UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in:	**Fys2160,**
Exam date:	**Friday 13 December 2019**
Exam time:	**09.00 - 13.00**
Number of pages:	**2**
Allowed material:	**Electronic calculator** **K. Rottman: Matematisk formelsamling** **Øgrim and Lian or Angell and Lian: Fysiske størrelser og enheter** **One A4 sheet of notes (both sides can be used)**

Make sure that the exam set is complete before you begin to solve the problems. You must justify your answers in the solutions.

Problem 1:

1.1 Suppose you roll a 6-sided die 5 times. What is the probability that you get the same side of the die exactly 3 times?

1.2 Define temperature and thermodynamic equilibrium between two systems in thermal contact.

1.3 Why does a pressure cooker cook food faster than a normal casserole?

1.4 You have a thermos with a gas tight piston enclosing a monatomic ideal gas inside the thermos. If you use the piston to compress the gas volume by 20% will the temperature of the gas increase, decrease or stay the same?

1.5 Inside a similar thermos you have equal masses of ice, liquid water and air at atmospheric pressure. The specific heat capacity of ice is twice that of water vapour and the specific heat capacity of water is twice that of ice. The air is saturated with water vapour and the partial pressure of water is $p_w = 0.61$ kPa. Now you pull the piston slowly to increase the gas volume by 20%. What will the pressure and temperature of the gas be?

Problem 2:

2.1 Consider an ideal gas of non-relativistic particles in two-dimensions (2D). What is the probability of a particle having velocity vector $\vec{v}$? (You do not need to write the normalization constant explicitly.)

2.2 Derive the Maxwell-Boltzmann expression for the speed distribution and show that it equals

$$D(v) = \left(\frac{m}{2\pi kT}\right) 2\pi v e^{-mv^2/(2kT)}, \tag{1}$$

2.3 Determine the most likely speed and show that it corresponds to the equipartition of energy.

Problem 3:

Consider a collection of N identical quantum harmonic oscillators at a given temperature T. Each quantum harmonic oscillator has an energy spectrum of allowed energies $\epsilon_n = nh\nu$, where $n = 0, 1, 2, \cdots$, h is the Planck's constant and ν is the fixed frequency of the oscillator.

3.1 Show that the partition function of a single quantum harmonic oscillator $Z_1(T, \nu)$ is given by

$$Z_1 = \frac{1}{1 - e^{-\beta h\nu}} \tag{2}$$

(Hint: Make use of the formula $\sum_{n=0}^{\infty} x^n = 1/(1-x), |x| < 1$).

3.2 Show that the average energy of a single quantum harmonic oscillator at a given temperature is

$$\langle \epsilon \rangle (T, \nu) = \frac{h\nu}{e^{\beta h\nu} - 1}. \tag{3}$$

Express this in terms of the Bose-Einstein distribution. Provide an example of a Bose system with this average energy and discuss it.

3.3 Calculate the total average energy and heat capacity of N independent harmonic oscillators as functions of frequency and temperature. Show that, in the high temperature limit, the energy and heat capacity follow the equipartition of energy theorem.

3.4 Calculate Helmholtz free energy of a single harmonic oscillator and then of N independent harmonic oscillators.

3.5 Calculate the entropy of N independent harmonic oscillators.

Problem 4:

4.1 The heat capacity is defined as the absorbed heat per unit increase in temperature $C \equiv \frac{Q}{\Delta T}$. Use the first law of thermodynamics to derive the expressions for the heat capacity at constant volume

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \tag{4}$$

and the heat capacity at constant pressure

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P \tag{5}$$

in terms of changes in the internal energy, U.

4.2 Use the thermodynamic identities of the internal energy $U(S, V)$ and the enthalpy $H(S, P)$ to show that

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \tag{6}$$

and

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P \tag{7}$$

4.3 From the thermodynamic identity of the Helmholtz energy $F(T, V)$ derive the Maxwell relation

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \tag{8}$$

4.4 Using the equipartition of energy, determine the internal energy of a monoatomic ideal gas in 3D, i.e. $U(T, N)$. Determine the heat capacities C_V and C_P for the ideal gas using 4.1, and show that $C_P - C_V = Nk$. Explain why the heat capacity is larger at constant pressure than at constant volume.