UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Eksamen i:	FYS2160
Eksamensdag:	1. desember 2017
Tid for eksamen:	1430–1830
Oppgavesettet er på:	3 sider
Vedlegg:	ingen
Tilatte hjelpemidler	Elektronisk kalkulator, godkjent for videregående skole Rottman: Matematisk formelsamling Øgrim og Lian: Fysiske størrelser og enheter To A4 ark med notater (det kan være skrift på begge sider av arket)

English version

You have to justify your answers

Oppgave 1

(a) The Gibbs free energy of two phases A and B of a pure substance are equal at the coexistence line in a phase diagram: $G_A = G_B$. Explain why the coexistence line is inclined.

$$\frac{dP}{dT} = \frac{S_A - S_B}{V_A - V_B} \tag{1}$$

where S_X is the entropy and V_x is the volume of the phase X.

(b) For "ordinary substances there are coexistence lines between gas-liquid, gas-solid, liquid-gas and between different crystalline solid phases. Why does only the coexistence line between gas and liquid end in a critical point? (All the other coexistence lines end in another coexistence line or in (TPV) equals zero or infinity:)

(c) Explain the principle of corresponding states.

(d) Do systems obeying the 1) ideal gas 2) van der Waals equation of state have a critical point?

For ideal mixtures the density, $N_i/V_i = N/V$ is independent of the mixture composition, $x_i = N_i/N$, where N_i is the number of particles and V_i is the volume of component i and $N = \sum_i N_i$ is the total number of particles and $V = \sum_i V_i$ is the volume of the mixture. The entropy change of component i in an ideal mixture only depends on the volume available for i before and after mixing:

$$\Delta S_i = kN_i(\ln V - \ln V_i) \tag{2}$$

(e) Show that the chemical potential for component i in an ideal mixture is

$$\mu_i(x_i, T) = \mu_i^*(T) + kT \ln x_i \tag{3}$$

Oppgave 2

A rubber band of length $L(T) = l_0(T) + l(T)$ is stretched with a force f at each end. When $f = 0$ the length is $L = l_0$ and when $f > 0$ the dilatation is $l > 0$.

(a) Write an expression for the thermodynamic identity of the rubber band.

(b) Show that the thermodynamic identity for the Helmholtz free energy, F, can be written

$$dF = fdl - SdT \tag{4}$$

(c) Show that the force f for a given dilatation l can be written as a derivative of Helmholtz free energy:

$$f = \left(\frac{\partial F}{\partial l}\right)_T \tag{5}$$

and by the entropy:

$$f = -T\left(\frac{\partial S}{\partial l}\right)_E . \tag{6}$$

We will now use a microscopic model of the rubber band made of a polymer. Assume the polymer being stretched consistes of N monomers of length δ The monomers are attached to each other andcan extend either to the right or to the left. That is, each monomer has two possible states

$$s_i = \begin{cases} +1 & \text{høyre} \\ \text{-1} & \text{venstre} \end{cases} \tag{7}$$

that are equally probable. When the rubber band is stretched some of the turns (change of direction between neighbours) are straightened out. The dilatation of the rubber band is then $l = \delta \sum_i s_i$.

(d) If N_h is the number of monomers directed to the right show that the multiplicity of the rubber band is

$$\Omega(N_h, N) = \binom{N}{N_h} \tag{8}$$

and use Stirling's approxmation $\ln(N!) \approx N \ln N - N$ to find the entropy.

(9)

(e) show that the force is given by

$$f = \frac{kT}{2\delta} \ln \frac{N\delta + l}{N\delta - l} \tag{10}$$

(f) Show that when the polymers are curled up ($l << N\delta$) the rubber band follows Hooke's law, that is, the dilatation is proportional to the force. Find the spring constant.

(g) If we suspend a weight in the rubber band and heat the rubber band, will the weight not move, move up or move down? Justify your answer.

We wish to understand the relation between the entropy and the dilatation of the rubber band to analyse the heat flow in/out of the rubber band. We still assume that ($l << N\delta$).

(h) Use a Maxwell relation to show that

$$\left(\frac{\partial S}{\partial f}\right)_T = -\frac{l}{T} \tag{11}$$

(i) When the rubber band is stretched slowly, will there be heat flowing into the band, out of the band or will there be no heat flow? Justify your answer.

(j) Draw a cyclic process in a $f - L$-diagram where the force $f = 0$ in state 1) and 3) and $f > 0$ in state 2). Along the path 1)→2) the rubber band is stretched slowly. Along the path 2)→3) the force is quickly reduced. Along the path 3)→1) the force on the rubber band is not changed. Indicate the heat flow, the type of process of each path and justify the shape of the path (straight, curved...)

The analysis of the rubber band model shows that the force is related to the entropy, that is, how ordered the polymer is. In order to test if the internal energy matters we wil make a model where each monomer has the energy $\delta\epsilon$ independent of s_i. This signifies that the (total internal) energy of the polymer in state i is $\epsilon_i = N\delta\epsilon$. We may now write the partition function as

$$Z = \sum_i e^{-\epsilon_i/kT} = \Omega(N_h, N)e^{-N\delta\epsilon/kT}. \tag{12}$$

(k) Use the partition function to find an expression for the force and compare to the expression we found for the microcanonical ensemble. Comment the result.

Oppgave 3

In this problem we will study a one dimensional electron gas. The energy states of an electron (a Fermion) in a one dimensional box of length L is given by

$$\epsilon_n = an^2 \ , \ n = 1, 2, \dots \ , \tag{13}$$

where

$$a = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 \ . \tag{14}$$

(a) Find the density of states $g(\epsilon)$ of the system.

(b) Find the Fermi-energy of the system.

We will now study the system at low temperatures. At low temperatures we may assume that $\mu \simeq \epsilon_F$ and that $kT \ll \epsilon_F$. We may approximate the heat capacity of the gas by

$$C_V \simeq g(\epsilon_F)\int_0^\infty (\epsilon - \epsilon_F)\frac{\partial f}{\partial T}d\epsilon \ , \tag{15}$$

where f is the Fermi-Dirac distribution function.

(c) Justify the expression for C_V. Use the expression to show that the heat capacity at low temperatures has the form $C = \alpha T^m$, and determine the constants α and m.

If we stretch the one dimensional electron gas a distance ΔL with a force K we perform the work $W = K\Delta L$ on the gas.

(d) Find the thermodynamic identity of the system

(e) Find an expression for the entropy S of the system.

(f) Find an expression for the energy U of the system.

(g) Find the equation of state of the system.

(h) *(This problem counts twice)* Sketch a Carnot cycle with this system as the medium in a $K - L$ diagram. Explain all the sub processesand find the efficiency of the machine.