UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: **Fys2160**

Exam date: **Wednesday January 17, 2024**

Exam time: **15.00 - 19.00**

Number of pages: **4**

Allowed material:

- **Electronic calculator**
- **K. Rottman: Matematisk formelsamling**
- **Øgrim and Lian or Angell and Lian: Fysiske størrelser og enheter**
- **One A4 sheet of notes (both sides can be used)**

Make sure that the exam set is complete before you begin to solve the problems. You must justify your answers in the solutions.

	water	ice
Density [kg/m^3]	1000	916.2
Speed of sound [m/s]	1500	3800
Latent heat of melting [kJ/kg]		334
Latent heat of vaporization [kJ/kg]	2260	
Heat capacity [kJ/kg K]	4.186	2.108
Thermal conductivity [W / m K]	0.54	2.22
	eV/K	J/K
Boltzmann constant	$8.62 \cdot 10^{-5}$	$1.38 \cdot 10^{-23}$

Table 1: Numerical values you may need.

Problem 1: Short Questions

(a) (7 points) Explain thoroughly in words and mathematical relations, starting from fundamental principles of thermodynamics, how the melting temperature of ice changes with pressure. Explain also the signs (positive or negative) of the properties in your explanation.

(b) (7 points) What is the change of entropy in 1 kg of H_2O at atmospheric pressure when the temperature is reduced from 10C to -10C?

(c) (3 points) The emissivity, ϵ of glass is about 0.9. You have a sheet with one black side, $\epsilon \approx 1$, and one shiny side $\epsilon \approx 0.1$ that is made to cover the windshield of your car. On warm, sunny days you can use the sheet to avoid heating your car when parked. On cold, clear winter nights you can use the sheet to slow down cooling of your car and windshield. Explain which side of the blanket should face outwards in each case and why.

(d) (4 points) An diatomic ideal gas in a 3D box undergoes a quasitatic isothermal compression in which its volume is reduced by 25% . Compute the work done on the system.

(e) (4 points) Using the same gas compression as above, compute the internal energy in the initial and final state. Discuss each step in your calculation. What is the change in the internal energy during the isothermal compression?

(f) (3 points) How much heat is exchanged during the isothermal compression discussed in the previous 2 questions? Discuss the heat exchange and entropy change during this process.

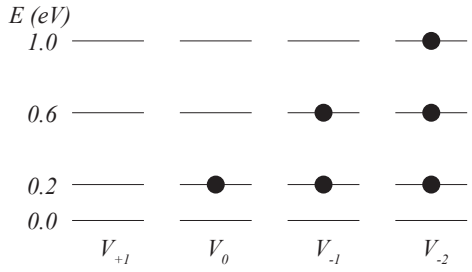


Figure 1: Illustration of how the occupation of various energy levels gives various charges for a vacancy.

Problem 2: Charged vacancies

Consider a crystalline lattice where the empty lattice sites are called vacancies. These vacancies can be occupied by electrons independently of each other. We assume that the vacancy can have four possible charge configurations with net charge $+1, 0, -1, -2$ when different levels are occupied by electrons. The energy levels are sketched in Figure 1.Let us introduce $\epsilon = 0.2$ eV, such that the energy levels are then 0, ϵ, 3ϵ , 5ϵ. Thus, each vacancy has four charge-states labelled as V_1, V_0, V_{-1}, V_{-2}.

Assume that the energy for the unoccupied vacancy is zero. Ignore the spin of the electron.

(a) (4 points) Compute the Gibbs sum $Z_G(\epsilon, T, \mu)$ of one vacancy as the Gibbs sum depending on energy level ϵ, temperature T and chemical potential μ. Show that it reduces to

$$Z_G = 1 + e^{\frac{\mu-\epsilon}{kT}} + e^{\frac{2(\mu-2\epsilon)}{kT}} + e^{\frac{3(\mu-3\epsilon)}{kT}} \ .$$

(b) (6 points) Compute the average occupation $\langle N \rangle$ of the charge state of the vacancy. Show that when the chemical potential for electrons, $\mu = 0.2$ eV and at room temperature, the average occupation reduces to

$$\langle N \rangle \simeq \frac{1}{2} \ . \tag{1}$$

(c) (3 points) What is the average charge state of the vacancy when the chemical potential is instead 0.6 eV?

Problem 3: Classical ideal gas particle in 2D

Consider a classical particle of mass m moving freely with a velocity $\vec{v}$ in a finite domain in the $x-y$ plane of area $A = L^2$. The particle is at thermal equilibrium with a thermal bath at temperature T.

(a) (6 points) Compute its partition function over all possible microstates determined by varying the velocity $\vec{v}$ and position $\vec{r}$ using this formula

$$Z_1 = \frac{m^2}{h^2} \int_A d\vec{r} \int d\vec{v} e^{-E/(kT)}$$

where h is the Planck's constant and E is the energy of the particle in a given microstate at temperature T.

(*Hint: Use the Gaussian integral* $\int_{-\infty}^{+\infty} dx\, e^{-ax^2} = \sqrt{\pi/a}, \qquad a > 0$)

(b) (3 points) Show that the partition function Z_1 computed above is determined by the thermal wavelength $\Lambda(T) = \sqrt{h^2/(2\pi mkT)}$, and reduces to the simple expression

$$Z_1(T) = \left(\frac{L}{\Lambda(T)} \right)^2$$

Considering that the partition function is the weighted sum of states, what does this expression imply?

(c) (2 points) Compute the Helmholtz free energy of the particle at temperature T.

(d) (2 points) Compute the entropy of the particle.

(e) (5 points) Consider now an ideal gas of N particles treated above. Compute the total Helmholtz free energy $F_N(T)$ and the pressure $P(T)$ of the ideal gas and comment the result.