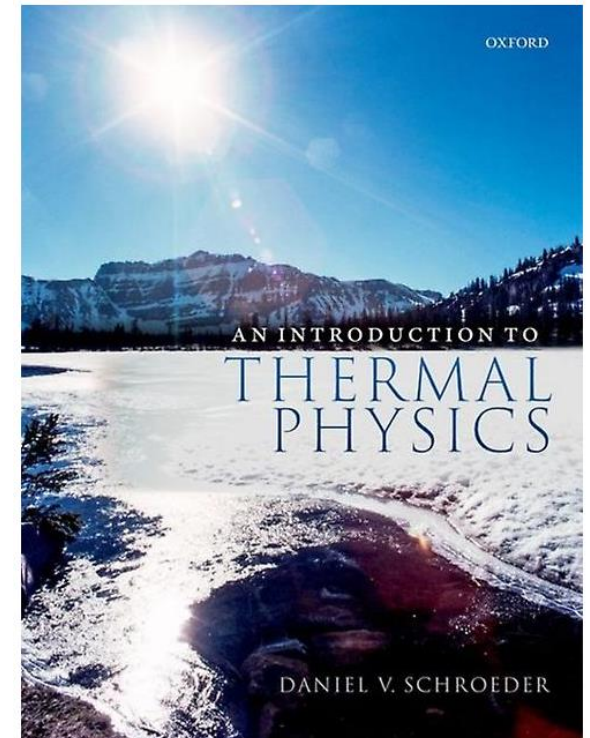
6 Boltzmann Statistics

7 Quantum Statistics

7.1 The Gibbs Factor

7.2 Bosons and Fermions

7.3 Degenerate Fermi Gases



Generalized thermodynamic identity and chemical potential

$$dU = TdS - PdV + \mu dN$$

Fixed U and V :

$$\mu = -T \left(\frac{\partial S}{\partial N} \right)_{U,V}$$

Fixed U and S :

$$\mu = P \left(\frac{\partial V}{\partial N} \right)_{U,S}$$

Fixed V and S :

$$\mu = \left(\frac{\partial U}{\partial N} \right)_{S,V}$$

Chemical potential is the amount by which a system's energy changes when one adds one particle and keeps the entropy and volume fixed. μ has units of energy.

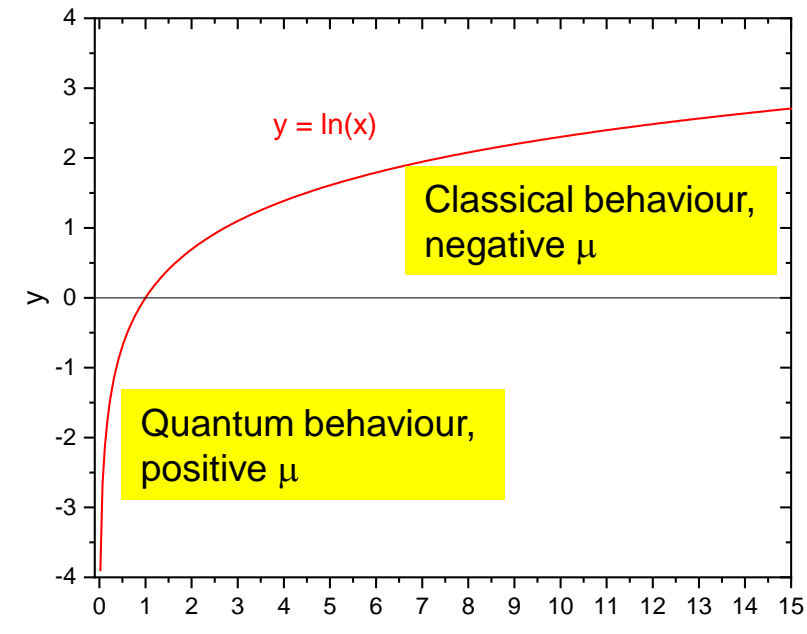
Chemical potential of ideal gas

Sackur-Tetrode equation:
$$S = Nk \left[\ln \left(\frac{V}{N} \left(\frac{4\pi m U}{3N h^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

$$\mu \equiv -T \left(\frac{\partial S}{\partial N} \right)_{U,V}$$

$$\mu = -kT \ln \left[\frac{V}{N} \left(\frac{2\pi m k T}{h^2} \right)^{3/2} \right]$$

$$\vartheta_Q = l_Q^3 = \left(\frac{h}{\sqrt{2\pi m k T}} \right)^3$$



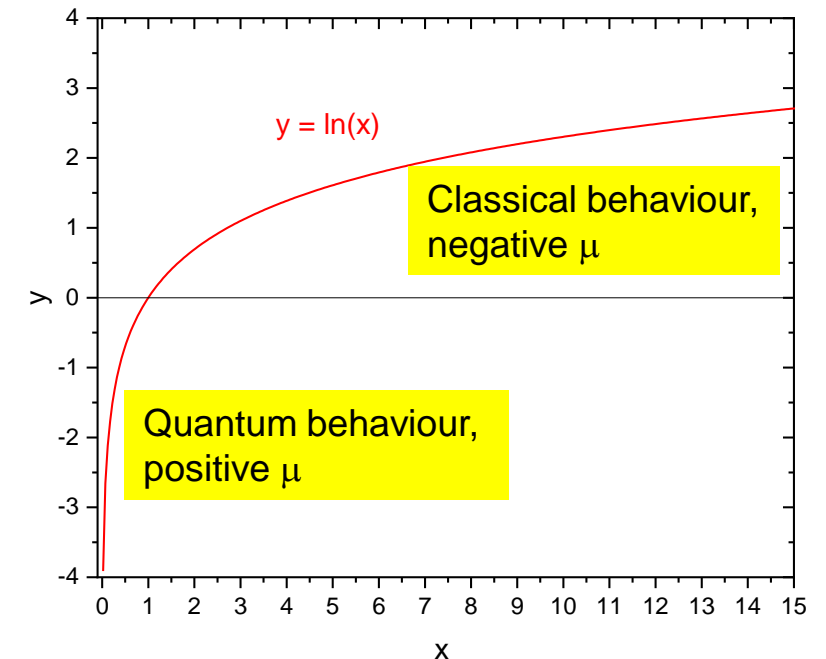
- The **sign of chemical potential depends on the ratio of the volume per one particle and the quantum volume ϑ_Q** . μ is **negative** for $\frac{V}{\vartheta_Q N} \gg 1$, or a non-dense systems with negligible probability for particles to occupy the same energy state. A large mass of particles results in a small ϑ_Q .
- Reduction in mass and decrease in temperature results in $\frac{V}{\vartheta_Q N} \ll 1$, **positive μ** and attempts of particles to occupy the same energy state.

Quantum volume and length

$$\mu = -kT \ln \left[\frac{V}{N} \left(\frac{2\pi m k T}{h^2} \right)^{3/2} \right]. \quad \vartheta_Q = l_Q^3 = \left(\frac{h}{\sqrt{2\pi m k T}} \right)^3$$

$$\frac{h}{\sqrt{2\pi m k T}} \xrightarrow{\quad} \frac{h}{\sqrt{2\pi m \epsilon}} \xrightarrow{\epsilon = \frac{p^2}{2m}} \frac{h}{p\sqrt{\pi}} \xrightarrow{p = \frac{\hbar h}{2\pi}} \frac{2\pi}{\hbar\sqrt{\pi}} \xrightarrow{\frac{2\pi}{\hbar} = \lambda_{dB}} \frac{\lambda_{dB}}{\sqrt{\pi}}$$

- For the air we breathe, the average distance between molecules is about 3 nm while the **average de Broglie wavelength** is less than 0.02 nm, so condition $\frac{V}{\vartheta_Q N} \gg 1$ is satisfied.
- For an electron at room temperature, **because of low mass**, the quantum volume is $\vartheta_Q = (4.3 \text{ nm})^3$, while the volume per conduction electron is roughly the volume of an atom, $(0.2 \text{ nm})^3$. Therefore, **electron gas** in metals at ambient conditions is **quantum gas** with $\frac{V}{\vartheta_Q N} \ll 1$.



Electron gas

The Nobel Prize in Physics 1906

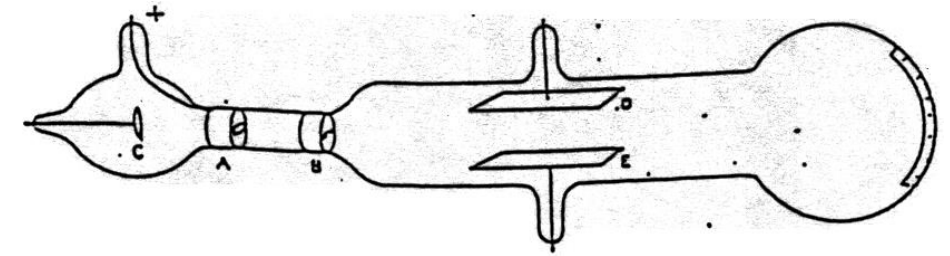
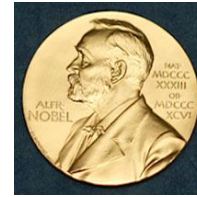
Joseph John Thomson

"in recognition of the great merits of his theoretical and experimental investigations on the **conduction of electricity** by gases"

'By carefully measuring how the cathode rays were deflected by electric and magnetic fields, Thomson was able to determine **the ratio between the electric charge (e) and the mass (m) of the rays**. Thomson's result was $e/m = 1.8 \cdot 10^{11}$ coulombs/kg.

The particle that J.J. Thomson discovered in 1897, the electron, is a constituent of all the matter we are surrounded by. All atoms are made of a nucleus and electrons. **He received the Nobel Prize in 1906 for the discovery of the electron, the first elementary particle.'**

<http://www.nobelprize.org/educational/physics/vacuum/experiment-1.html>



https://en.wikipedia.org/wiki/J._J._Thomson

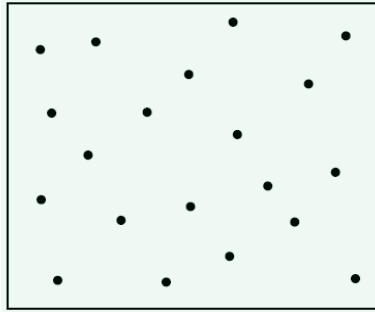


Cavendish Laboratory

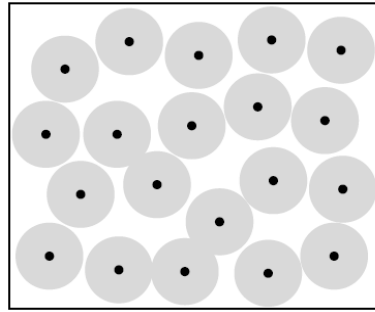


Electron gas in vacuum

https://en.wikipedia.org/wiki/Vacuum_tube



Normal gas



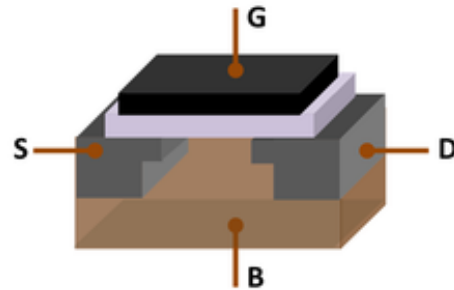
DVS

Quantum
electron gas



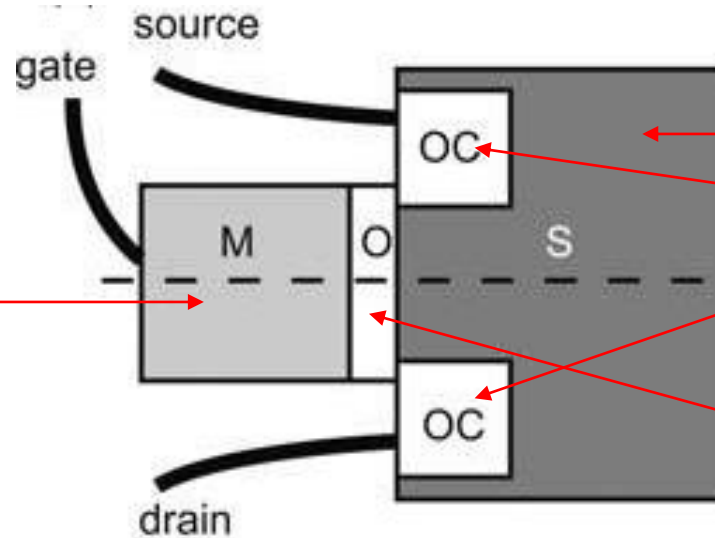
“The simplest vacuum tube, the **diode** (i.e. **Fleming valve**), invented in 1904 by **John Ambrose Fleming**, contains only a heated **electron-emitting cathode** and an anode. Electrons can only flow in one direction through the device—from the cathode to the anode. Adding one or more **control grids** within the tube allows the **current between the cathode and anode to be controlled by the voltage on the grids.**”

Solid-state transistors



Si-MOSFET

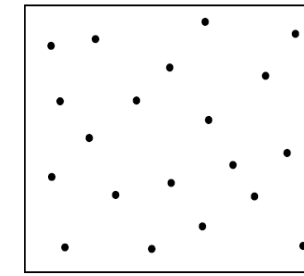
Metallic gate



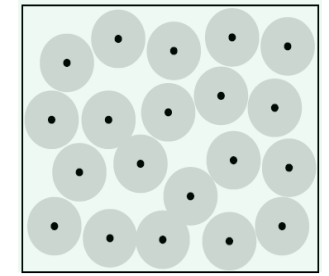
p-doped Si

Ohmic contacts,
n-doped

Oxide, SiO₂



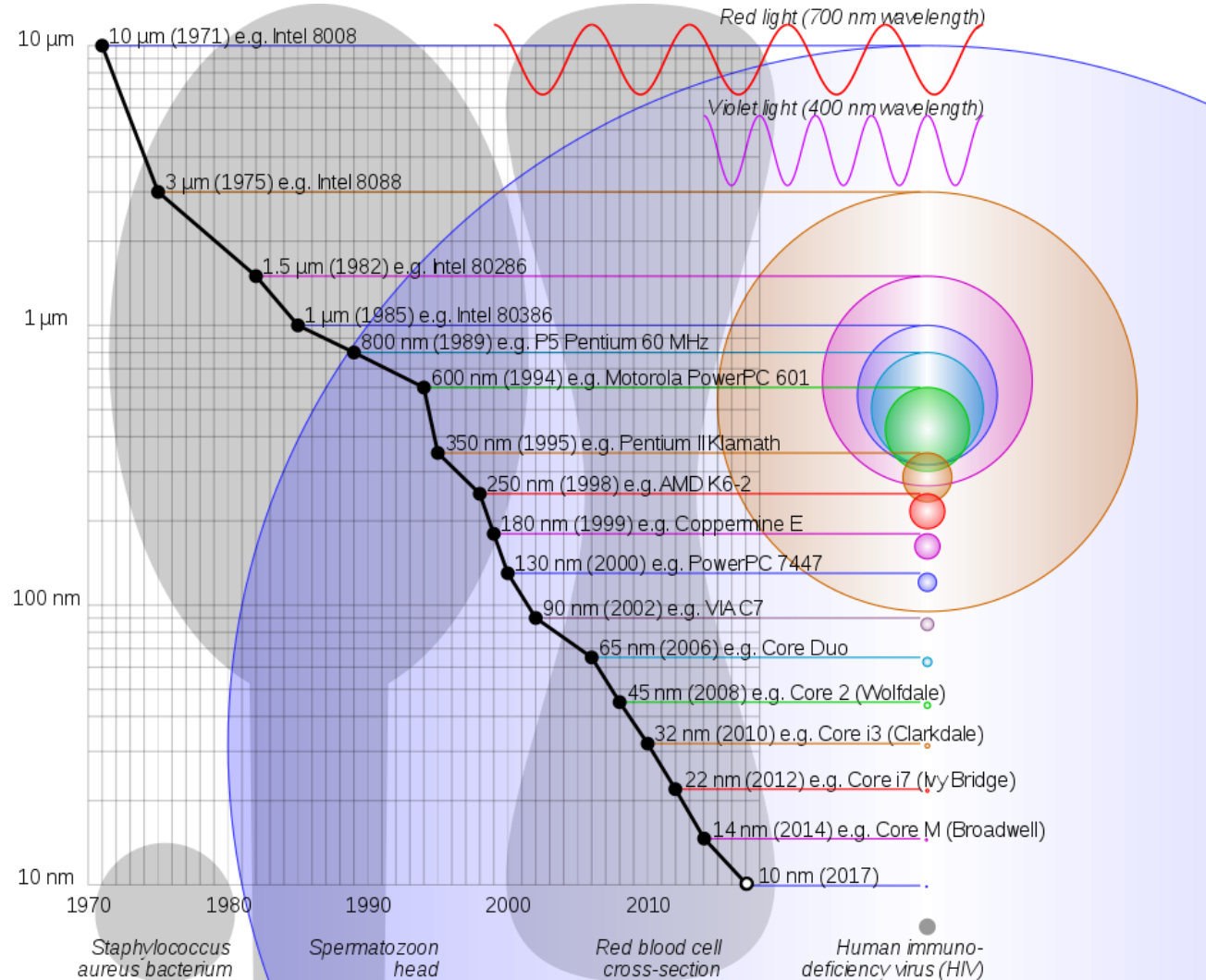
Normal gas



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DVS

Progress in miniaturisation



Semiconductor manufacturing processes



10 μm	– 1971
6 μm	– 1974
3 μm	– 1977
1.5 μm	– 1982
1 μm	– 1985
800 nm	– 1989
600 nm	– 1994
350 nm	– 1995
250 nm	– 1997
180 nm	– 1999
130 nm	– 2001
90 nm	– 2004
65 nm	– 2006
45 nm	– 2008
32 nm	– 2010
22 nm	– 2012
14 nm	– 2014
10 nm	– 2017
7 nm	– ~2018
5 nm	– ~2020

Half-nodes

V • T • E

A black and white portrait of a middle-aged man with short, dark hair and glasses. He is wearing a dark suit jacket over a light-colored, vertically striped shirt and a dark, patterned tie. He is seated and holding a thick book or a stack of papers in his left hand, which is resting on his lap. He is looking directly at the camera with a neutral expression. The background is a plain, light-colored wall.

Transistor count

curve shows transistor count doubling every two years

4004, 8008, 8080, 8085, 8086, 8088, 80186, 80286, 80386, 80486, Pentium, Pentium II, Pentium III, AMD K5, AMD K6, AMD K6-III, AMD K7, AMD K8, Barton, Atom, Core 2 Duo, Core i7 (Quad), Six-Core Opteron 2400, Quad-Core Itanium Tukwila, 8-Core Xeon Nehalem-EX, Quad-core z196, 8-core POWER7, 10-Core Xeon Westmere-EX, 16-Core SPARC T3, Six-Core Xeon 7400, Six-Core Core i7, Dual-Core Itanium 2, AMD K10, POWER6, Itanium 2 with 9MB cache, AMD K10, Cell, Core 2 Duo

2,300, 10,000, 100,000, 1,000,000, 10,000,000, 100,000,000, 1,000,000,000, 2,600,000,000

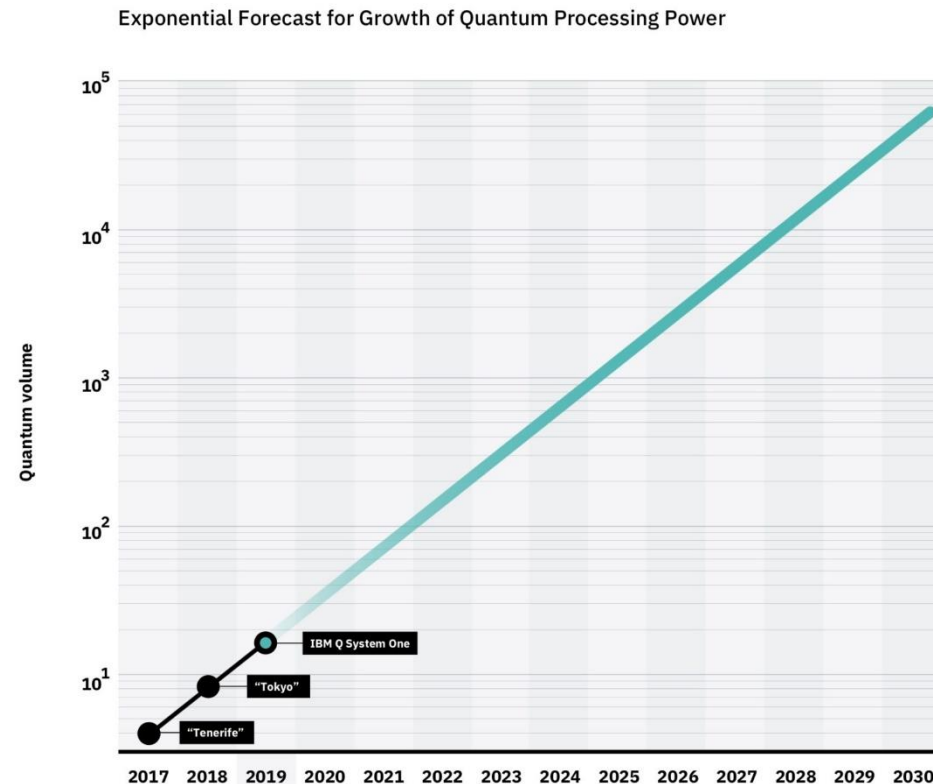
1971, 1980, 1990, 2000, 2011

Date of introduction

http://en.wikipedia.org/wiki/Moore's_law

Moor's law 2.0

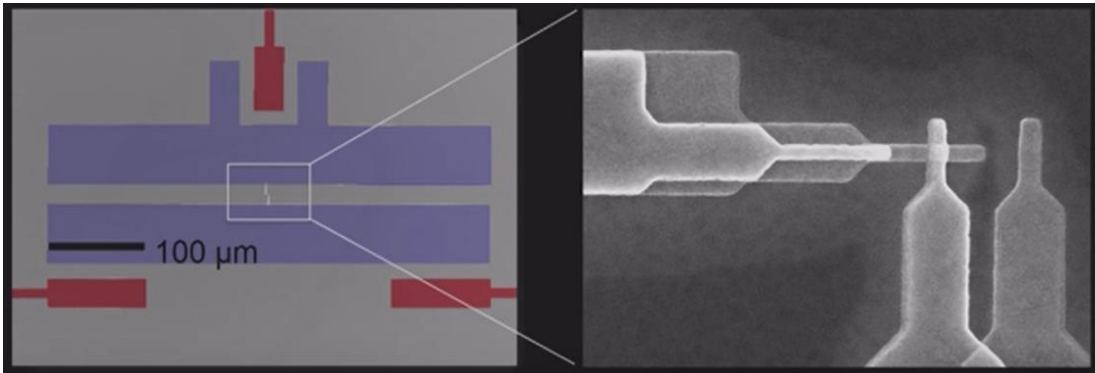
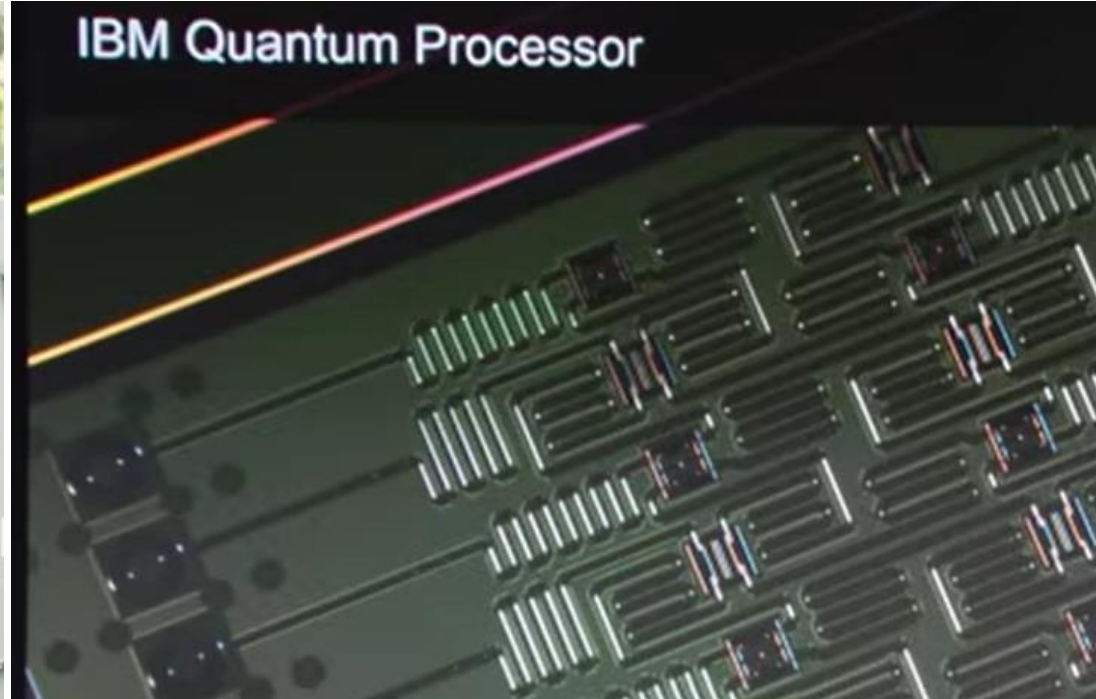
BOSTON, March 4, 2019 /PRNewswire/ -- At the 2019 [American Physical Society March Meeting](#), IBM (NYSE: [IBM](#)) unveiled a new scientific milestone, announcing its highest quantum volume to date.



IBM has doubled the power of its quantum computers annually since 2017.

https://newsroom.ibm.com/2019-03-04-IBM-Achieves-Highest-Quantum-Volume-to-Date-Establishes-Roadmap-for-Reaching-Quantum-Advantage#assets_all

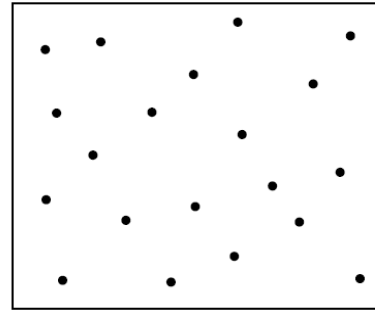
Superconductivity and quantum computing



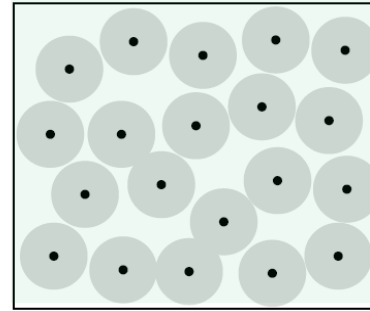
<https://techcrunch.com/2017/11/10/ibm-passes-major-milestone-with-20-and-50-qubit-quantum-computers-as-a-service/>

<https://www.youtube.com/watch?v=yy6TV9Dntlw>

Bosons and fermions



Normal gas



DVS

Quantum
electron gas

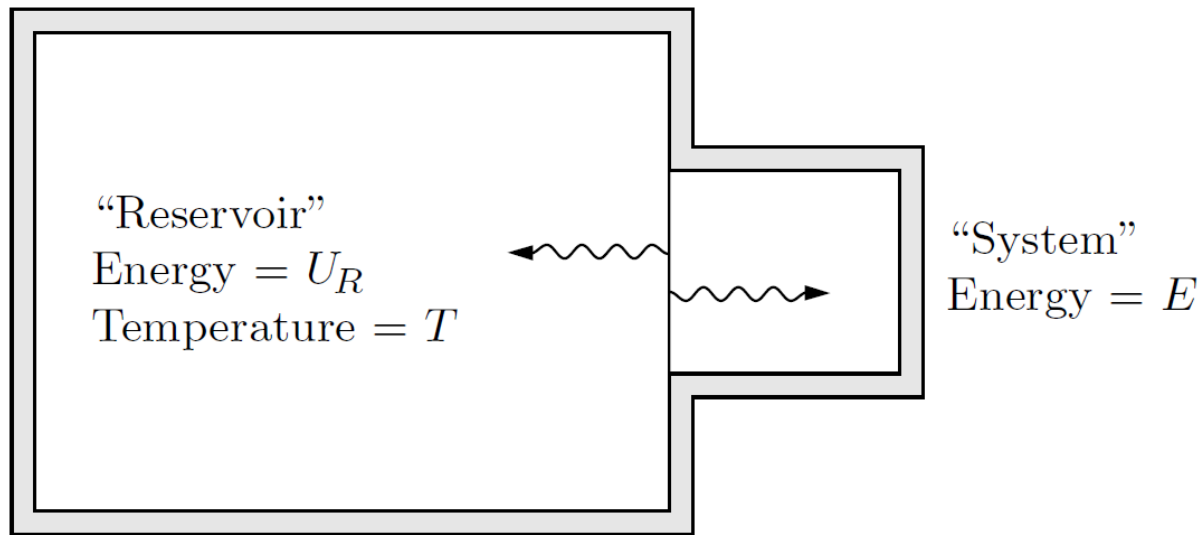
- For a dense system, particles that try to occupy the same state can be divided in two groups.
- Particles that can share a state with another are called bosons. Examples: photons and helium-4 atoms.
- Particles that cannot share a state with another are called fermions. Examples: electrons, protons, neutrons and helium-3 atoms.
- Particles with integer spin (0, 1, 2, etc., in units of $\hbar/2\pi$) are bosons.
- Particles with half-integer spin ($1/2$, $3/2$, etc.) are fermions.

Is electron gas composed of fermions or bosons?

Microcanonical, canonical, and grand canonical ensembles

In isolated systems or **microcanonical** ensemble, all allowed microstates had the same probability, i.e. “trivial” probability distribution. In **canonical** ensemble, members are assigned to states according to the Boltzmann probability distribution. It considers system in thermal contact with a much larger “reservoir” at some well-defined temperature allowing exchange of energy. **Grand canonical** ensemble allows exchange of particles too.

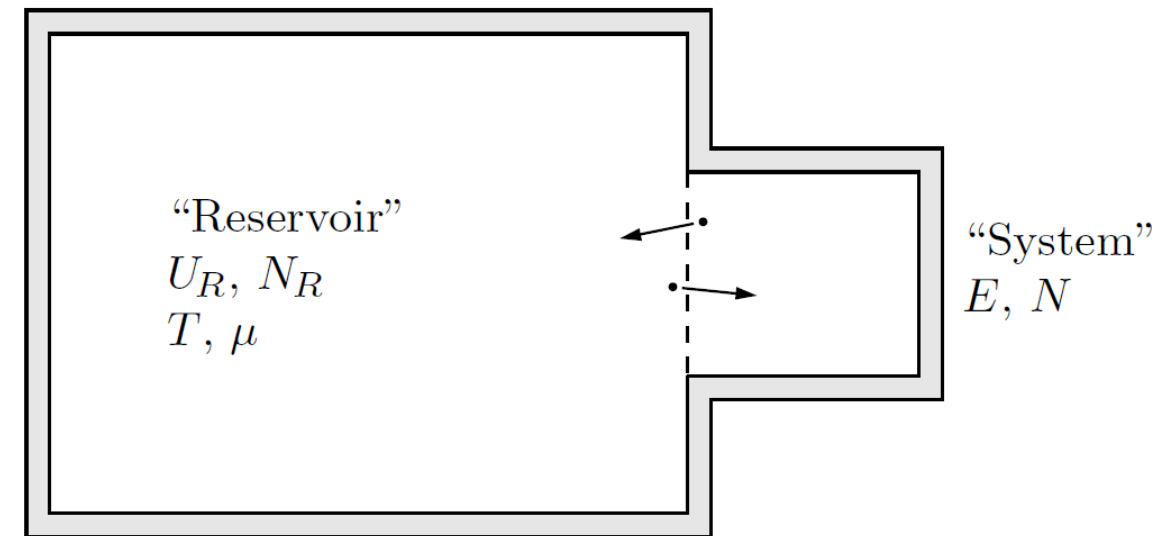
Boltzmann and Maxwell distributions



DVS

Canonical ensemble

Fermi-Dirac and Bose-Einstein distributions



DVS

Grand canonical ensemble

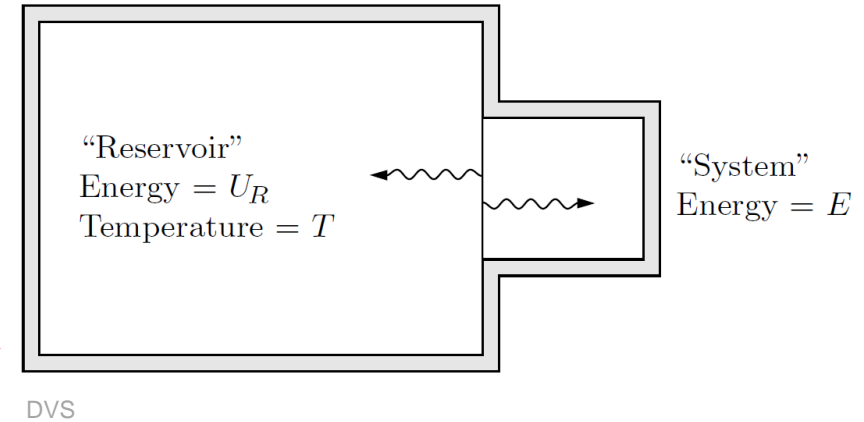
Black-body radiation - Planck Distribution

Boltzmann statistics

Boltzmann statistics calculates probability of the system in the contact with reservoir having energy E . This probability is proportional to multiplicity of reservoir:

$$P(E) = C\Omega_R(E)$$

$$\Omega_R(E) = A\Omega_R(0) \quad S_R(E) = k \ln \Omega_R(0) + k \ln A$$



$$\Delta S_R = k \ln A \quad \Delta U = T\Delta S - P\Delta V + \mu\Delta N$$

$$E = -\Delta U_R = -T\Delta S_R \quad \Delta S_R = -\frac{E}{T}$$

$$A = e^{-E/kT}$$

$$P(E) = AC\Omega_R(0)$$

$$P(s) = \frac{1}{Z} e^{-\frac{E(s)}{kT}}$$

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

Boltzmann distribution

A is Boltzmann factor $e^{-\frac{E}{kT}}$

$$P(E) = e^{-E/kT} C\Omega_R(0) = \frac{1}{Z} e^{-E/kT}$$

Transition to Gibbs statistics

Boltzmann statistics calculates probability of the system in the contact with reservoir having energy E . This probability is proportional to multiplicity of reservoir:

$$P(E) = C\Omega_R(E)$$

$$\Omega(E) = A\Omega_R(0) \quad S_R(E) = k \ln \Omega_R(0) + k \ln A$$

$$\Delta S_R = k \ln A \quad \Delta U = T\Delta S - P\Delta V + \mu\Delta N$$

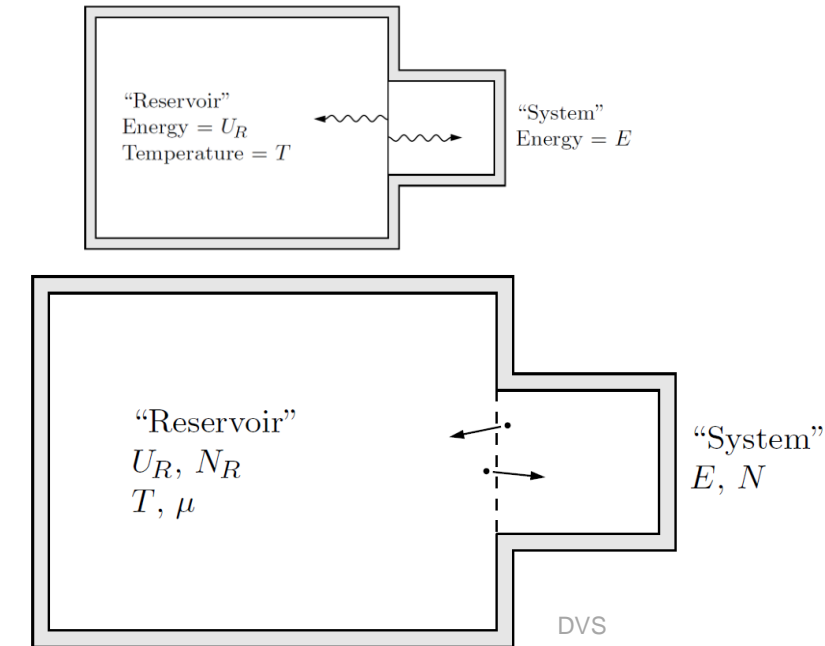
$$E = -\Delta U_R = -T\Delta S_R - \mu\Delta N_R \quad \Delta S_R = -\frac{E - \mu N}{T}$$

$$A = e^{-(E - \mu N)/kT}$$

$$\mathcal{P}(E) = A C \Omega_R(0)$$

Gibbs distribution

$$A \text{ is Gibbs factor } e^{-\frac{E - \mu N}{kT}} \quad \mathcal{P}(E) = e^{-(E - \mu N)/kT} C \Omega_R(0) = \frac{1}{\mathcal{Z}} e^{-(E - \mu N)/kT}.$$



Grand canonical ensemble

Gibbs factor

$$e^{-\frac{E(s) - \mu N(s)}{kT}}$$

Probability distribution

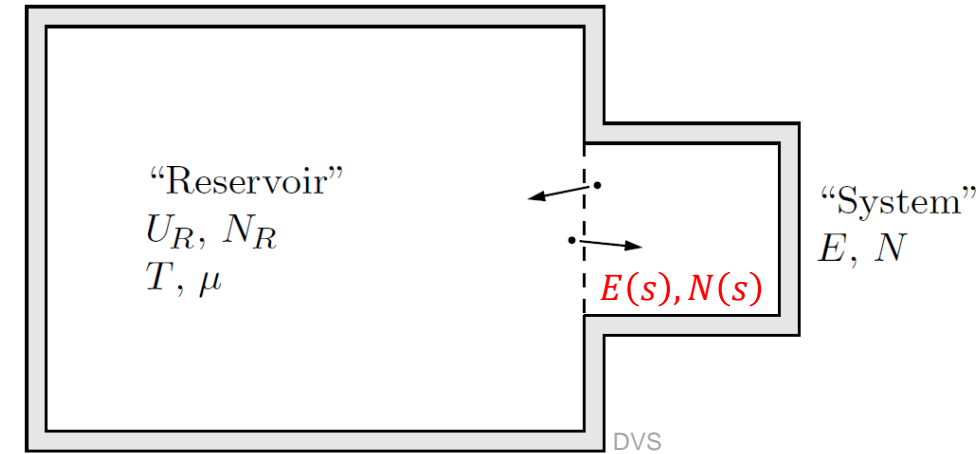
$$\mathcal{P}(s) = \frac{1}{\mathcal{Z}} e^{-\frac{E(s) - \mu N(s)}{kT}}$$

\mathcal{Z} is the grand partition function

$$\mathcal{Z} = \sum_s e^{-\frac{E(s) - \mu N(s)}{kT}}$$

If more than one type of particle is present in the system, then the μN term in equations becomes a sum over species of $\mu_i N_i$.

Grand canonical ensemble allows exchange of both energy and particles and has well defined **chemical potential μ** .



The grand partition function for this single-site occupation by oxygen of hemoglobin site. It has just two terms:

$$\mathcal{Z} = 1 + e^{-\frac{\epsilon - \mu}{kT}}$$

$$\epsilon = -0.7 \text{ eV}$$

$$\mu = -0.6 \text{ eV}$$

$$\mathcal{Z} = 1 + e^{\frac{0.1 \text{ eV}}{kT}} \approx 41$$

The probability of site occupation is $40/(40+1) = 98\%$.

With CO 100 times less abundant, it drops to 25%.

Grand canonical distribution for quantum particles

Gibbs factor

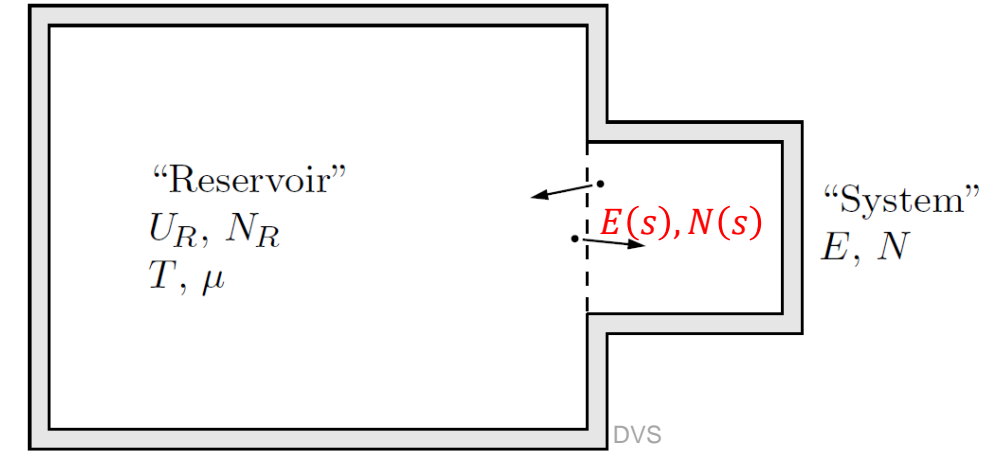
$$e^{-\frac{E(s) - \mu N(s)}{kT}}$$

Probability distribution

$$\mathcal{P}(s) = \frac{1}{\mathcal{Z}} e^{-\frac{E(s) - \mu N(s)}{kT}}$$

Grand partition function

$$\mathcal{Z} = \sum_s e^{-\frac{E(s) - \mu N(s)}{kT}}$$



Bose - Einstein distribution

$$\bar{n}_{BE} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1} \quad (\epsilon = hf)$$

Fermi-Dirac distribution

$$\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$$

Planck distribution

$$\bar{n} = \frac{1}{e^{\frac{hf}{kT}} - 1}$$

Number of photons

Planck distribution is Bose - Einstein distribution with chemical potential equal to zero. This comes from the fact that photons can be created or destroyed in any quantity. Their total number is not conserved. If one imposes μ by grand canonical distribution, this can only be done with $\mu = 0$.

The chemical potential for a gas of photons in a box is zero.

Fermi-Dirac distribution

Main idea is to consider a system as a **state for single-particles** and find average number of particles in this state. The **energy** when the state is occupied by a single particle is ϵ . When the state is unoccupied, its energy is 0. If it is **occupied by n particles**, the energy is $n\epsilon$. The probability of the state being occupied by n particles is:

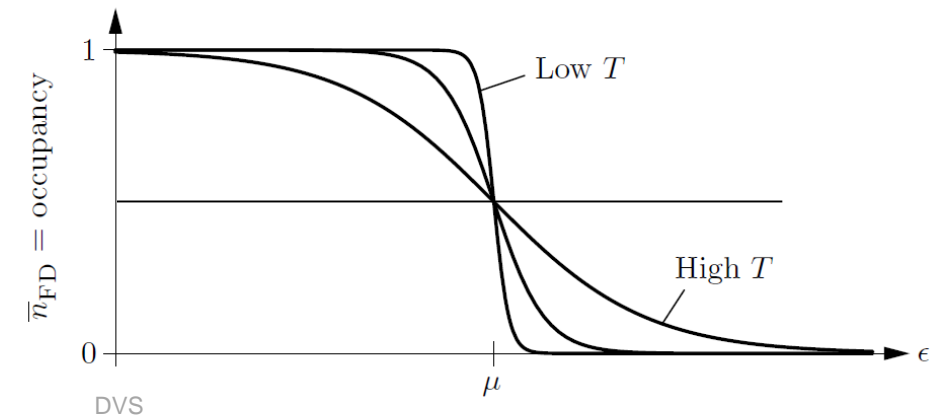
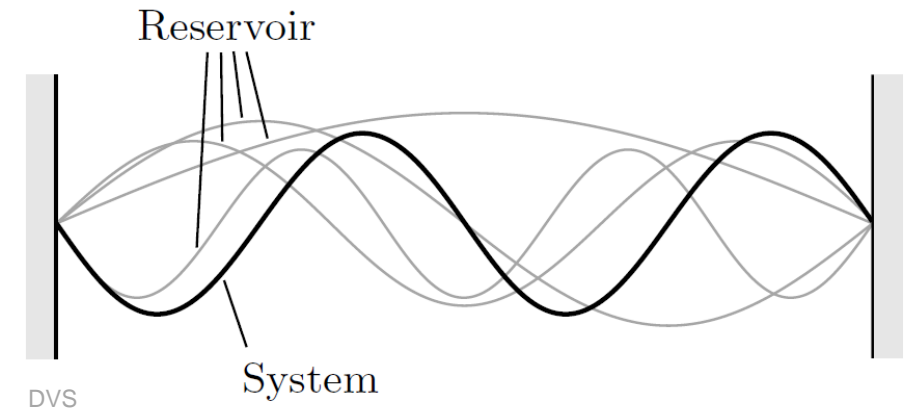
$$\mathcal{P}(n) = \frac{1}{\mathcal{Z}} e^{-\frac{n\epsilon - \mu n}{kT}} = \frac{1}{\mathcal{Z}} e^{-\frac{n(\epsilon - \mu)}{kT}}$$

If the particles are fermions, then n can only be 0 or 1, so the grand partition function is: $\mathcal{Z} = 1 + e^{-\frac{\epsilon - \mu}{kT}}$.

The average number of particles in the state or the occupancy of the state is then:

$$\bar{n} = \sum_n n \mathcal{P}(n) = 0 \cdot \mathcal{P}(0) + 1 \cdot \mathcal{P}(1) = \frac{e^{-\frac{\epsilon - \mu}{kT}}}{1 + e^{-\frac{\epsilon - \mu}{kT}}} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}.$$

It is the **Fermi-Dirac distribution**: $\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$.



Bose-Einstein distribution

If the particles are bosons, then n can be any nonnegative integer, so the grand partition function is:

$$\mathcal{Z} = 1 + e^{-\frac{\epsilon - \mu}{kT}} + e^{-\frac{2(\epsilon - \mu)}{kT}} + \dots = 1 + e^{-\frac{\epsilon - \mu}{kT}} + \left(e^{-\frac{\epsilon - \mu}{kT}}\right)^2 + \dots = \frac{1}{1 - e^{-\frac{\epsilon - \mu}{kT}}}$$

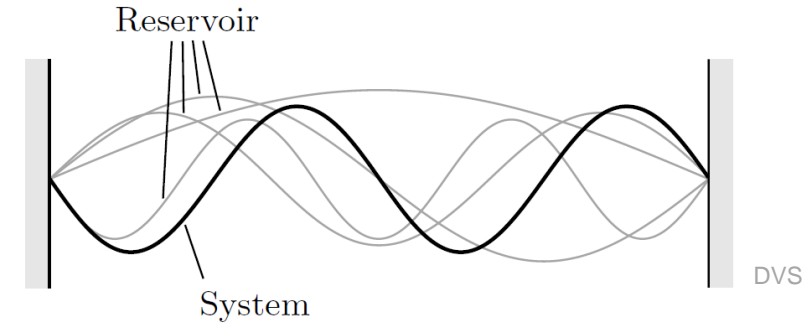
$$\mathcal{P}(n) = \frac{1}{\mathcal{Z}} e^{-\frac{n\epsilon - \mu n}{kT}} = \frac{1}{\mathcal{Z}} e^{-\frac{n(\epsilon - \mu)}{kT}}$$

The average number of particles in the state or the occupancy of the state is then:

$$\bar{n} = \sum_n n \mathcal{P}(n) = 0 \cdot \mathcal{P}(0) + 1 \cdot \mathcal{P}(1) + 2 \cdot \mathcal{P}(2) + \dots = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1}.$$

This is **Bose-Einstein distribution**: $\bar{n}_{BE} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1}.$

Like the Fermi-Dirac distribution, the Bose-Einstein distribution goes to zero when $\epsilon \gg \mu$.
Unlike the Fermi-Dirac distribution, it goes to infinity as ϵ approaches μ from above.



Comparison of distributions

For the Boltzmann distribution: $P(s) = \frac{1}{Z_1} e^{-\frac{\epsilon}{kT}}$ $\mu = -kT \ln \left(\frac{Z_1}{N} \right)$

$$\bar{n}_{Boltzmann} = \frac{1}{Z_1} N e^{-\frac{\epsilon}{kT}} = e^{-\frac{\epsilon}{kT}} e^{\frac{\mu}{kT}} = e^{-\frac{(\epsilon - \mu)}{kT}}$$

$$F = -kT \ln(Z)$$

$$\mu = + \left(\frac{\partial F}{\partial N} \right)_{T,V}$$

$$Z = \frac{Z_1^N}{N!} = \frac{Z_1^N}{N!}$$

$$\ln N! \approx N(\ln N - 1)$$

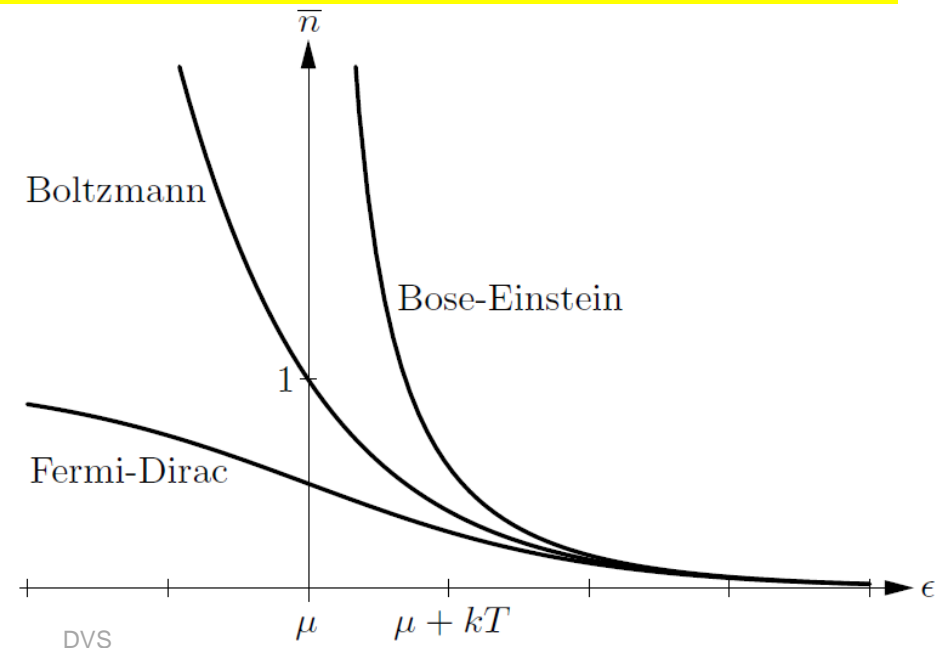
$$\bar{n}_{Boltzmann} = e^{-\frac{(\epsilon - \mu)}{kT}}$$

$$\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$$

$$\bar{n}_{BE} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} - 1}$$

When $\epsilon \gg \mu$, the exponent is very large, one can neglect the 1 in the denominator of Fermi-Dirac and Bose-Einstein distributions, and both are reduced to the Boltzmann distribution. The precise condition for the three distributions to agree is: $\epsilon - \mu \gg kT$.

To apply the distributions to any particular system, **one needs to know what the energies of all the states are.**

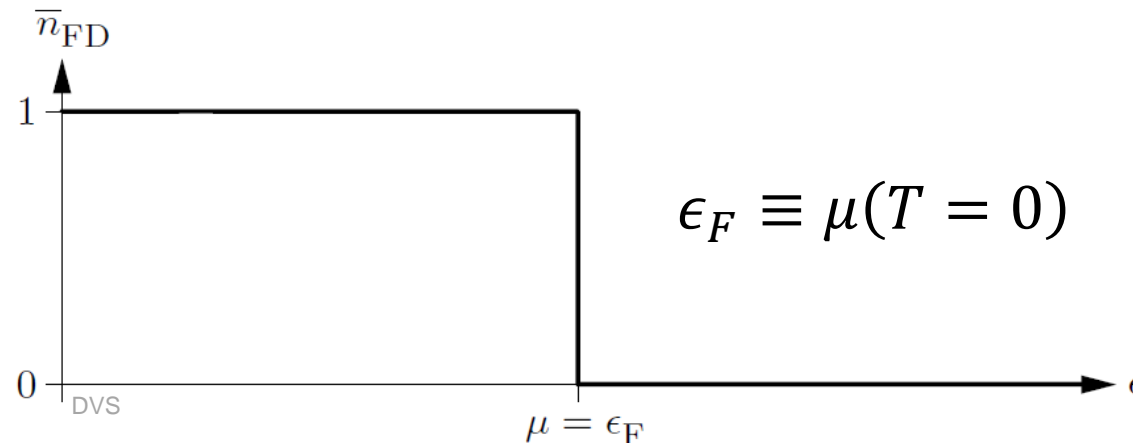


Degenerate Fermi gas

- Gas of fermions is degenerate when **nearly all states below μ are occupied and nearly all states above μ are unoccupied**, which typically happens at a low temperatures $kT < \epsilon - \mu$.
- At zero temperature, Fermi-Dirac distribution function is a step function. It equals **1 for all states with $\epsilon < \mu$ and equals 0 for all states with $\epsilon > \mu$** .
- As a boundary of filled state at $T = 0$, μ is also called Fermi energy: ϵ_F .
- The value of ϵ_F is determined by the total number of electrons.
- All electron states are filled**, from the lowest available state to ϵ_F .
- μ is change in total energy** at zero temperature when **one particle is added** to the system.

$$\bar{n}_{FD} = \frac{1}{e^{\frac{\epsilon - \mu}{kT}} + 1}$$

$$\mu = \left(\frac{\partial U}{\partial N} \right)_{S,V}$$



Counting quantized states in 3D:

$$\epsilon_F = \frac{h^2}{8m} \left(\frac{3N}{\pi V} \right)^{\frac{2}{3}}$$

Properties of degenerate Fermi gas

- The average energy of the electrons is $3/5$ the Fermi energy: $U = 3/5\epsilon_F$. Fermi energy for conduction electrons in a typical metal is a few electron-volts. This is much larger than the average thermal energy of a particle at room temperature, $kT \approx 1/40 \text{ eV}$, which means electron gas in metals is a degenerate Fermi gas.
- The condition $\epsilon_F \gg kT$ comes from the condition $V/\vartheta_Q \ll N$, which means that quantum statistics is important for the electron gas.
- The large, comparable with kT , Fermi energy justifies approximation of $T \approx 0$.
- Using the formula $P = -(\partial U/\partial V)_{S,N}$, the degeneracy pressure $P = \frac{2U}{3V}$ is found to be few billion N/m^2 , sufficient to withstand electrostatic forces and hold the electrons inside the metal. This pressure does not come from the electrostatic repulsion between the electrons. It arises purely from the quantum exclusion principle.
- All electron states are filled, from the lowest available state to ϵ_F .

Fermi gas at small nonzero temperatures

- At **finite temperature T** , normal particles would get energy about kT . However, degenerate electron gas is special. Most of the electrons cannot acquire such energy, because all the states that they might jump in are already occupied.
- The only electrons that can acquire some energy (thermal) are those that are already within kT of the Fermi energy. Only they can jump up into unoccupied states above ϵ_F .
- The number of electrons that can be affected by the increase in T is proportional to T . This number must also be proportional to N . Thus, the **additional energy at finite T** is doubly proportional to T : $\Delta U(T) \propto NkT \cdot kT$.
- Coefficient proportionality can be guessed from dimensionality units. It must have unit of one over energy, and the only energy available in this model is ϵ_F .
- Knowing this, allows to calculate **heat capacity** of electron gas. It is going to zero as $T \rightarrow 0$.

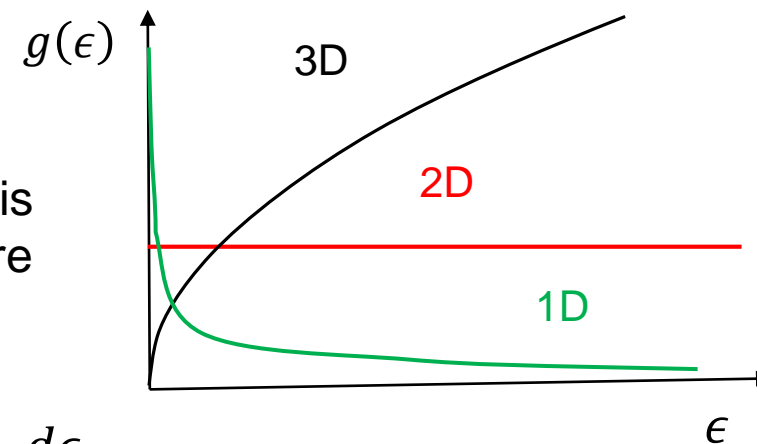
$$U = \frac{3}{5}N\epsilon_F + A \frac{NkT \cdot kT}{\epsilon_F} \quad A = \frac{\pi^2}{4} \quad C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{\pi^2 Nk^2 T}{2\epsilon_F}$$

Density of states

When the energy states are filled, they are **typically not equidistant** (the exception is 2D electron gas). For 3D electron gas, the density of states, or the number of energy states per unit of energy, follows equation:

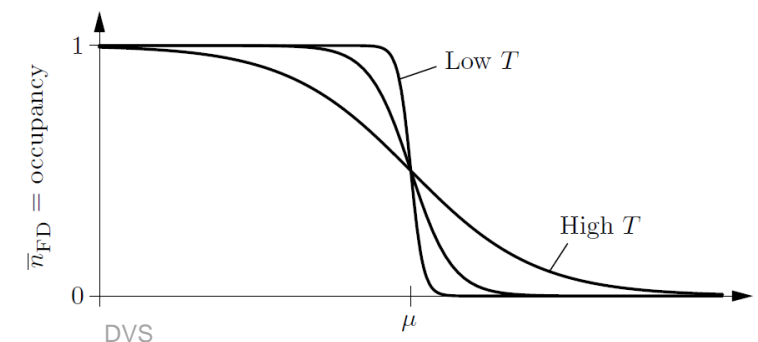
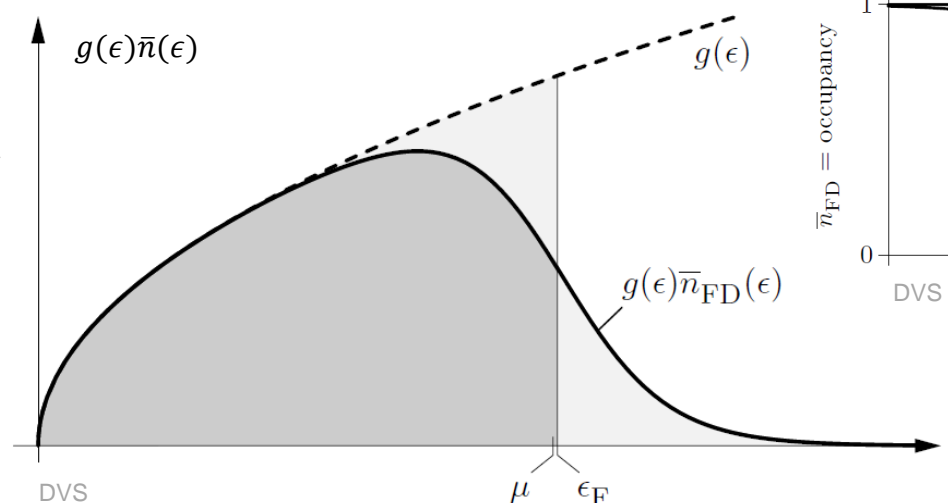
$$g(\epsilon) = \frac{\pi(8m)^{3/2}}{2h^3} V \sqrt{\epsilon}$$

The derivation of the energy dependence of density of states is straightforward from the quantization of momentum in units of h/L , where L is size of electron box.



The number of electrons is then: $N = \int_0^\infty g(\epsilon) \bar{n}(\epsilon) d\epsilon = \int_0^\infty g(\epsilon) \frac{1}{e^{\frac{\epsilon-\mu}{kT}} + 1} d\epsilon$

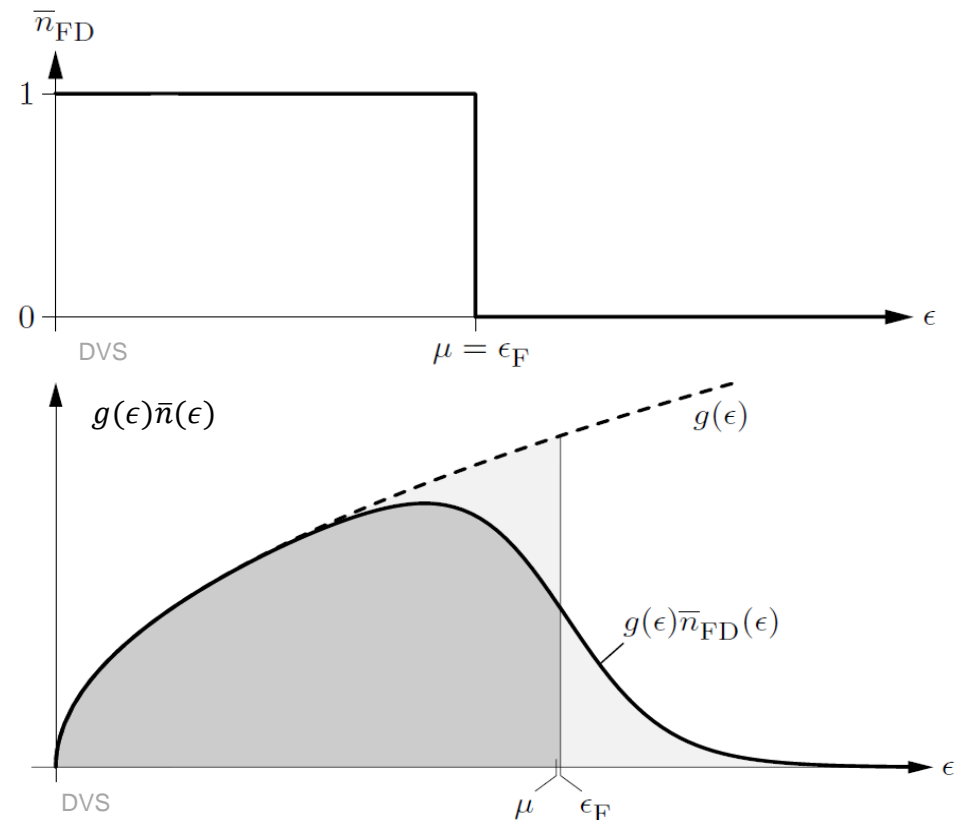
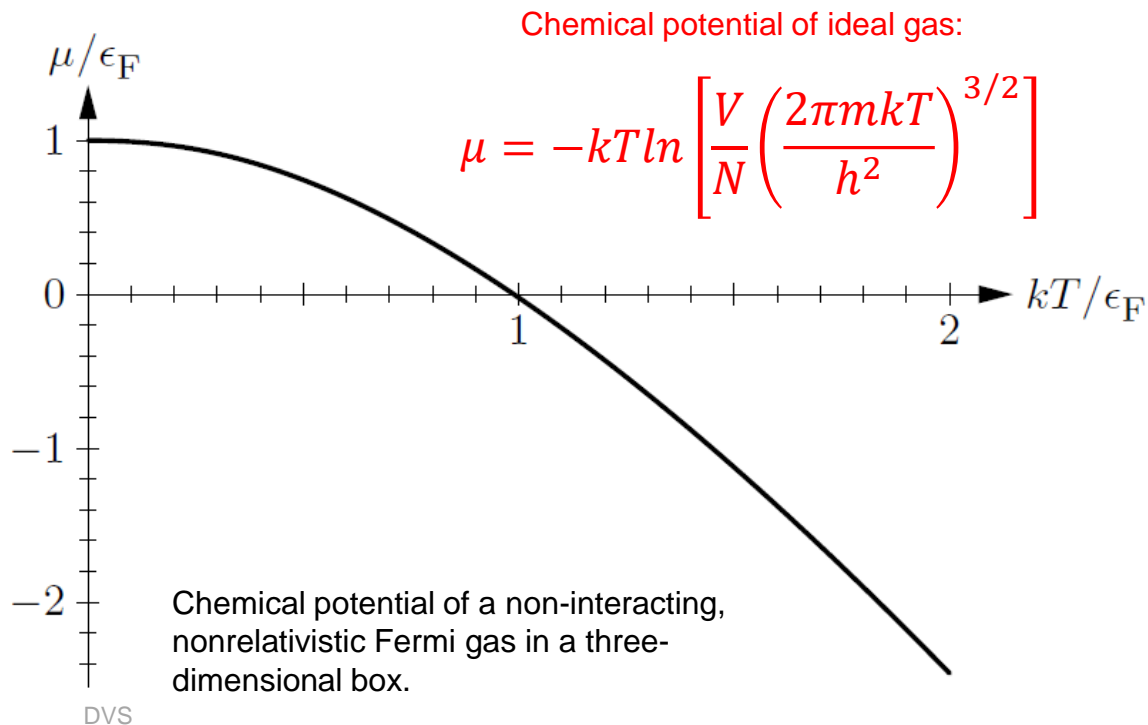
At finite temperature, $\mu \neq \epsilon_F$. Because increasing T does not change total number of fermions, the two lightly shaded areas must be equal. Since $g(\epsilon)$ is greater above ϵ_F than below, this means that the **chemical potential decreases with increase of T** .



The chemical potential, μ , is the point where the probability of a state being occupied is exactly 1/2.

Chemical potential of degenerate Fermi gas

- The chemical potential, μ , is the point where the probability of a state being occupied is exactly 1/2.
- At $T = 0$, $\mu = \epsilon_F$.
- The chemical potential decreases with increase of T .
- At high temperatures, μ becomes negative and approaches the form for an ordinary gas obeying Boltzmann statistics.

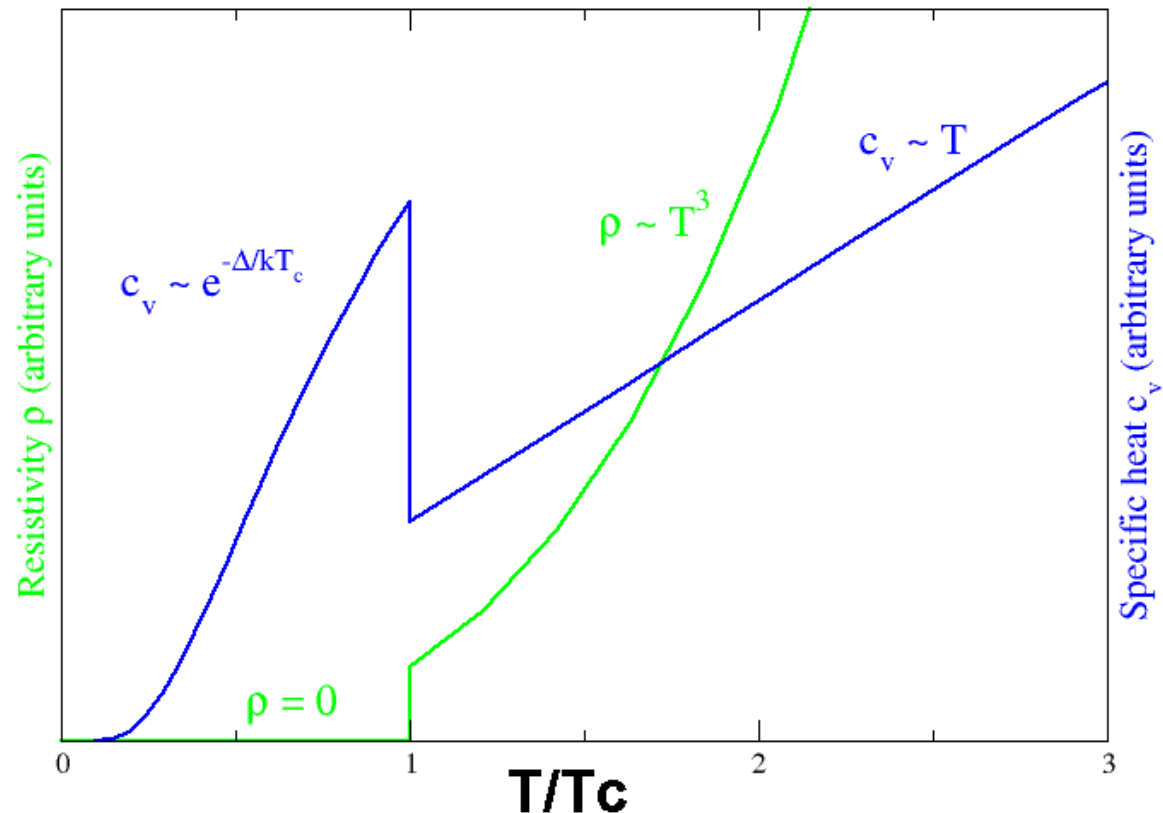


Bose-Einstein condensation and superconductivity

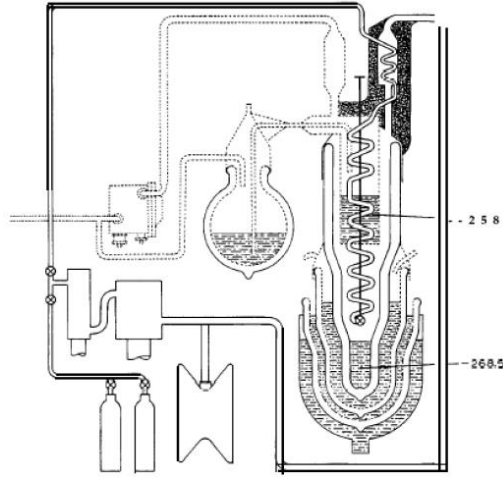
- Superconductivity is the result of **Bose-Einstein condensation** taking place when fermions form bosons being united into Cooper pairs.
- As a result, electron gas acquires property of **superfluidity** dropping **resistance to absolute zero**.
- Superconductors have **unique quantum properties** allowing multiple uses in **modern technology**.

Fermi gas:
$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{\pi^2 N k^2 T}{2 \epsilon_F}$$

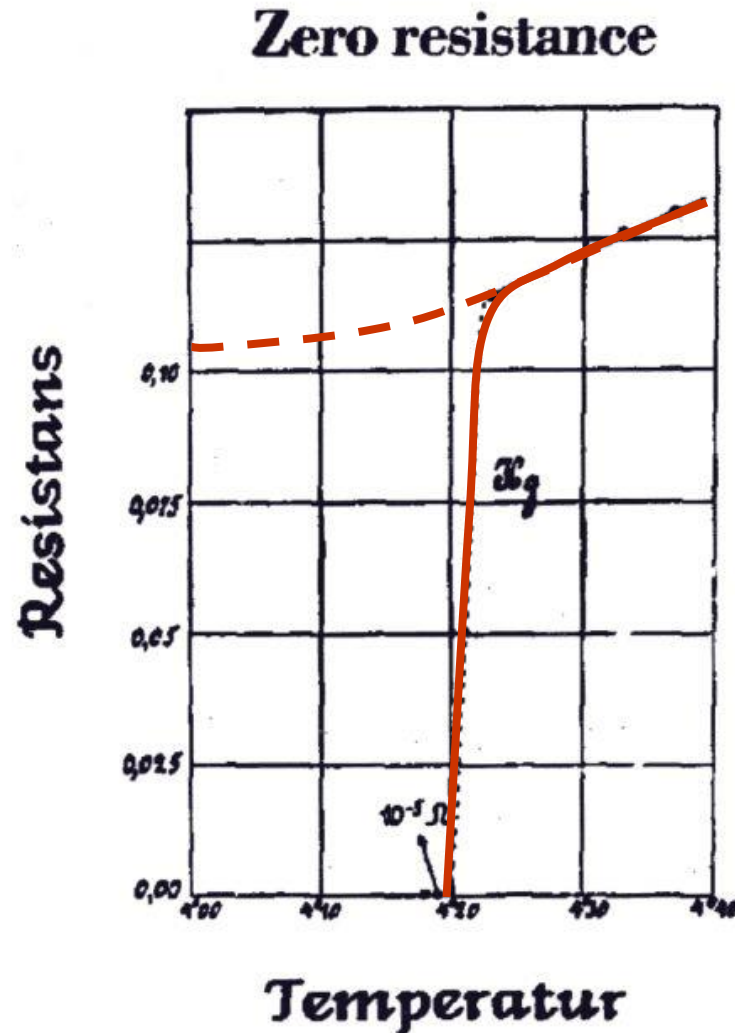
Superconductor:



Discovery of superconductivity



- Discovered by **Kamerlingh Onnes** in 1911 during first low temperature measurements to liquefy helium
- Whilst measuring the resistivity of “pure” Hg he noticed that the electrical resistance dropped to zero at 4.2K



1913

General properties

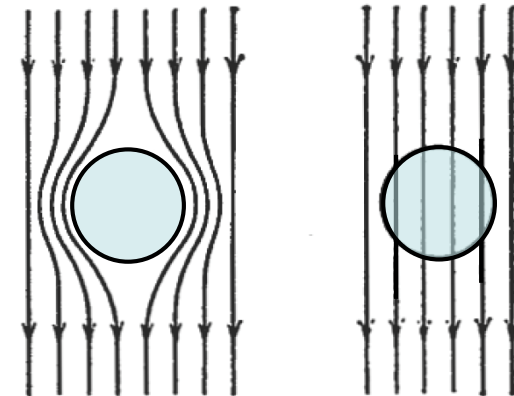
- Zero resistance (*Kammerlingh-Onnes, 1911*) at $T < T_c$. The temperature T_c is called the *critical* one.
- Superconductivity can be destroyed also by an external magnetic field H_c which is also called the *critical* one (*Kammerlingh-Onnes, 1914*). Empirically,

$$H_c(T) = H_c(0) [1 - (T/T_c)^2] .$$

- If the superconductivity is destroyed by a current the critical current is just the one which produces the field H_c at the surface (the *Silsby rule*).
- The *Meissner-Ochsenfeld effect* (1933)



Magnetic field is expelled
from the superconductor



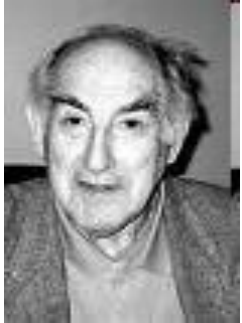
Ideal conductor! Ideal diamagnet!

Ginzburg-Landau Theory (1950)

$$F(T, M) = F(T, 0) + a(T - T_{Cur})M^2 + bM^4 + c|\nabla M|^2$$

Order parameter?

Hint: wave function of Bose condensate (complex!)



2003

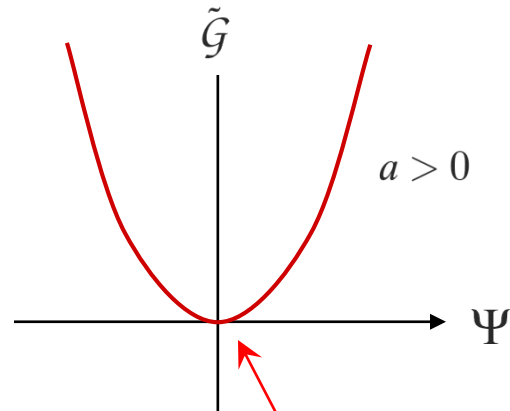
$$|\Psi|^2 \ll 1$$



$$\mathcal{G}_s = \mathcal{G}_n + a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \dots$$

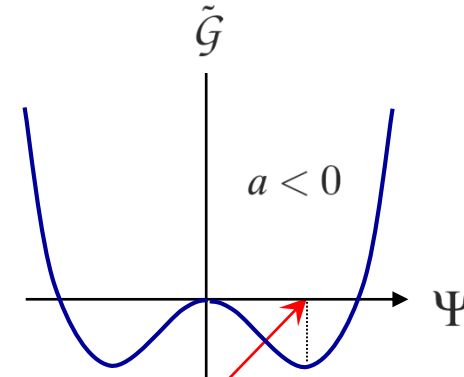


1962



$a > 0$

$$\tilde{\mathcal{G}} \equiv \mathcal{G}_s - \mathcal{G}_n$$



$a < 0$

a should change the sign at the transition point

Introduce $\tau = \frac{T - T_c}{T_c}$. Near T_c , $|\tau| \ll 1$: $a = \alpha\tau$, $\alpha > 0$.

$$\begin{aligned} \Psi &= 0 & \text{at } T > T_c, \\ |\Psi|^2 &= -(\alpha/b)\tau = |\Psi_0|^2 & \text{at } T < T_c \end{aligned}$$

Microscopic mechanism



John Bardeen, Leon Cooper, and Robert Schrieffer



1972

Mechanism of pairing – phonon-mediated attraction

‘The electrons are bound into **Cooper pairs**... Therefore, in order to break a pair, one has to change energies of all other pairs. This means there is an energy gap for single-particle excitation, unlike in the normal metal (where the state of an electron can be changed by adding an arbitrarily small amount of energy). **The energy gap is most directly observed in tunnelling experiments** and in reflection of microwaves from superconductors.’

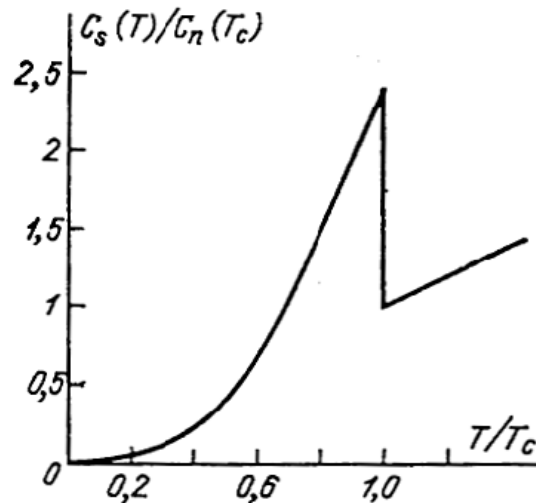
‘The theory describes superconductivity as a microscopic effect caused by a **condensation of Cooper pairs into a boson-like state**. The theory is also used in nuclear physics to describe the pairing interaction between nucleons in an atomic nucleus.’

https://en.wikipedia.org/wiki/BCS_theory

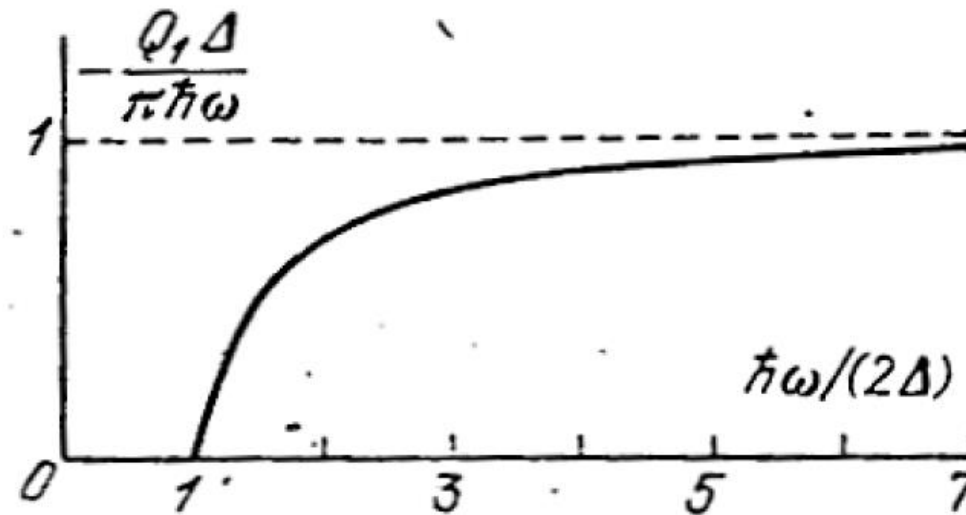
Energy gap in superconductors

The gap in the quasiparticle energy spectrum leads to crucial consequences.

It is the gap that determines most of thermal, magnetic, and electrical properties of superconductors.



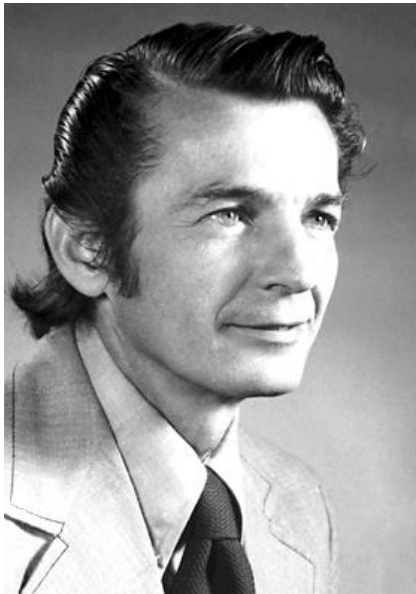
Specific heat



Microwave absorption

The Nobel Prize in Physics 1973

Leo Esaki, Ivar Giaever, Brian D. Josephson



[How Quantum Tunneling Works - by Ivar Giaever - YouTube](#)

Ivar Giaever

Prize motivation: "for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively"

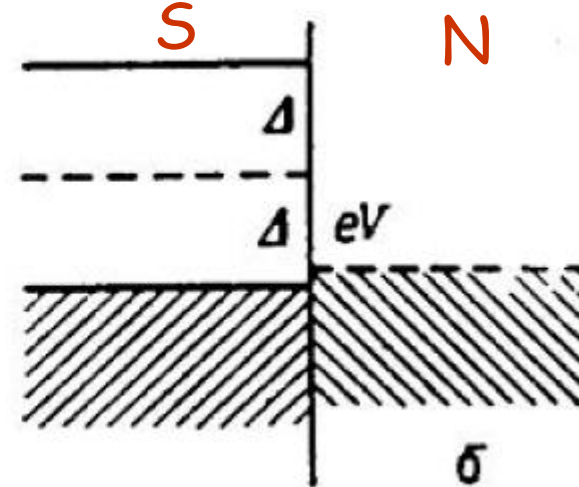
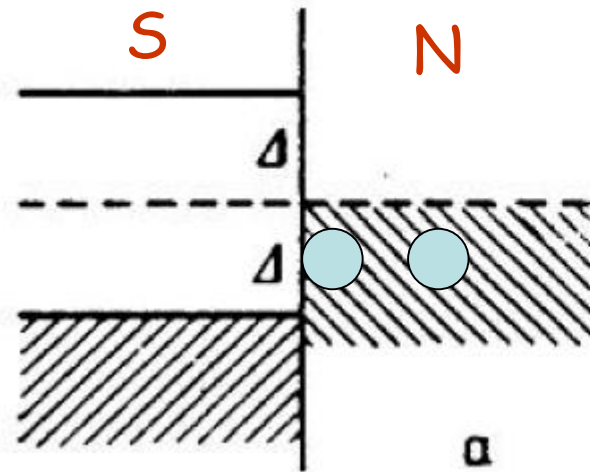
Field: condensed matter physics, semiconductors



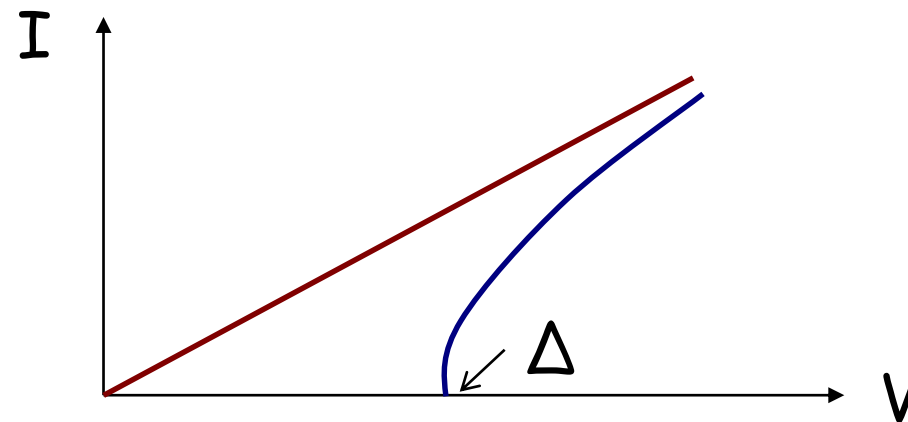
1973

Effect of energy gap in superconductors

At the S-N interface:



No single-electron tunneling possible until
 $eV \leq \Delta$





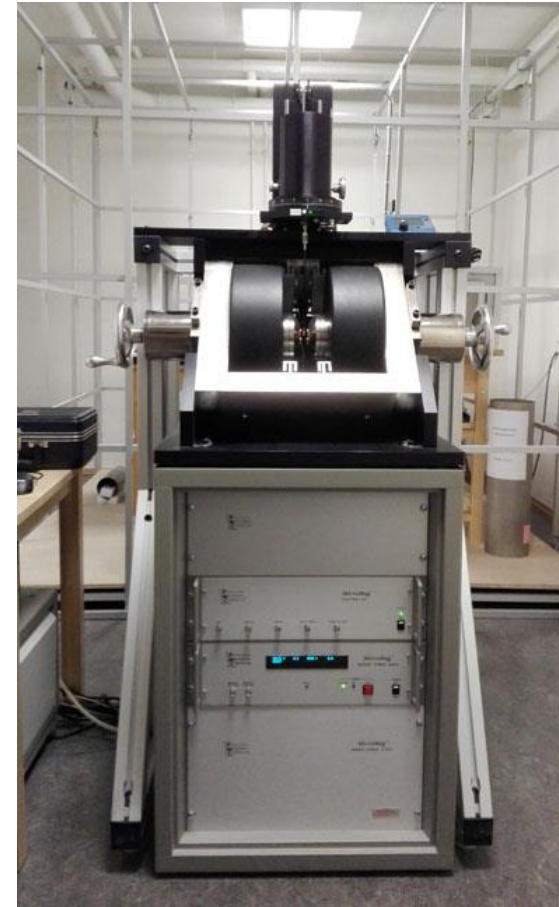
<http://www.mn.uio.no/geo/om/aktuelt/aktuelle-saker/2016/geomagnetisme.html>

Official opening on 7 September 2016

Instruments for Paleomagnetic Measurements and Rock Magnetic Analyses



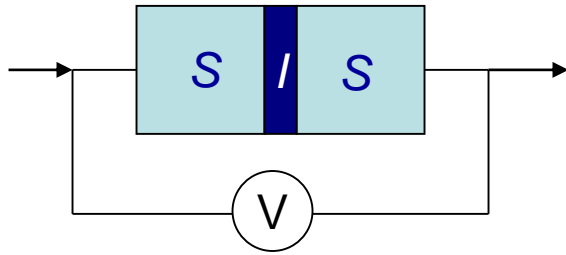
AGICO JR-6A Spinner Magnetometer



Lake Shore PMC MicroMag 3900
Vibrating Sample Magnetometer (VSM)

Josephson effects

Is it possible to convey Cooper pairs between superconductors?



Weak link - two superconductors divided by a thin layer of insulator or normal conductor

What is the resistance of the junction?

For small currents, the junction is a superconductor!

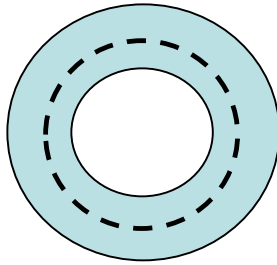
Reason - order parameters overlap in the weak link



1973

B. Josephson

Quantization of magnetic flux



Long hollow
cylinder

The current
inside is zero

$$\Psi = |\Psi|e^{i\chi}$$

$$|\Psi| = \text{const} \quad \longrightarrow \quad \mathbf{j} = -\frac{e\hbar}{m}|\Psi|^2 \left[\nabla\chi + \frac{2e}{\hbar}\mathbf{A} \right]$$



Alexei A. Abrikosov



2003

$$\underbrace{\oint \frac{\mathbf{j}}{|\Psi|^2} d\mathbf{l}}_0 = -\frac{e\hbar}{m} \left[\underbrace{\oint \nabla\chi d\mathbf{l}}_{-2\pi k} + \frac{2e}{\hbar c} \underbrace{\oint \mathbf{A} d\mathbf{l}}_{\text{flux}} \right]$$

$$\Phi = k\Phi_0, \quad \Phi_0 = \pi\hbar c/e = 2.07 \cdot 10^{-7} \text{G} \cdot \text{cm}^2.$$

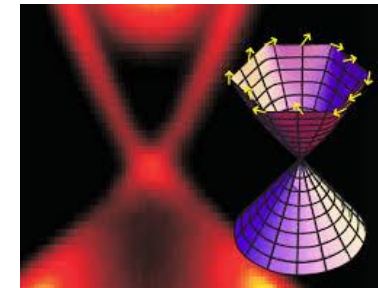
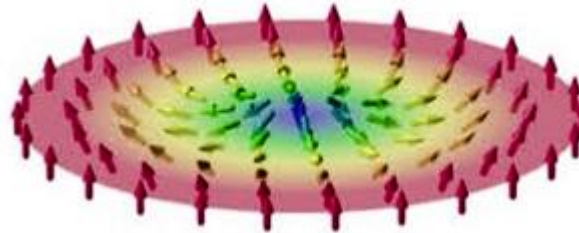
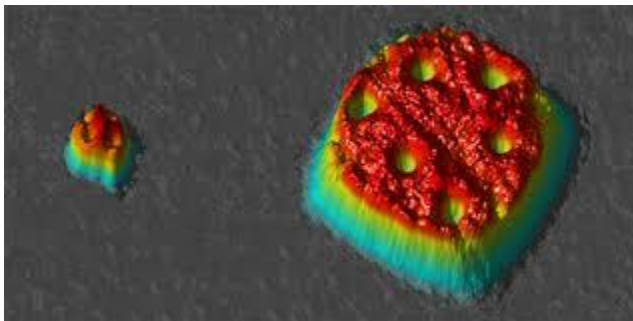
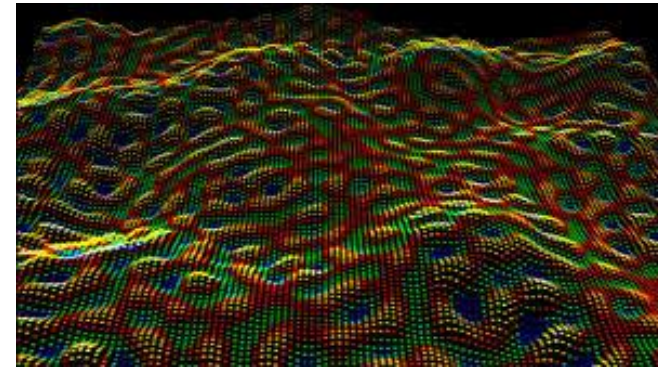
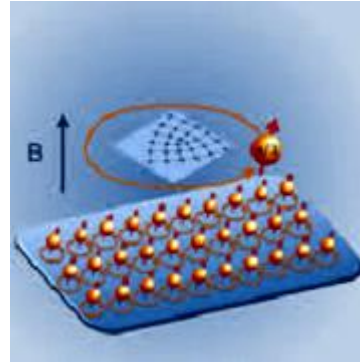
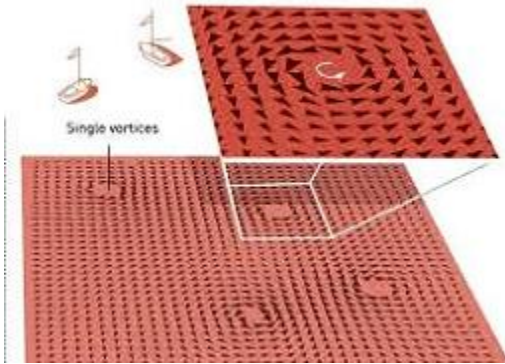
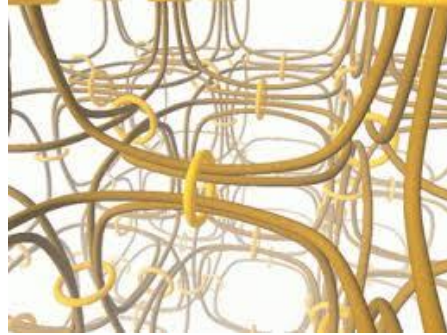
<http://en.wikipedia.org/wiki/SQUID>

Physics Nobel Prize 2016

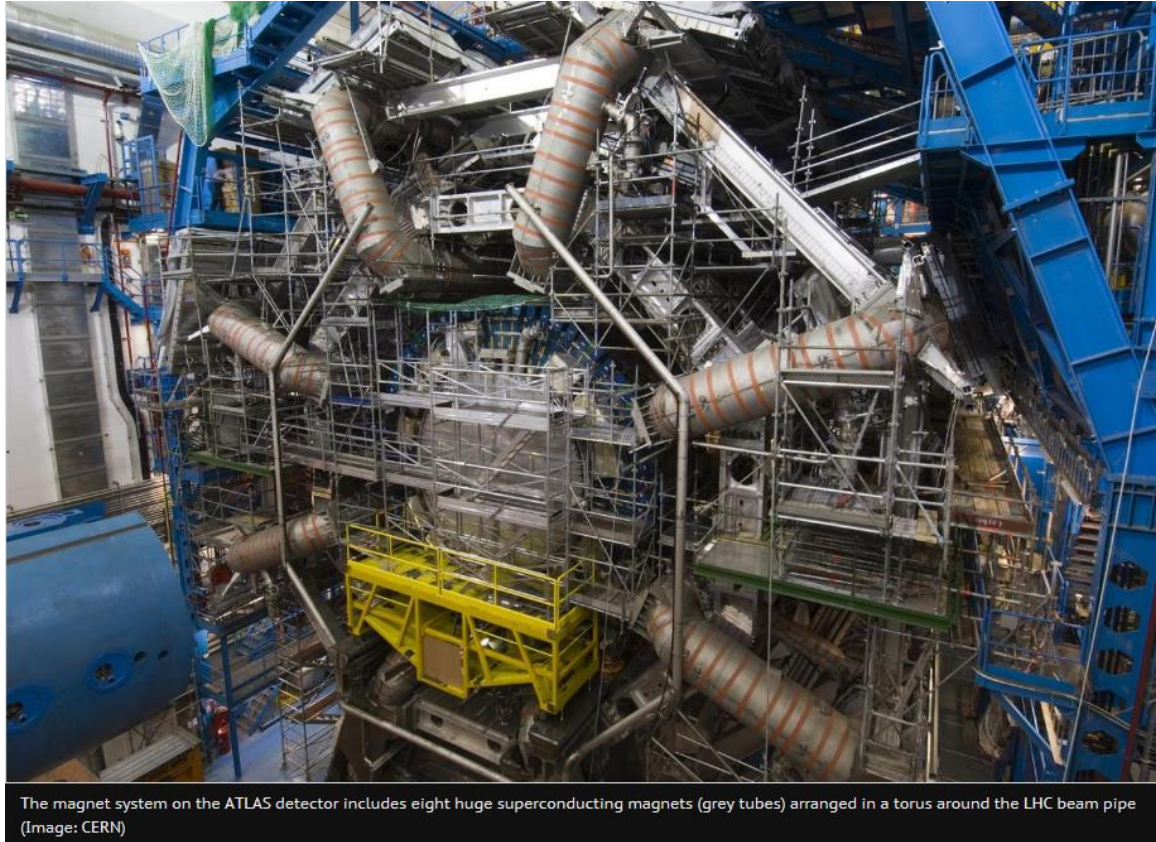
Given for topological phase transitions and topological phases of matter



David Thouless, Duncan Haldane and Michael Kosterlitz



Superconductivity in relativistic heavy ion collisions



The magnet system on the ATLAS detector includes eight huge superconducting magnets (grey tubes) arranged in a torus around the LHC beam pipe (Image: CERN)

The Large Hadron Collider (LHC) is currently operating at the energy of 6.5 TeV per beam. At this energy, the trillions of particles circle the collider's 27-kilometre tunnel 11,245 times per second. The magnet system on the ATLAS detector includes eight huge superconducting magnets (grey tubes) arranged in a torus around the LHC beam pipe (Image: CERN).

All the magnets on the LHC are superconducting. There are 1232 main dipoles, each 15 metres long and weighing in at 35 tonnes. If normal magnets were used in the 27 km-long LHC instead of superconducting magnets, the accelerator would have to be 120 kilometres long to reach the same energy.

Superconductivity in cancer therapy



Joseph Minervini: “Using superconductivity in a cyclotron design can reduce its mass an order of magnitude from conventional, resistive magnet machines,”

Making cancer treatment more accessible:

Alexey Radovinsky, Joe Minervini, Phil Michael, and Leslie Bromberg of the Plasma Science and Fusion Center MIT collaborates on a smaller, lighter delivery system for proton-beam radiotherapy.

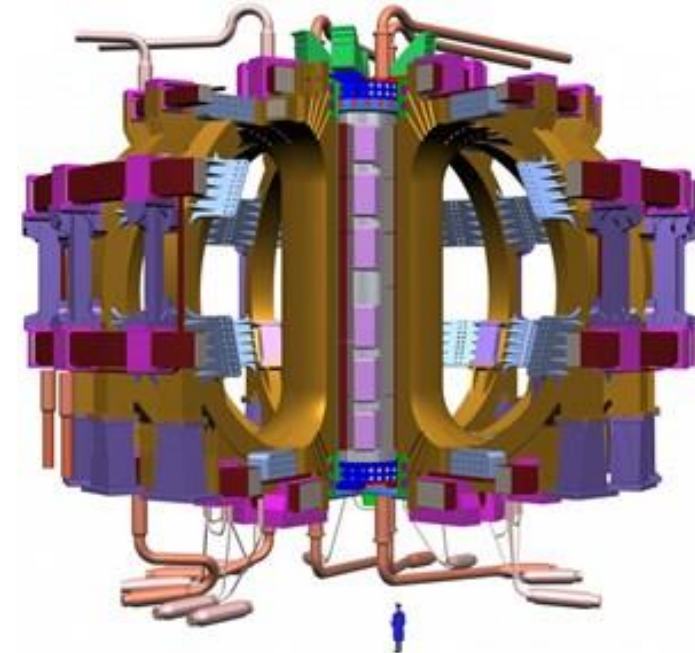
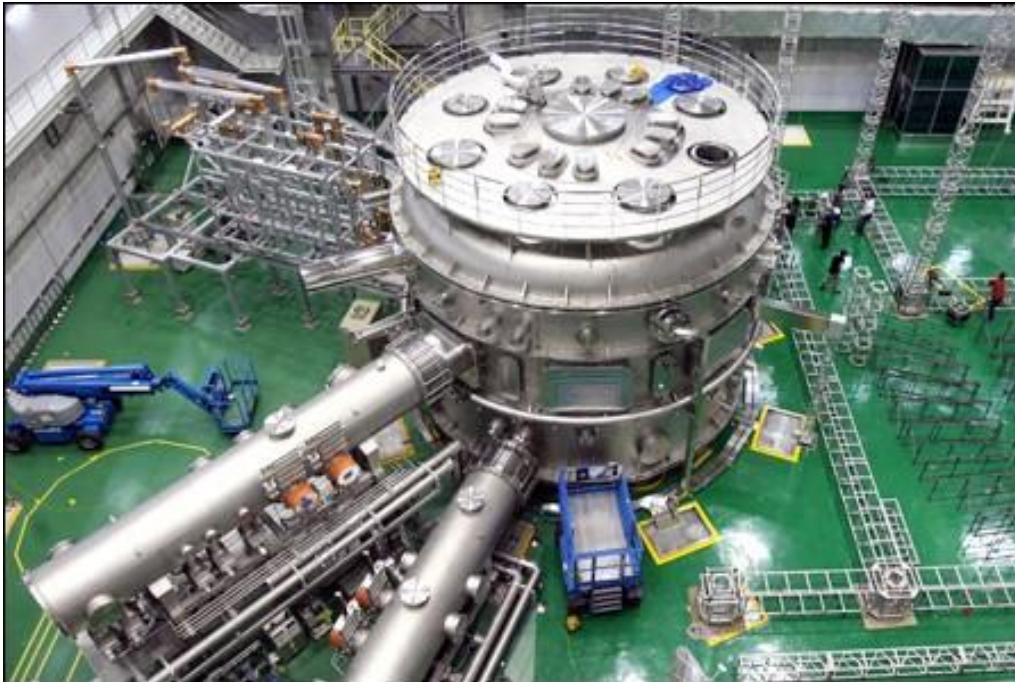


<http://thesilicongraybeard.blogspot.no/2015/07/techy-tuesday-using-superconductors-to.html>
<http://news.mit.edu/2015/making-cancer-treatment-more-accessible-0604>

Superconductivity in thermonuclear energy, ITER

‘ITER (International Thermonuclear Experimental Reactor, and is also Latin for "the way") is an international nuclear fusion research and engineering megaproject, which will be the world's largest magnetic confinement plasma physics experiment.’

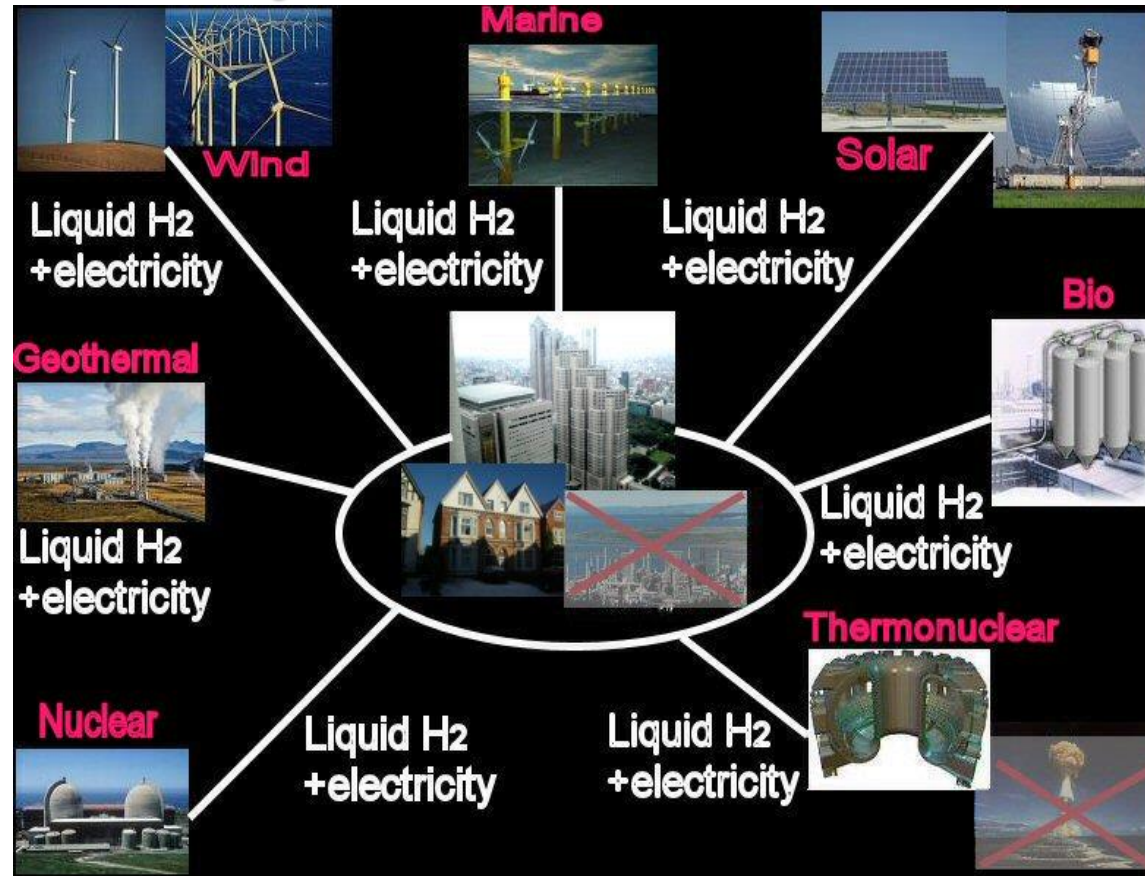
<https://en.wikipedia.org/wiki/ITER>



‘Without **superconductivity**, ITER would go from being a "net energy positive" machine to a "net energy negative" machine.’

<https://www.iter.org/newsline/146/408>

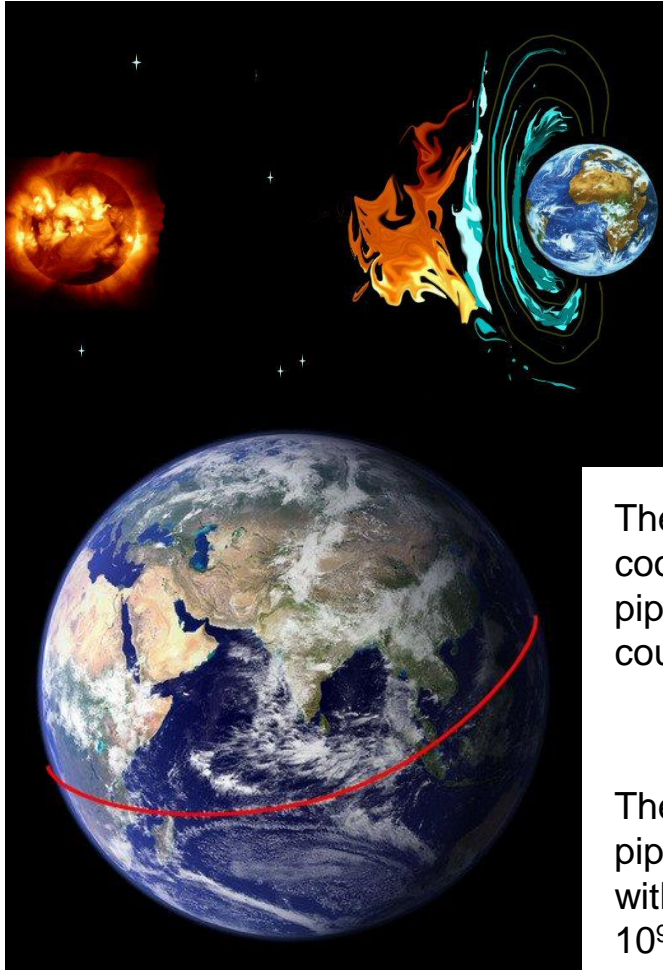
Superconductivity and renewable energy sources



- Hydrogen and electricity can easily be produced by renewable energy sources solving simultaneously problem of energy storage.
- Hydrogen can release full potential of superconductivity starting with building infrastructure for hydrogen economy.

Global applications of superconductivity

Magnetic field protection of Earth during poles reversal



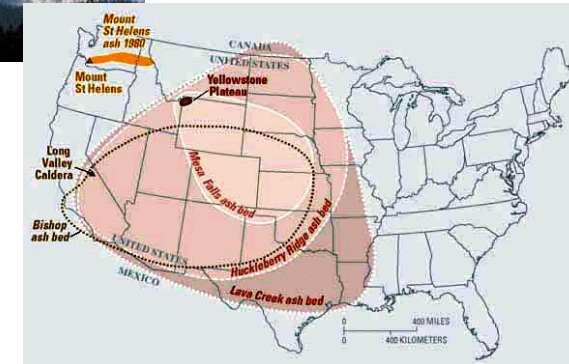
The liquid hydrogen-cooled superconducting pipeline encircling planet could be built

The superconducting pipeline would need to withstand a current of 10^9 A

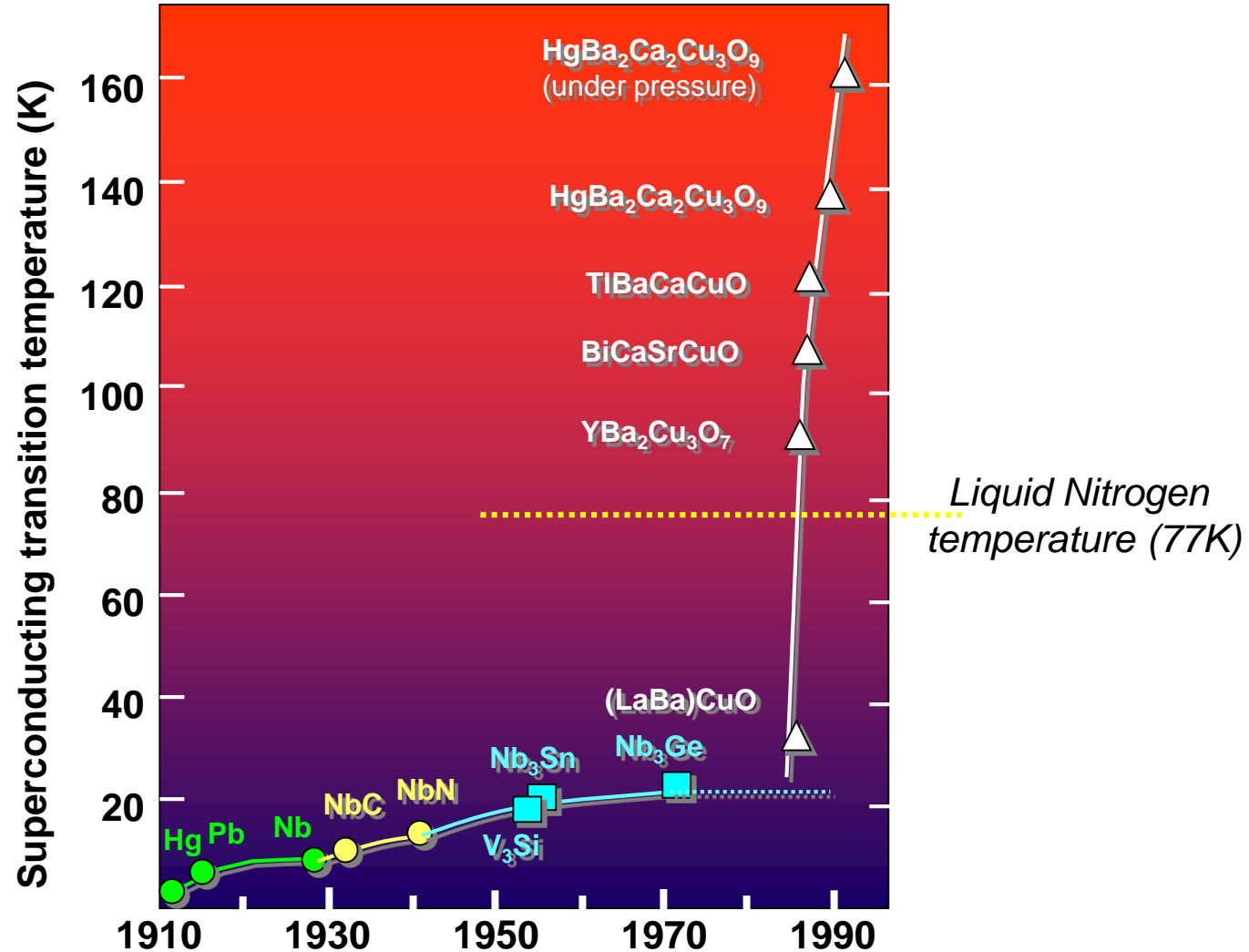
Prevention of super-volcano eruption



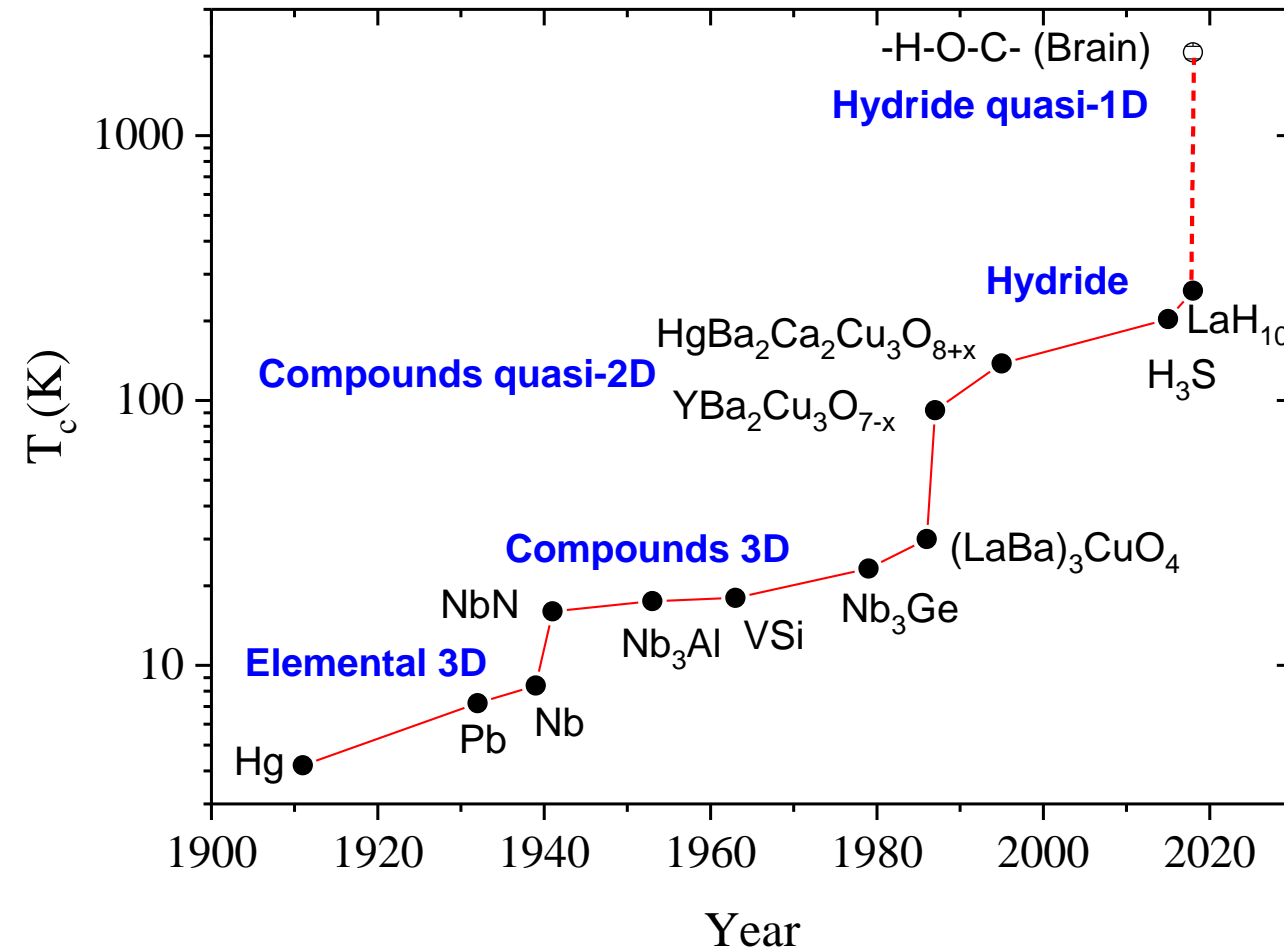
The superconducting pipeline encircling super-volcano could be used to extract energy and prevent its eruption



Critical temperature of superconductors




Progress in critical temperature



Possible superconductivity in the brain

<https://link.springer.com/article/10.1007/s10948-018-4965-4>


Springer Link



[Journal of Superconductivity and Novel Magnetism](#)
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Possible Superconductivity in the Brain

Authors [Authors and affiliations](#)

P. Mikheenko 

Review Paper
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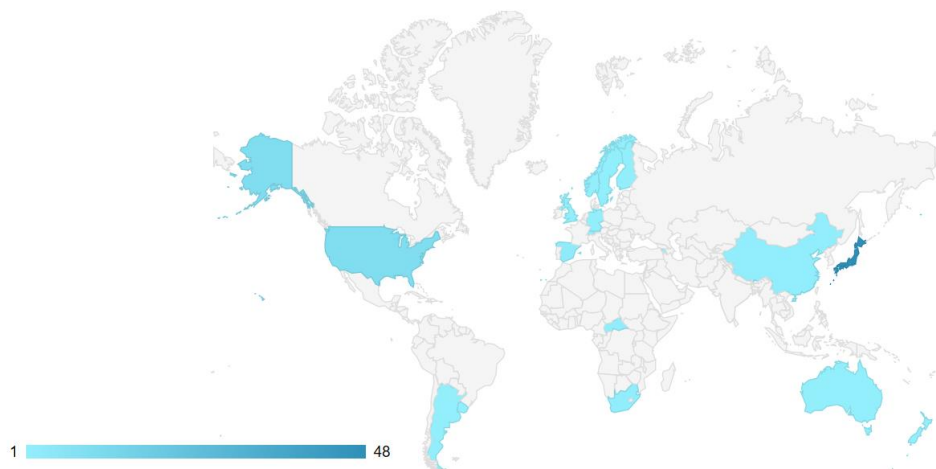
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<https://www.altmetric.com/details/52746868#score>

Summary

- **Electron gas** in solid materials is **quantum gas** at room temperature.
- The application of the formalism of **grand canonical ensemble** is needed for the description of electron gas.
- **Fermi-Dirac distribution** provides a good description of standard electron gas.
- At high temperatures, Fermi-Dirac distribution **merges with Boltzmann distribution**, as quantum volume becomes smaller than the volume for one particle of gas.
- **Chemical potential of electron gas changes from positive to negative** with the increase of temperature.
- **Bose-Einstein condensation** is possible in electron gas. It leads to the phenomenon of **superconductivity**.
- Superconductivity is intensively used in practical applications, and can form a basis of **renewable energy economy**.