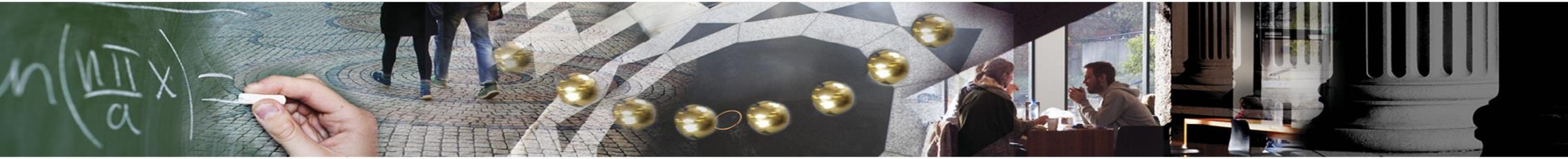

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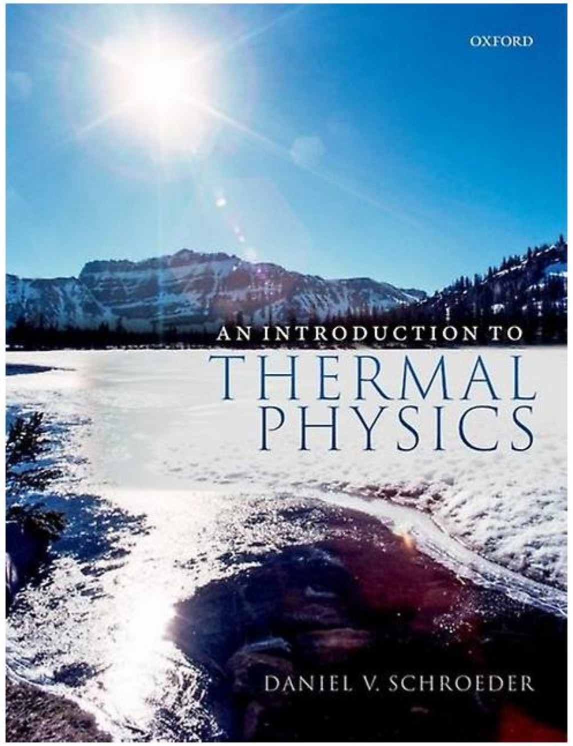
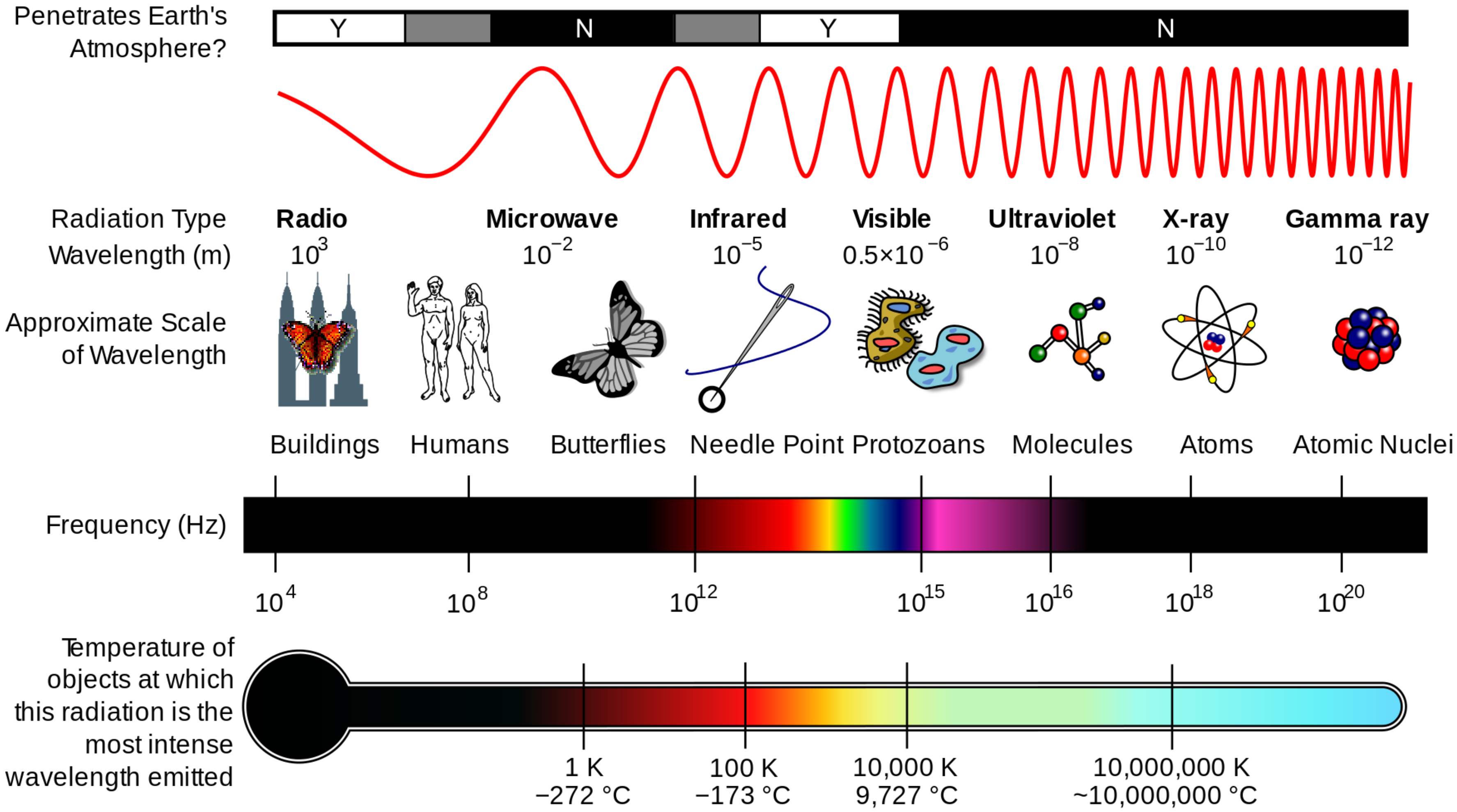
Blackbody radiation



7.4 Blackbody Radiation

Electromagnetic blackbody radiation

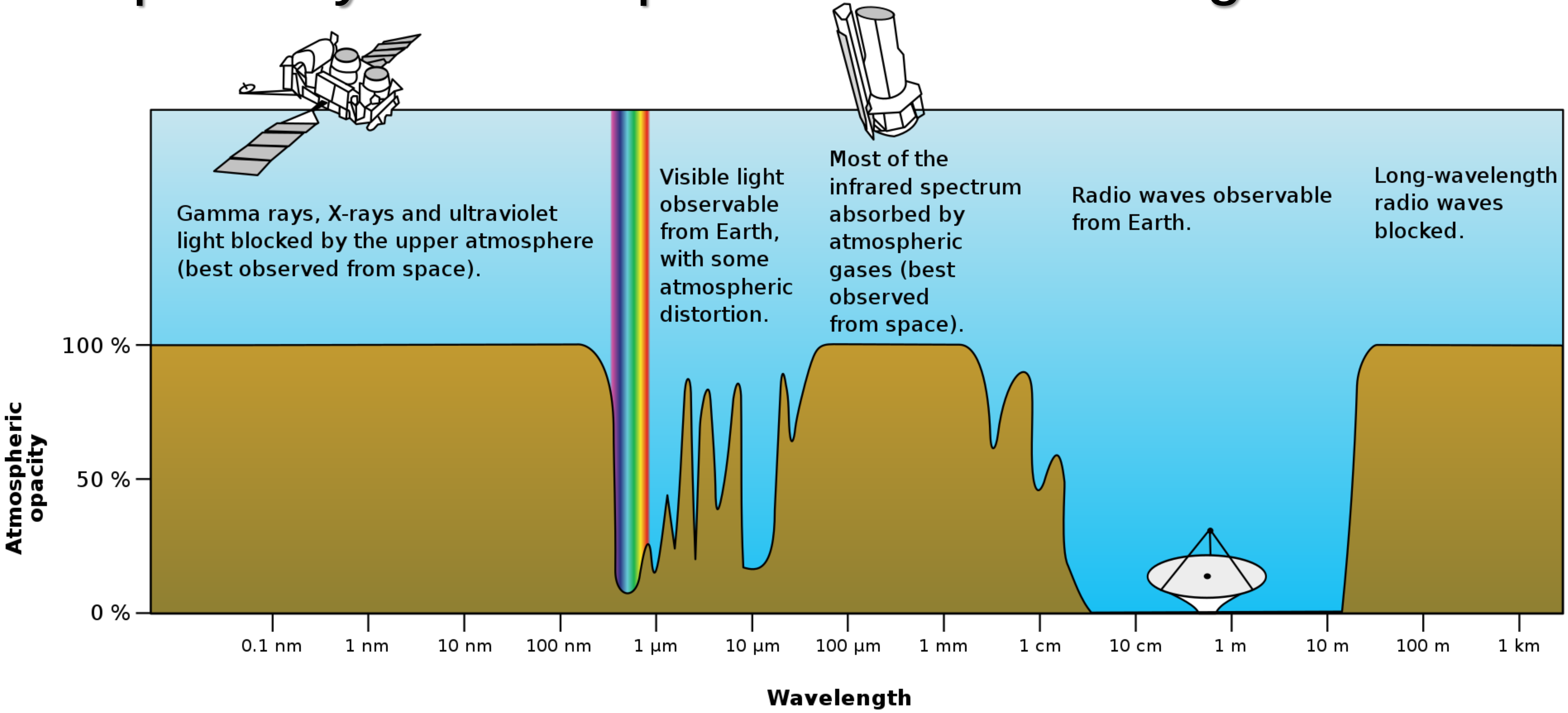
Blackbody radiation is the thermal electromagnetic radiation within or surrounding a body in thermodynamic equilibrium with its environment.



https://en.wikipedia.org/wiki/Electromagnetic_spectrum#/media/File:EM_Spectrum_Properties_edit.svg

Inductiveload, NASA

Transparency of atmosphere for electromagnetic radiation

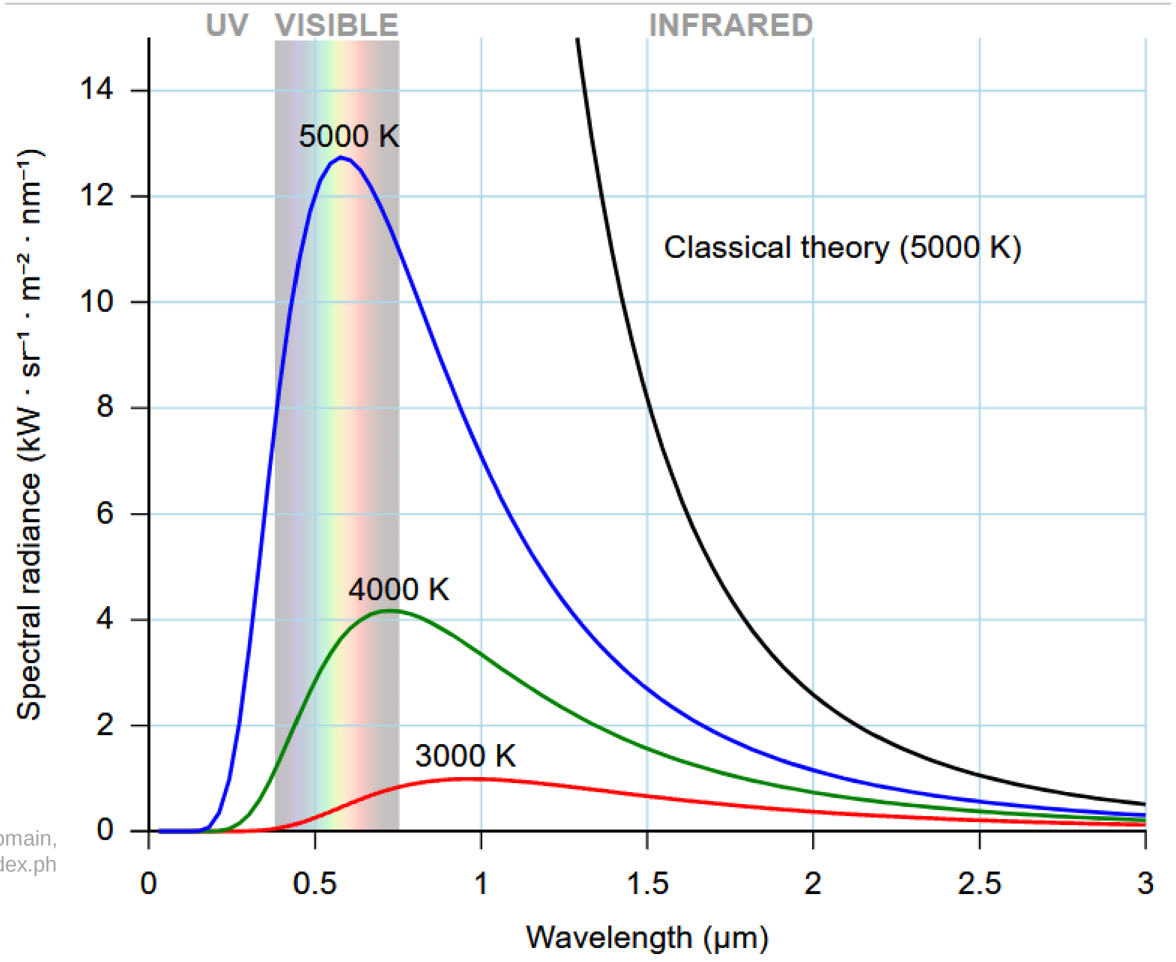


https://upload.wikimedia.org/wikipedia/commons/3/34/Atmospheric_electromagnetic_opacity.svg

Transparency of atmosphere is different for different wavelengths.

Blackbody radiation

- A blackbody is an idealized non-reflective body, which has specific spectrum of wavelengths λ inversely related to their intensity (at high λ).
- The spectrum of wavelengths depend only on the body's temperature.



$$\epsilon = \hbar\omega = \frac{hc}{\lambda}$$

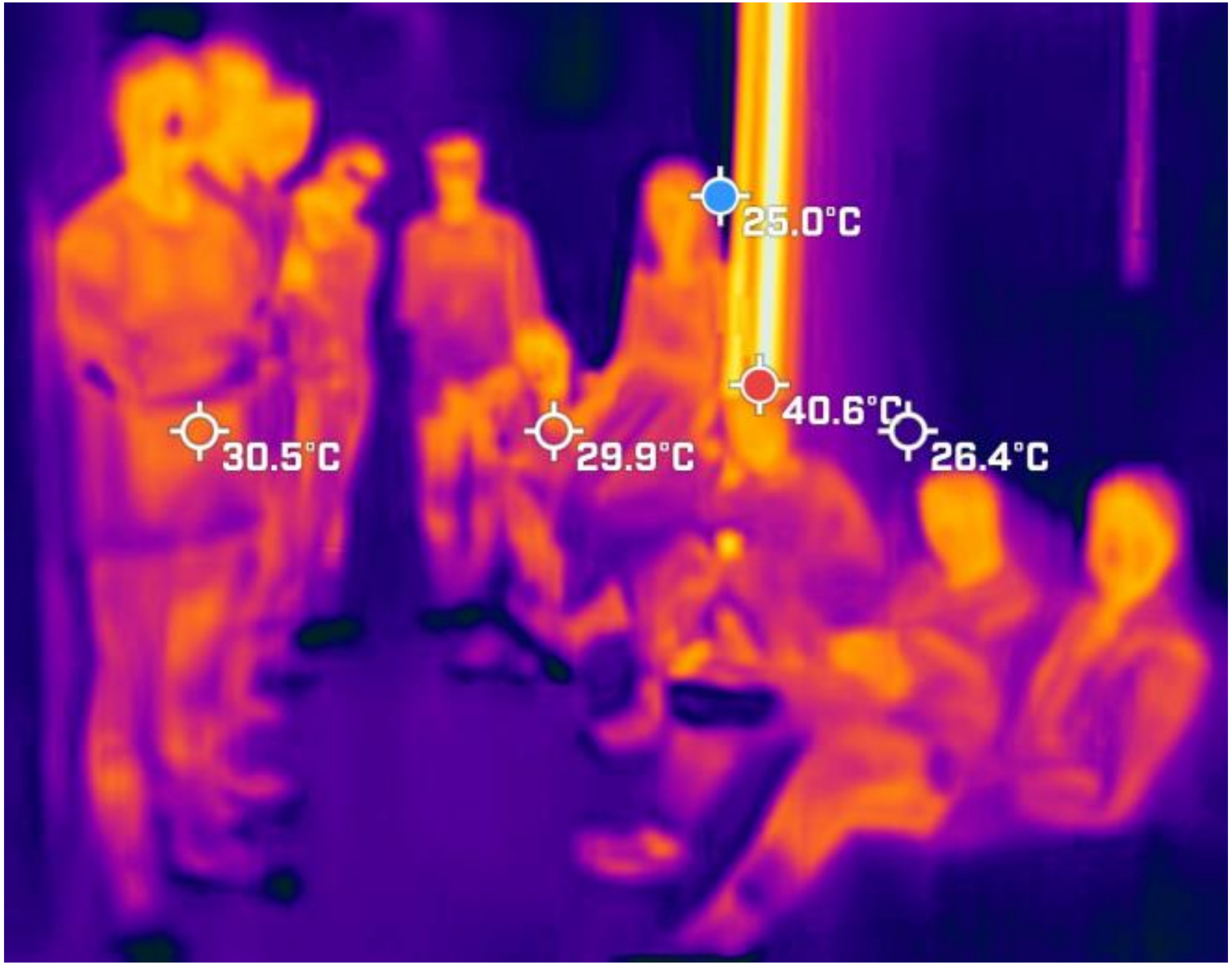
$$\hbar = \frac{h}{2\pi}$$

h is Planck constant
$h = 6.626\ 10^{-34}\ J/Hz$

Are we emitting visible light?

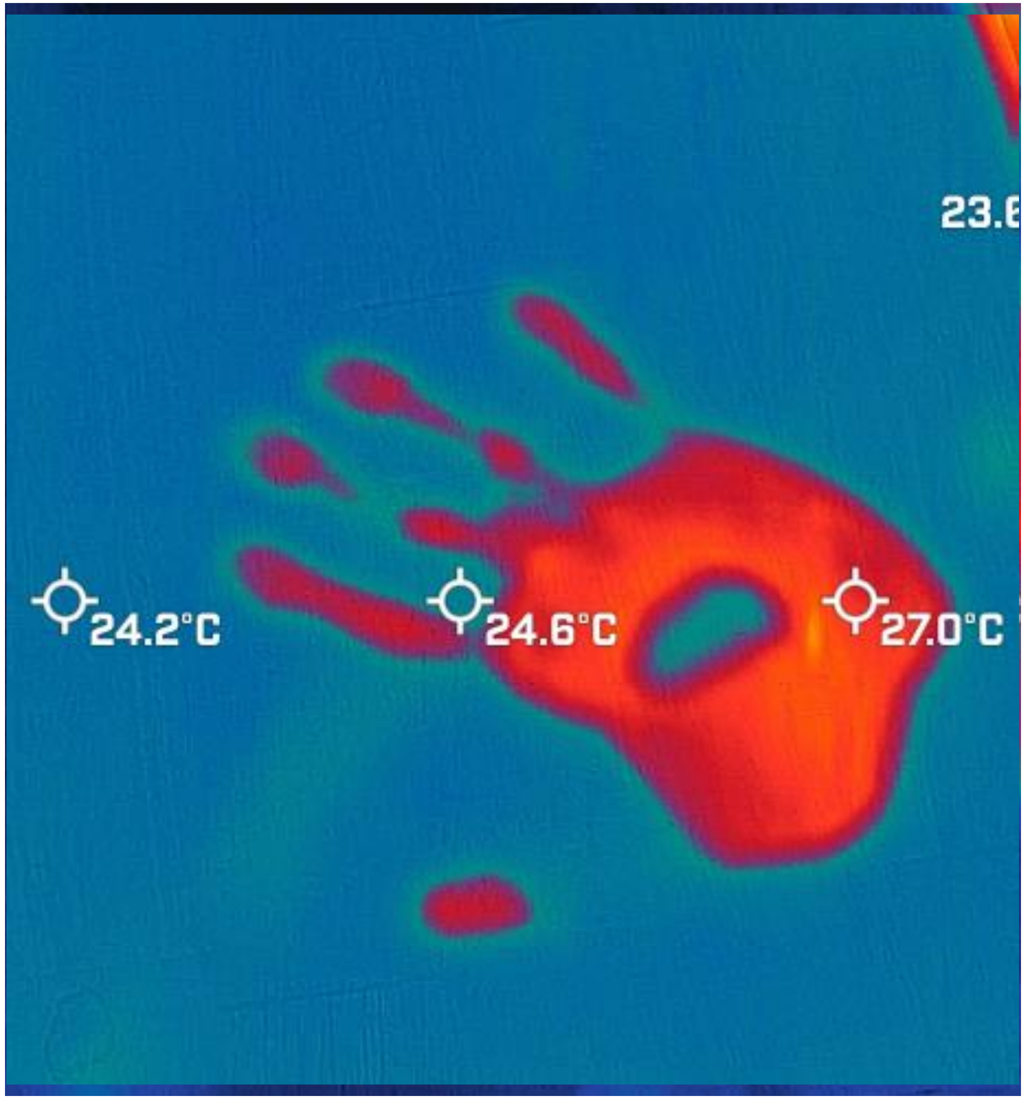
FYS2160students radiation

FYS2160students radiation

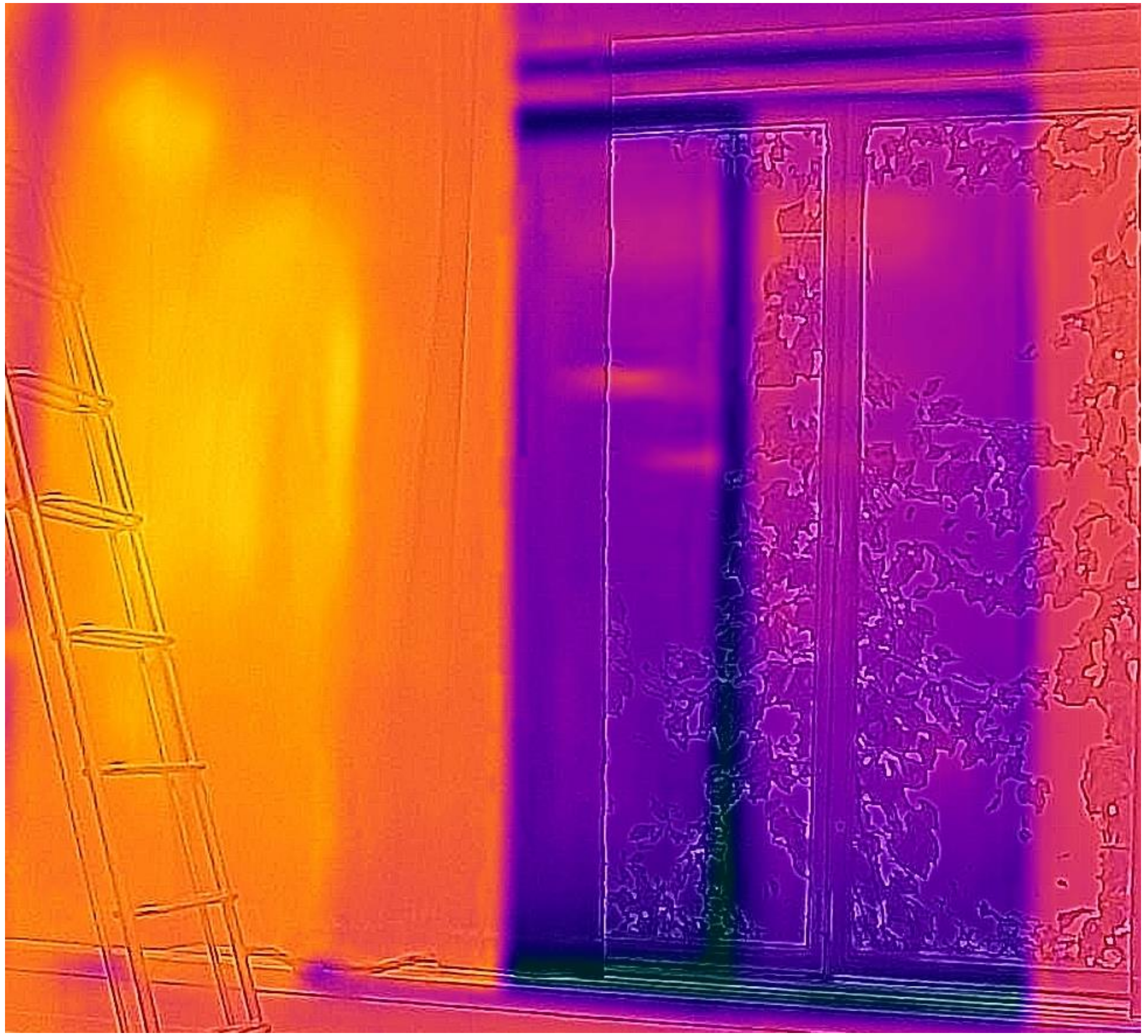


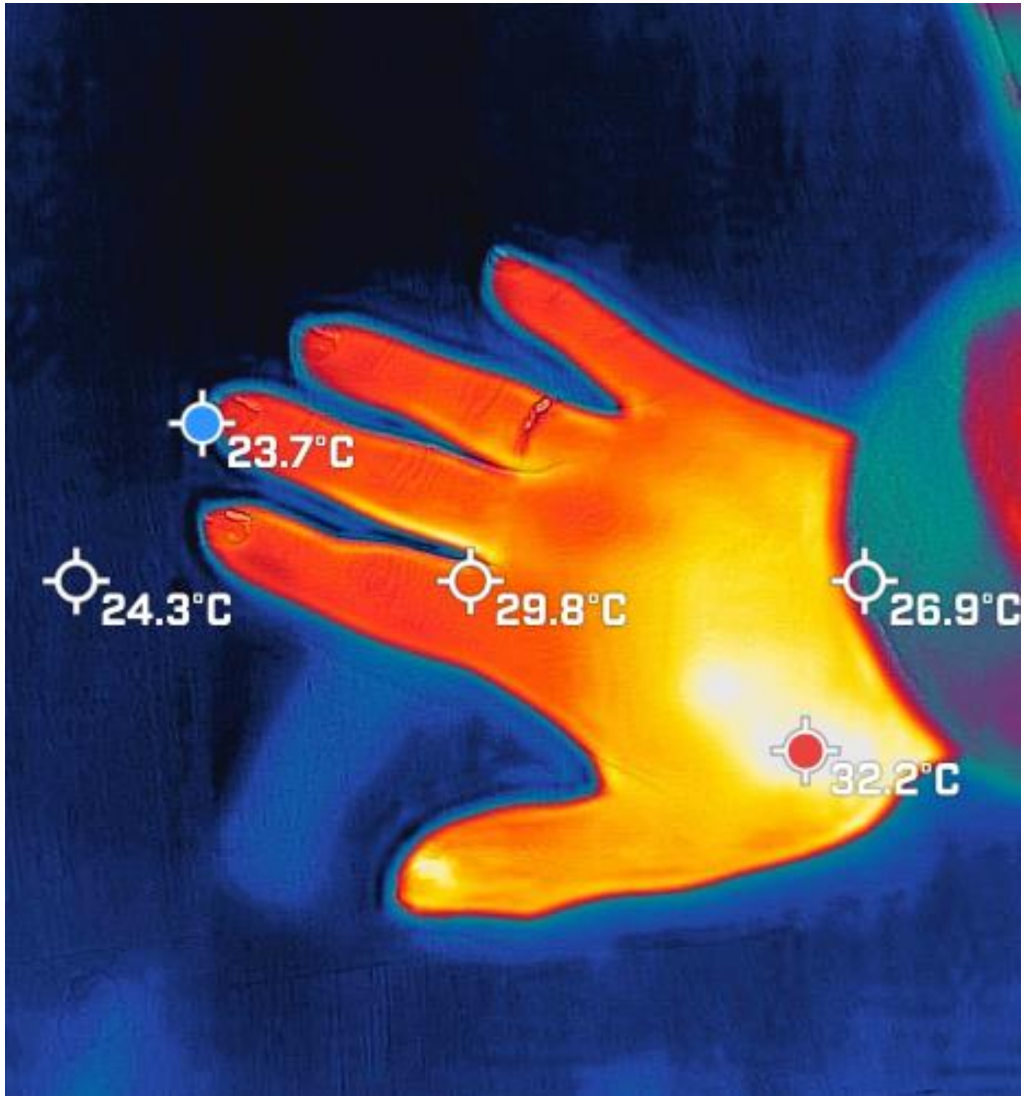
Dark matter

Detection of “crime”

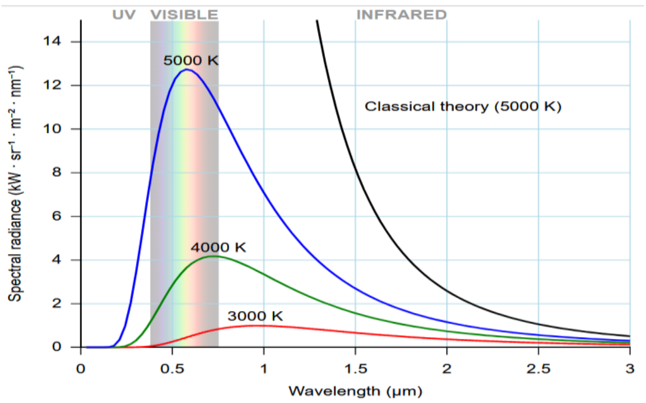


Detection of “crime”



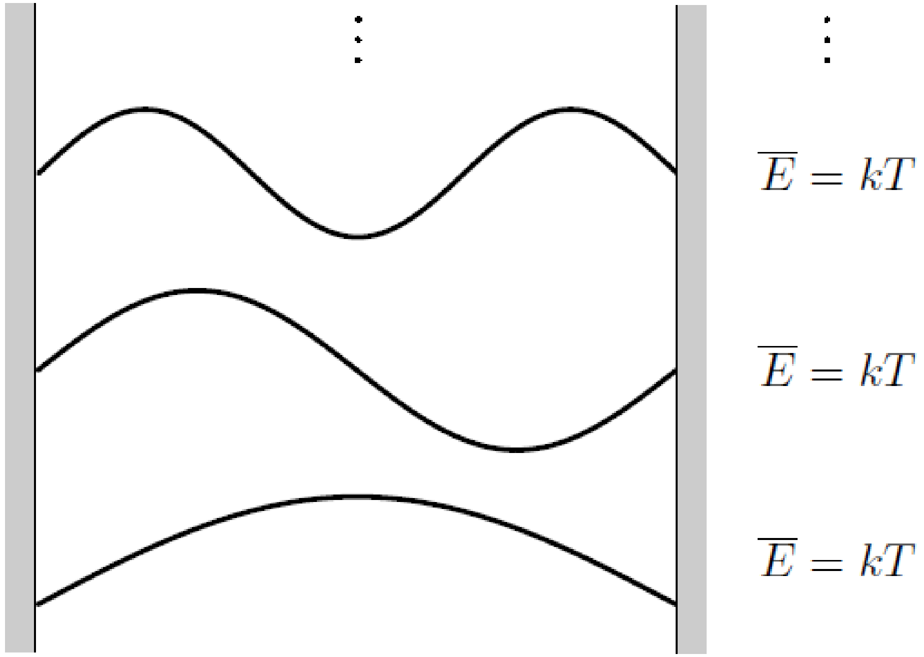


Ultraviolet Catastrophe



By Darth Kule - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=10555337

- In classical physics, electromagnetic radiation inside a box is treated as a combination of various standing-wave patterns.
- Each standing-wave pattern ($n\frac{\lambda}{2} = l$, n is integer number, l is length of the box) behaves as a harmonic oscillator with frequency $f = c/\lambda$, two degrees of freedom and with an average thermal energy of $2 \cdot 1/2 \cdot kT$.
- Since the total number of oscillators in the electromagnetic field is infinite, the total thermal energy should also be infinite.
- This paradox is named ultraviolet catastrophe because the infinite energy would come mostly from very short wavelengths.



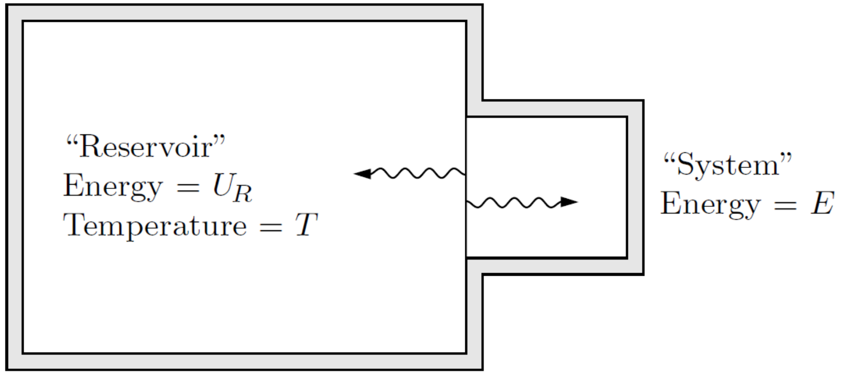
DVS

- Attempts to solve this paradox led to the birth of quantum mechanics.

Boltzmann statistics

Boltzmann statistics calculates probability of the system in the contact with reservoir having energy E. This probability is proportional to multiplicity of reservoir:

$$P(E) = C\Omega_R\ (E)$$



DVS

$$\Omega_R(E) = A\Omega_R\ (0) \quad S_R(E) = k\ \ln\Omega_R(0) + k\ln A$$

$$\Delta S_R = k\ln A \qquad \Delta U\ = T\Delta S - P\Delta V + \mu\Delta N$$

$$P(s) = \frac{1}{Z}e^{-\frac{E(s)}{kT}}$$

$$E\ = -\Delta U_R = -T\Delta S_R \qquad \Delta S_R = -\frac{E}{T}$$

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

$$A = e^{-E/kT} \qquad P(E) = AC\Omega_R\ (0)$$

Boltzmann distribution

A is Boltzmann factor $e^{-\frac{E}{kT}}$

$$P(E) = e^{-E/kT}C\Omega_R\ (0) = \frac{1}{Z}e^{-E/kT}.$$

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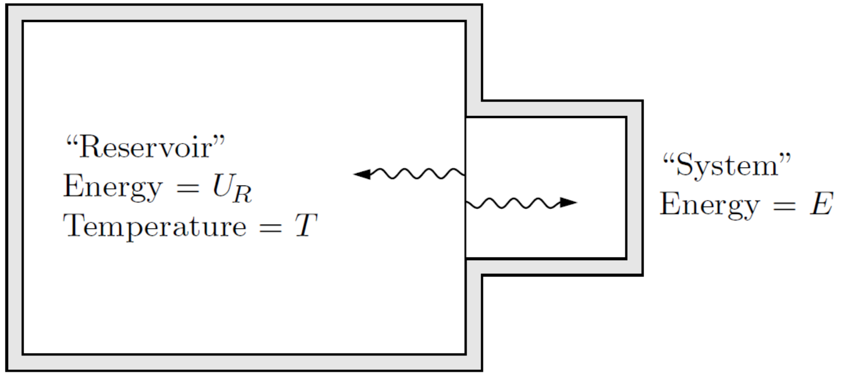
Planck Distribution

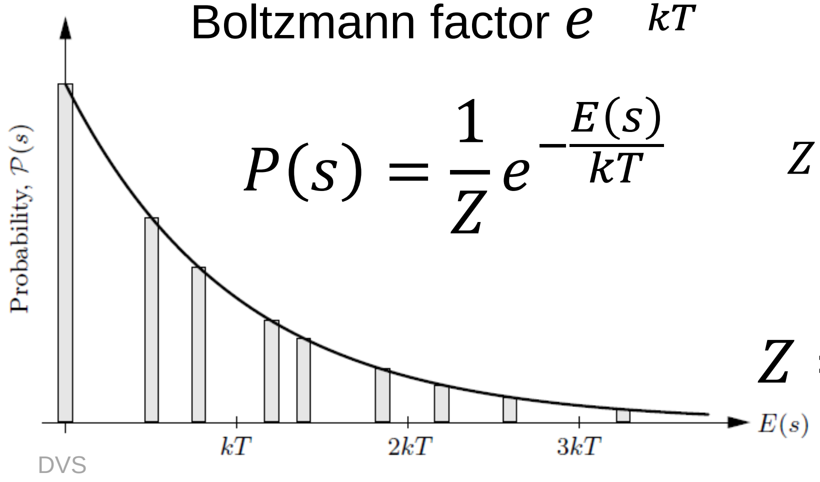
Canonical ensemble



The immediate solution of the paradox comes from Boltzmann distribution for quantized electromagnetic waves named Planck distribution, in which energy comes in units hf.

Boltzmann factor $e^{-\frac{E(s)}{kT}}$

$$E(s) = 0, hf, 2hf, \ldots.$$



$$P(s) = \frac{1}{Z} e^{-\frac{E(s)}{kT}}$$

Z is the partition function, $\beta = \frac{1}{kT}$

$$Z = \sum_s e^{-\frac{E(s)}{kT}} = \frac{1}{1 - e^{-\beta hf}}$$

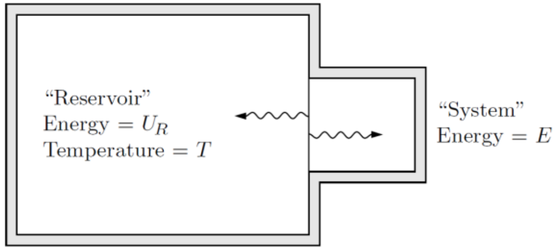
Average energy

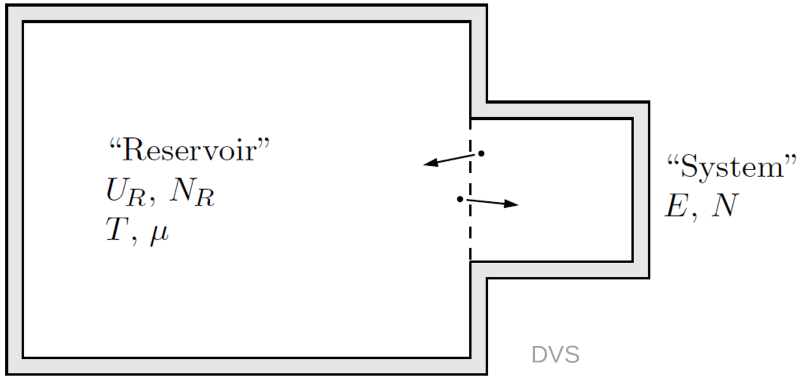
$$\bar{E} = \frac{1}{Z} \sum_s E(s) e^{-\frac{E(s)}{kT}}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{hf}{e^{\frac{hf}{kT}} - 1}$$

- According to Planck distribution, short-wavelength modes with $hf \gg kT$ are exponentially suppressed.
- The total number of electromagnetic oscillators that strongly contribute to the energy inside the box is finite.
- The ultraviolet catastrophe does not occur.
- Energy quantization is the necessary condition and it is the size of the energy units, compared to kT, that provides the exponential suppression factor. Elementary particles in this distribution are named photons.

Transition to Gibbs statistics





Boltzmann statistics calculates probability of the system in the contact with reservoir having energy E. This probability is proportional to multiplicity of reservoir:

$$P(E) = C\Omega_R(E)$$

$$\Omega(E) = A\Omega_R(0) \qquad S_R(E) = k\ln\Omega_R(0) + k\ln A$$

$$\Delta S_R = k\ln A \qquad \Delta U = T\Delta S - P\Delta V + \mu\Delta N$$

$$E = -\Delta U_R = -T\Delta S_R - \mu\Delta N_R \qquad \Delta S_R = -\frac{E-\mu N}{T}$$

$$A = e^{-(E-\mu N)/kT} \qquad \mathcal{P}(E) = AC\Omega_R(0)$$

Gibbs distribution

A is Gibbs factor $e^{-\frac{E-\mu N}{kT}}$

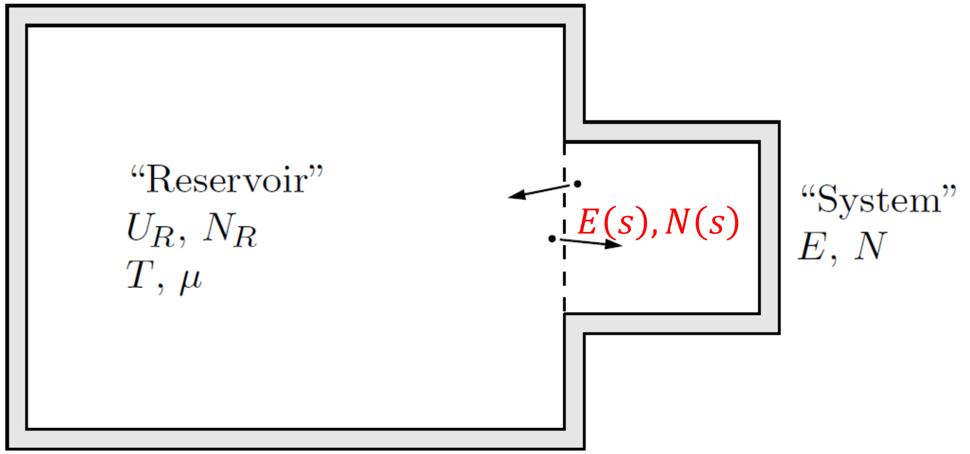
$$\mathcal{P}(E) = e^{-(E-\mu N)/kT}C\Omega_R(0) = \frac{1}{\mathcal{Z}}e^{-(E-\mu N)/kT}$$

Grand canonical distribution for photons?

Gibbs factor $e^{-\frac{E(s)-\mu N(s)}{kT}}$

Probability distribution $\mathcal{P}(s) = \frac{1}{\mathcal{Z}} e^{-\frac{E(s)-\mu N(s)}{kT}}$

Grand partition function $\mathcal{Z} = \sum_s e^{-\frac{E(s)-\mu N(s)}{kT}}$



DVS

$$\bar{E} = \frac{hf}{e^{\frac{hf}{kT}} - 1}$$

Planck distribution

$$\bar{n} = \frac{1}{e^{\frac{hf}{kT}} - 1}$$

Bose - Einstein distribution

$$\bar{n} = \frac{1}{e^{\frac{hf-\mu}{kT}} - 1} \quad (\epsilon = hf)$$

Energy of one particle is hf Average number of particles or photons

Planck distribution is Bose - Einstein distribution with chemical potential equal to zero. This comes from the fact that photons can be created or destroyed in any quantity. Their total number is not conserved.

The chemical potential for a gas of photons in a box is zero.

Planck Spectrum

Integrating energy over all states and taking into account momentum quantization and specific dispersion law for phonons : $\epsilon = pc$, where p is momentum and c is speed of light, it can be shown that the total energy per unit of volume U/V as function of ϵ is:

$$\frac{U}{V} = \int_0^{\infty} \frac{8\pi\epsilon^3/(hc)^3}{e^{\epsilon/kT} - 1} d\epsilon .$$

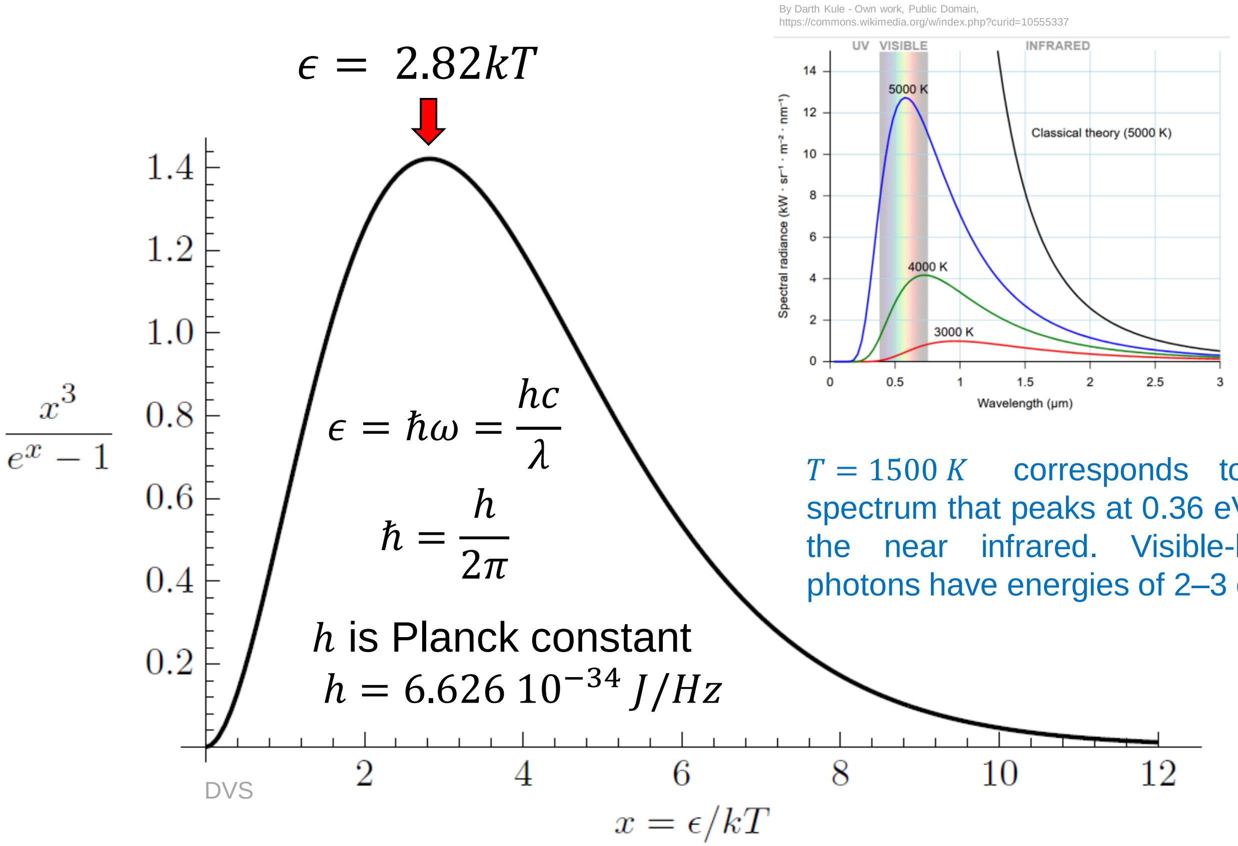
The integrand is the energy density per photon energy, or the spectrum of the pl

$$u(\epsilon) = \frac{8\pi\epsilon^3}{(hc)^3(e^{\epsilon/kT} - 1)} .$$

Introducing $x = \epsilon/kT$,

$$\frac{U}{V} = \frac{8\pi(kT)^4}{(hc)^3} \int_0^{\infty} \frac{x^3}{e^x - 1} dx .$$

$\epsilon = \ 2.82kT$



$$\epsilon = \hbar\omega = \frac{hc}{\lambda}$$

$$\hbar = \frac{h}{2\pi}$$

h is Planck constant

$h = 6.626\ 10^{-34}\ J/Hz$

$T = 1500\ K$ corresponds to a spectrum that peaks at 0.36 eV, in the near infrared. Visible-light photons have energies of 2–3 eV.

The spectrum peaks at $x = \ 2.82$, or $\epsilon = \ 2.82kT$. Higher temperatures give higher photon energies: **Wien's law.**

Density of photon states

Density of states $D(\epsilon) = dN/d\epsilon$ at energy $\epsilon = pc$ can be calculated using momentum space p or $\hbar \boldsymbol{k}$

If the sample is cube with dimensions $L \times L \times L$, k in each of three directions is quantised in units of $\frac{2\pi}{L}$ or p in units of $\hbar \boldsymbol{k} = \frac{2\pi\hbar}{L} = \frac{h}{L}$.

The total number of states n at a maximum momentum p is $N = \frac{4\pi}{3} p^3 / \left(\frac{h}{L}\right)^3$.

Since $\epsilon = pc$, total number of states is: $N = \frac{4\pi}{3} p^3 / \left(\frac{h}{L}\right)^3 = \frac{4\pi V}{3} \epsilon^3 / (hc)^3$

The density of states for one polarisation is then: $D(\epsilon) = dN/d\epsilon = V(4\pi\epsilon^2)/(hc)^3$

Total energy of photons

Total energy is:

$$U = 2\int_0^{\epsilon_{max}} \bar{n}_{pt}\epsilon D(\epsilon)d\epsilon$$

(2 is due to two different polarizations)

The average number of photons with energy ϵ is:

$$\bar{n}_{pt} = \frac{1}{e^{\frac{\epsilon}{kT}} - 1}$$

$$D(\epsilon) = V(4\pi\epsilon^2)/(hc)^3$$

$$U = 2\int_0^{\infty} \frac{V(4\pi\epsilon^3 d\epsilon)/(hc_s)^3}{e^{\frac{\epsilon}{kT}} - 1} \Rightarrow x = \frac{\epsilon}{kT} \Rightarrow \frac{U}{V} = \frac{8\pi(kT)^4}{(hc)^3}\int_0^{\infty} \frac{x^3 dx}{e^x - 1}$$

$$\int_0^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15} \Rightarrow \frac{U}{V} = \frac{8\pi^5(kT)^4}{15(hc)^3}$$

Total energy of photon gas

$$\frac{U}{V} = \frac{8\pi^5 (kT)^4}{15(hc)^3}$$

- The energy density is proportional to fourth power of the temperature. If the temperature is doubled, the amount of electromagnetic energy inside increases by a factor of $2^4 = 16$.
- Total electromagnetic energy inside a typical oven is quite small. At cookie-baking temperature of about 460 K, the energy per unit volume is 3.5 10^{-5} J/m^3. This is tiny compared to the thermal energy density of the air inside the oven of about 10^3 J/m^3.

The formula for energy density can be obtained as: $U = (constant)\frac{VkT}{(\lambda)^3},$

λ is temperature-dependent de Broglie wavelength for the photons, $\lambda = h/p = hc/\epsilon = hc/kT$

Ice melting enthalpy is 3.3355 10^5 J/kg

The heat of vaporization of water is about 2.260 10^6 J/kg

Air has a density of approximately 1.225 kg/m^3

Heat capacity and entropy of photon gas

Total energy density: $\frac{U}{V} = \frac{8\pi^5 (kT)^4}{15(hc)^3}$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 4aT^3, \qquad a = \frac{8\pi^5 k^4 V}{15(hc)^3}.$$

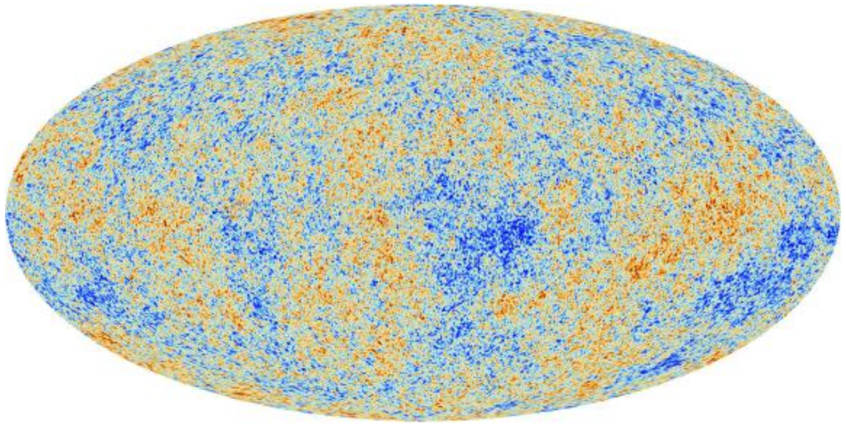
- Heat capacity of photon gas is proportional to T^3
  - Entropy of photon gas is: $S = \frac{4}{3}aT^3$

$$dU = TdS - PdV + \mu dN \qquad \frac{dU}{dT} = C_V = T\frac{dS}{dT}$$

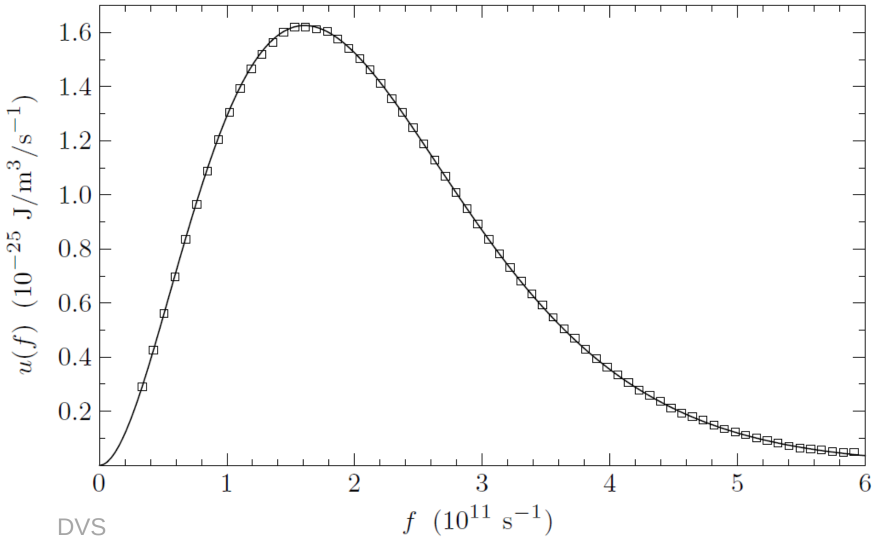
$$S(T) = \int_0^T \frac{C_V(T')}{T'}\,dT' = 4a\int_0^T (T')^2\,dT' = \frac{4}{3}aT^3$$

$$S = \frac{32\pi^5 Vk}{45}\left(\frac{kT}{hc}\right)^3$$

The Cosmic Background Radiation



- The radiation that fills the entire observable universe has an almost perfect thermal spectrum at a temperature of 2.73 K.
- The radiation is thought to be left from a time when the universe was filled with ionized gas that interacted strongly with electromagnetic radiation. At that time, the temperature was more like 3000 K.
- Since universe has expanded a thousand-fold in all directions, the photon wavelengths have been stretched according to Doppler effect preserving the shape of the spectrum, but shifting the effective temperature down to 2.73 K.
- The peak of the spectrum corresponds to wavelengths of about a millimeter, in the far infrared. These wavelengths don't penetrate Earth atmosphere. The spectrum is best measured from the satellites above atmosphere.



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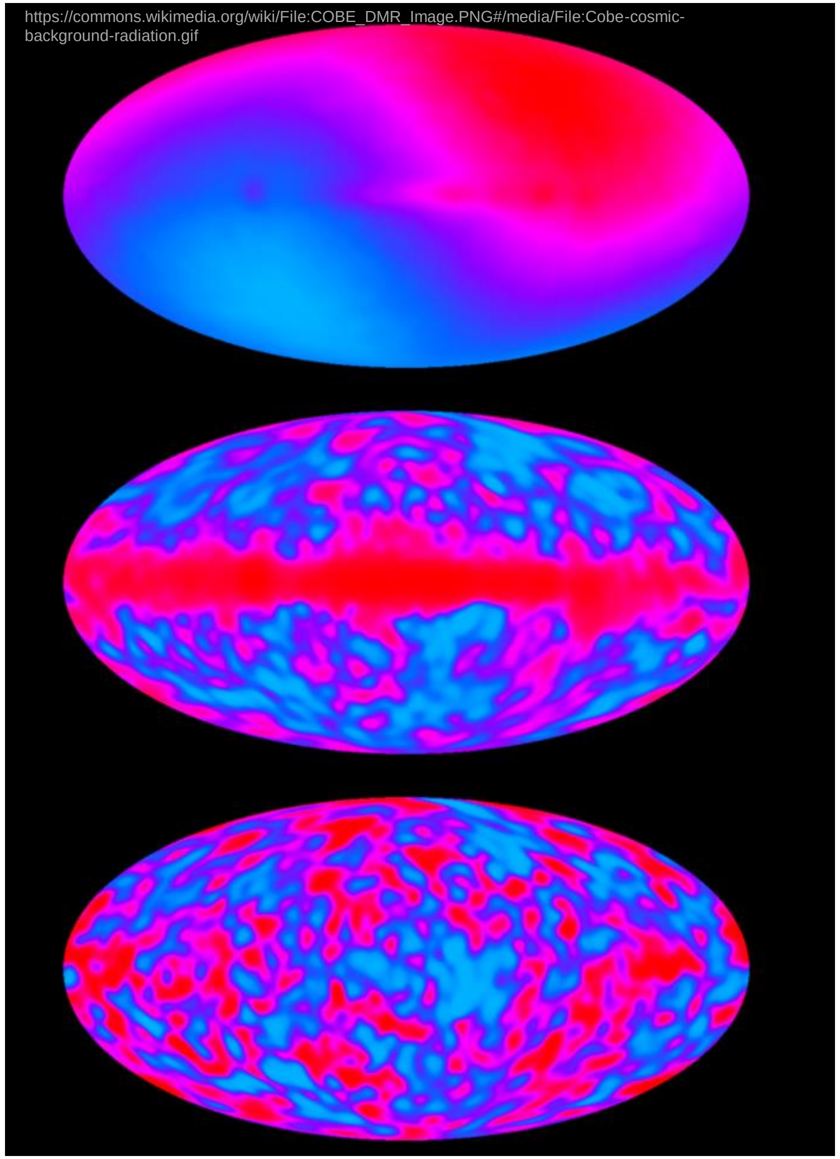
Spectrum of the cosmic background radiation, as measured by the Cosmic Background Explorer satellite.

Properties of Cosmic Background Radiation

$$\frac{U}{V} = \frac{8\pi^5 (kT)^4}{15(hc)^3} \qquad S = \frac{32\pi^5 V k}{45}\left(\frac{kT}{hc}\right)^3$$

- The total energy in the cosmic background radiation is only 0.26 MeV/m^3. It is much smaller than average energy density of ordinary matter (a proton per cubic meter on cosmic scale) of 1000 MeV/m^3.
- In contrast to energy, the entropy of the background radiation (1.5 10^9)k is much bigger than that of ordinary matter, which is a few k per cubic meter.



1 eV =1.6 10^{-19} J

First 2-years Cosmic Background Explorer satellite data: In the pictures, the plane of the Milky Way Galaxy is horizontal across the middle of them. Top: uncorrected temperature map; middle: corrected for the dipole term due to velocity of observer; bottom: further corrected image removing the contribution of our galaxy.

Photons escaping from a surface

- All photons travel at the same speed regardless of their wavelengths. Therefore, low-energy photons will escape through a hole in an oven with the same probability as high-energy photons, and the spectrum of the photons coming out of hole must be the same as the spectrum of the photons inside box.
- Detailed geometrical calculations for the oven show that the power emitted per unit area $\frac{\mathbb{W}}{A}$ is $\frac{cU}{4V}$. With accuracy of ¼ it could be derived just considering units of available parameters.

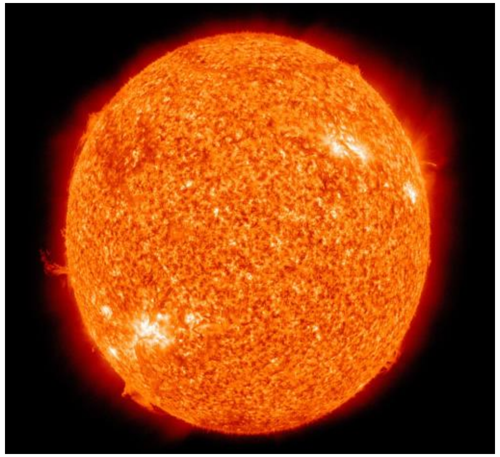
$$\frac{\mathbb{W}}{A} = \frac{cU}{4V} \qquad \frac{U}{V} = \frac{8\pi^5 (kT)^4}{15(hc)^3} \qquad \frac{\mathbb{W}}{A} = \sigma T^4 \qquad \sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67\ 10^{-8} \frac{W}{m^2 K^4}$$

Stefan's law
discovered empirically in 1879

Stefan-Boltzmann constant

- **Stefan's law** for photons applies to photons emitted by any nonreflecting ("**black**") surface at temperature T. Such radiation is therefore called **blackbody radiation**.

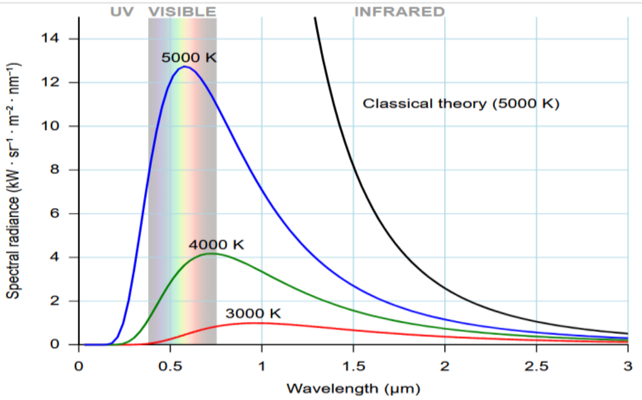
The Sun and the Earth



Stefan's law $\frac{\mathbb{W}}{A} = e\sigma T^4$ $\sigma = 5.67\ 10^{-8} \frac{W}{m^2 K^4}$

[The Sun's Magnetic Field is about to Flip | NASA]

- The Stefan's law can be applied to sun as radiating object.
- Earth receives from the sun radiation of 1370 W/m^2 known as the **solar constant** S_k. From earth's radius (6370 km) and the distance from the sun of 150 million kilometers, sun's total energy output or luminosity $\mathbb{W}$ is 3.9 10^{26} watts.
- The sun's radius (≈700 000 km) is a little over 100 times the earth's. Its surface area is then 6.1 10^{18} m^2.
- Assuming an emissivity e equal to one, the temperature of the sun ($T = \left(\frac{\mathbb{W}}{Ae\sigma}\right)^{1/4}$) is 5800 K.
- This gives maximum of spectrum at a photon energy of $\epsilon = 2.82\,kT = 1.41\,eV$, which corresponds to a wavelength of 880 nm, in the near infrared. This agrees well with experiment. Therefore, **much of the sun's energy is emitted as visible light.**
- The sun's spectrum is approximately given by the Planck formula.
- A tiny fraction of the sun's radiation is absorbed by the earth, warming the surface to a temperature suitable for life.



Equilibrium surface temperature of Earth

- On average, the earth is in thermal equilibrium. It also emits radiation into space, approximately at the same rate as it is receiving it from sun. The balance between absorption and emission gives a way to estimate the earth's equilibrium surface temperature.
- First approximation is to consider earth as a perfect blackbody at all wavelengths. The power absorbed by earth is the **solar constant** $S_k$times the earth's cross-sectional area as viewed from the sun, πR^2, where R is earth's radius. The emission is from all surface: $4\pi R^2$.The equilibrium condition is:

$$S_k \pi R^2 = 4\pi R^2 \sigma T^4 \qquad T = \left(\frac{S_k}{4\sigma}\right)^{1/4} = \left(\frac{1370\ \frac{W}{m^2}}{4 \cdot 5.67\ 10^{-8}\ \frac{W}{m^2 K^4}}\right)^{1/4} = 279\ K$$

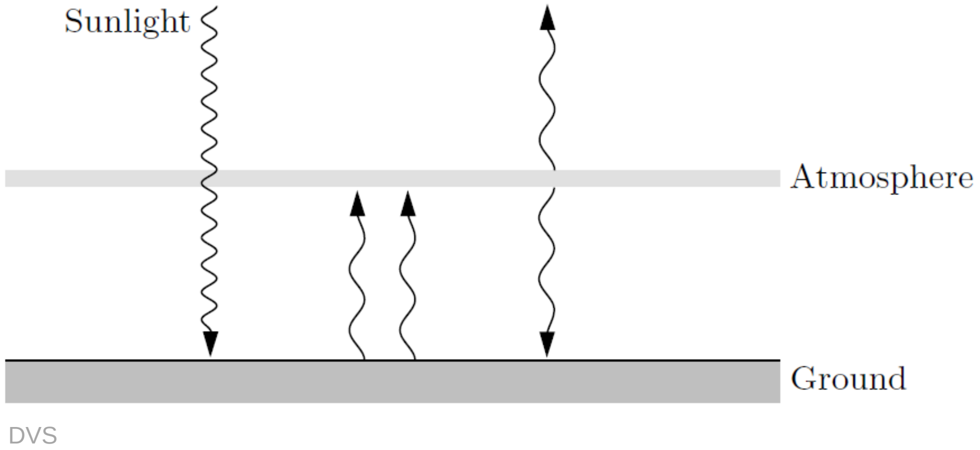
The estimated value is very close to the measured average temperature of $288\ K$ $(15\ °\mathrm{C})$.

- However, the earth is not a perfect blackbody, and about 30% of the sunlight is reflected directly back into the space. This would decrease average temperature down to $255\ K$.
- The counterbalancing (although not perfect) effect is the reflection of radiation from opaque atmosphere.

Greenhouse effect

$$T = \left(\frac{S_k}{4\sigma}\right)^{1/4}$$

The atmosphere can be modelled in single-layer approximation:



- In the single-layer approximation, surface of the planet receives twice the energy that is provided by the sun.
- According to this equation for T, the surface temperature would increase by a factor of $2^{1/4}$, i.e. to $303\ K$.
- Since atmosphere is not a single layer, its complex effect leads to measured $288\ K$.
- Evidently **due to human activity, earth is out of equilibrium and slowly warming**. This could have strong consequences for the climate and influence human population as a whole.
- The advanced modelling of the climate clarifying greenhouse effect is awarded Nobel Prize in Physics 2021.

UiO University of Oslo

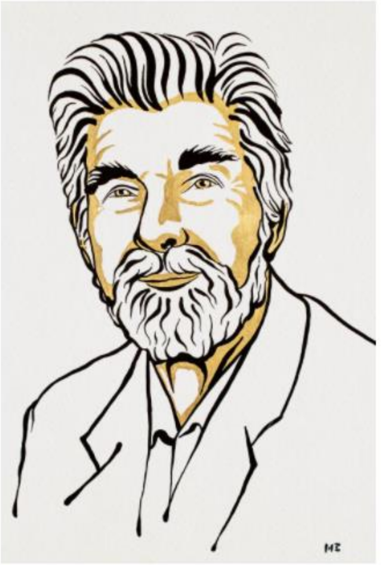
Nobel Prize in Physics 2021

https://www.nobelprize.org/prizes/physics/2021/summary/

Ill. Niklas Elmehed © Nobel Prize Outreach

Syukuro Manabe

Prize share: 1/4



Ill. Niklas Elmehed © Nobel Prize Outreach

Klaus Hasselmann

Prize share: 1/4

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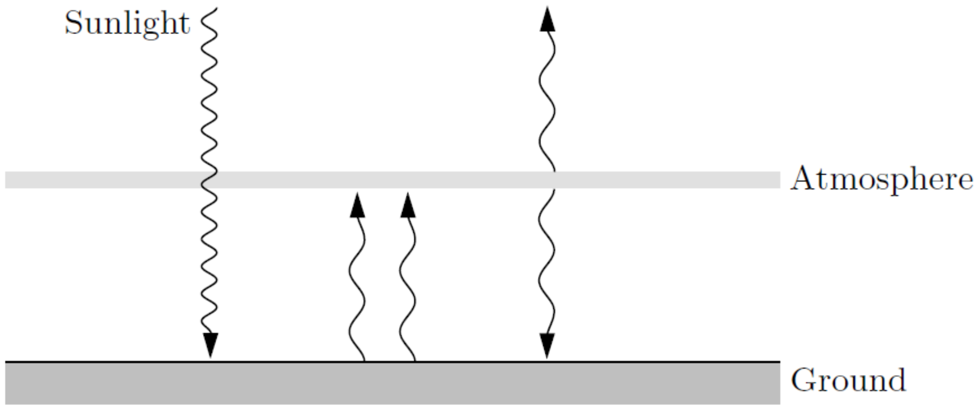
Giorgio Parisi

Prize share: 1/2

The Prize is awarded "for groundbreaking contributions to our understanding of complex systems" with one half jointly to Syukuro Manabe and Klaus Hasselmann "for the **physical modelling of Earth's climate, quantifying variability and reliably predicting global warming**" and the other half to Giorgio Parisi "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales."



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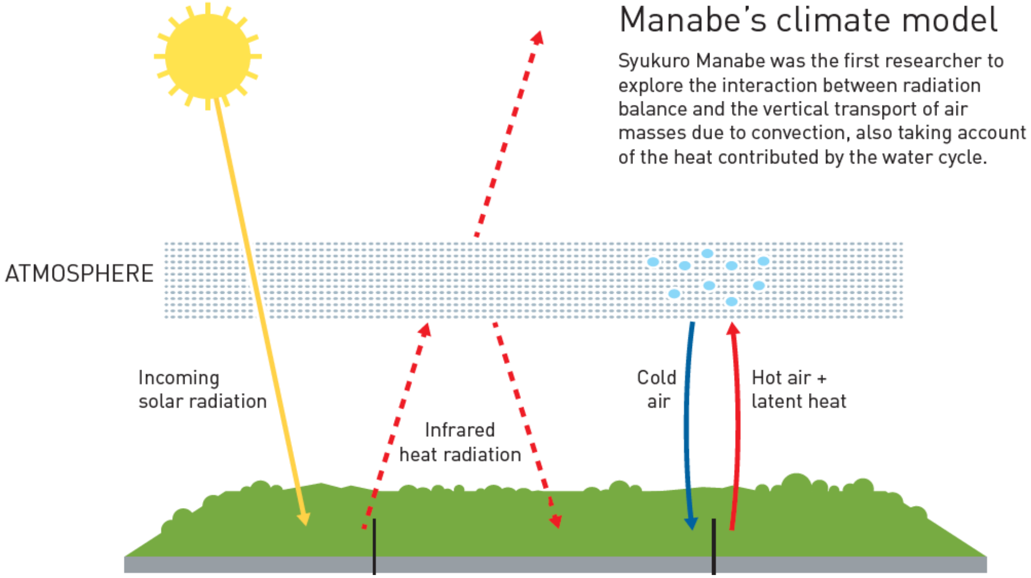
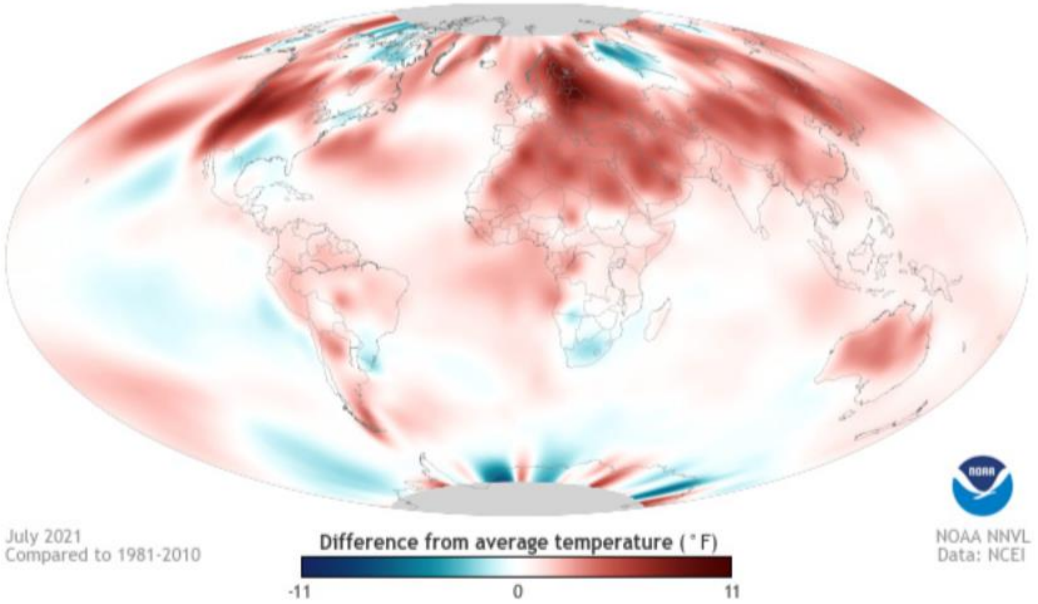
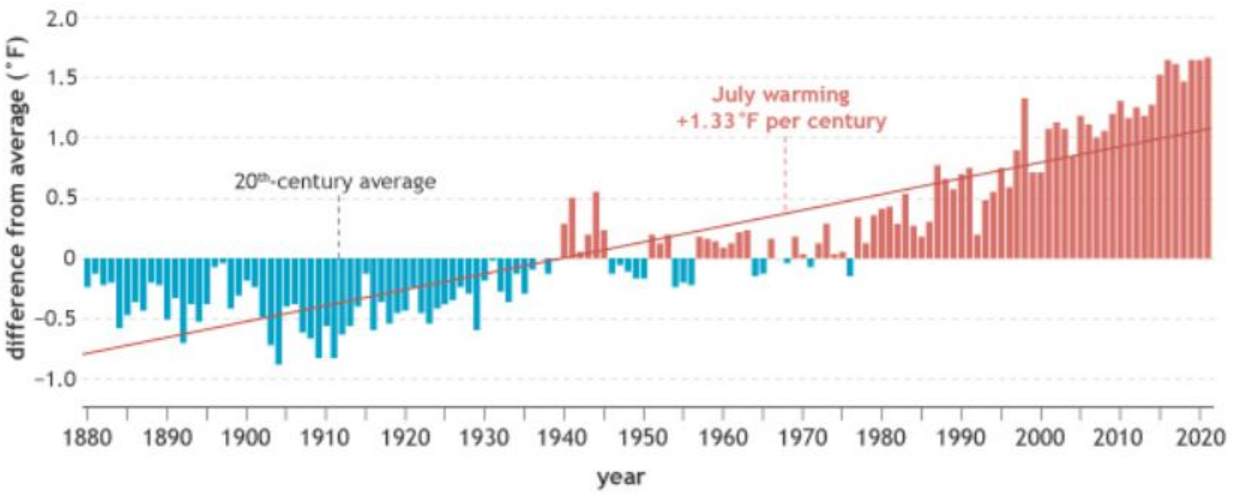


Greenhouse effect 2021

The July 2021 global surface temperature was $0.93\,°C$ above the 20th-century average of $15.8\,°C$. It was the highest for July in the 142-year record. This value was $0.01\,°C$ higher than the previous record set in 2016, and repeated in 2019 and 2020.



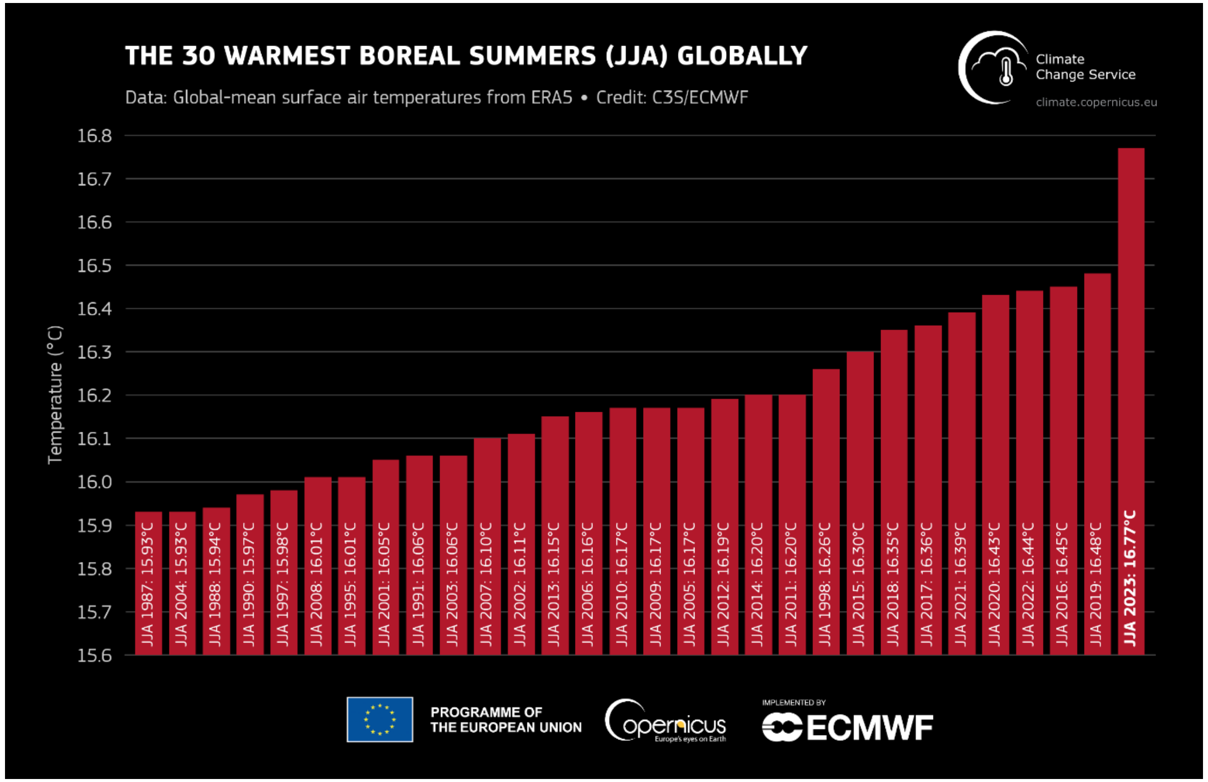


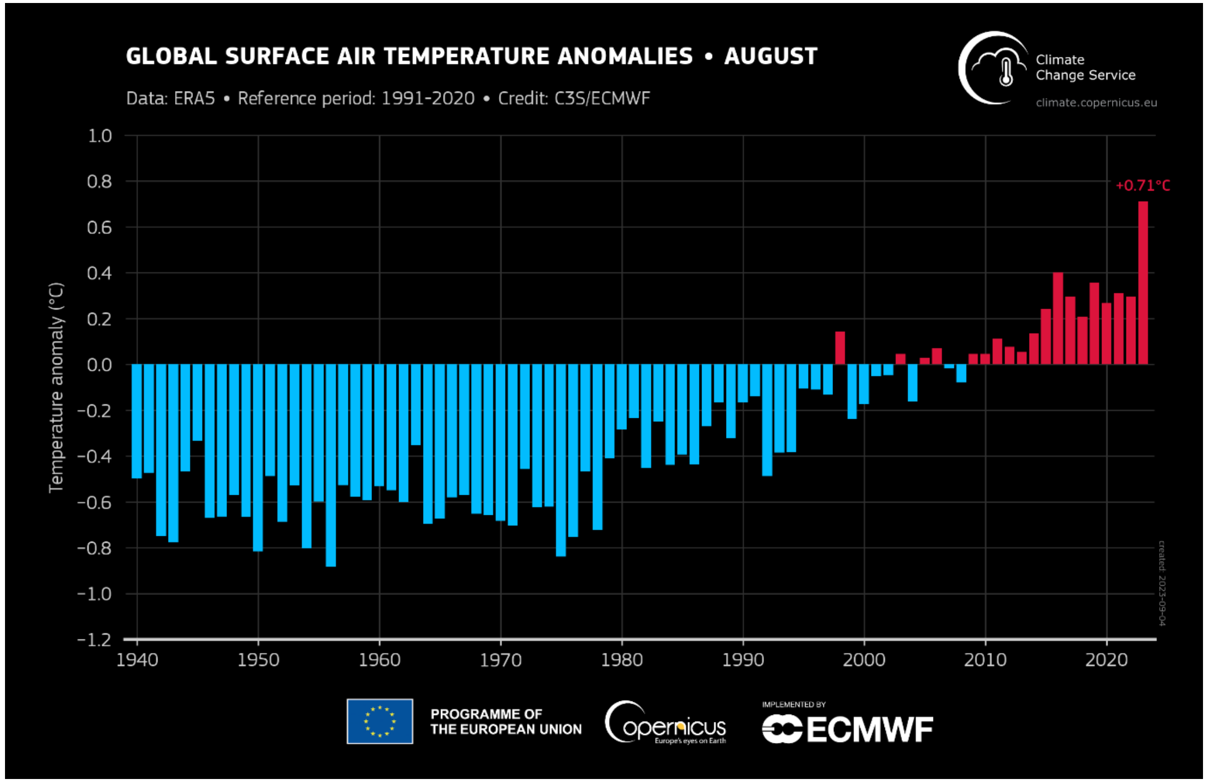
Global July temperatures compared to average (1880-2021)

https://www.climate.gov/news-features/understanding-climate/earths-hottest-month-was-record-hot-2021

There are several consequences and scenarios (mostly negative) for global warming.

Global warming





[Summer 2023: the hottest on record | Copernicus]

Summary

- Blackbody radiation is electromagnetic radiation from non-reflective body with specific spectrum of wavelengths defined by its temperature.
- Boltzmann distribution for quantized electromagnetic waves (Planck distribution) is a solution for paradox of ultraviolet catastrophe.
- Planck distribution is Bose - Einstein distribution with chemical potential equal to zero. Chemical potential of photons is zero.
- The energy density of photon gas is proportional to fourth power of the temperature.
- The radiation that fills the entire observable universe has an almost perfect thermal spectrum at a temperature of 2.73 K.
- Sun heats Earth to average temperature of 288 K (15 ℃).
- Greenhouse effect that accelerates warming of the planet is consequence of the human activity.