Approximation for $N!$ = $N \cdot (N-1)(N-2) \ldots$

$$\ln N! = \ln\left(N \cdot (N-1) \cdot (N-2) \ldots\right)$$

$$= \ln N + \ln N-1 + \ln N-2 \ldots = \sum_{n=1}^{N} \ln n$$

When $N \to \infty$ $\quad \Sigma \to \int$

$$\ln N! \simeq \int_0^N \ln n \, dn = \left[n \ln n - n\right]_0^N$$

$$\simeq N \ln N - N$$

$$N! \simeq N^N e^{-N}$$

Stirlings approximation

$$\Rightarrow \ln N! \simeq \ln\sqrt{2\pi N} + N(\ln N - 1)$$

$$N! \simeq \sqrt{2\pi N} N^N e^{-N}$$

N part $\quad$ 

$$\Omega(N,k) = \binom{N}{k}$$

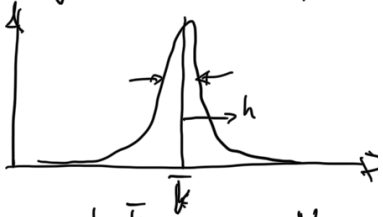
$$P(N,k) = \frac{\Omega(N,k)}{\Omega_{tot}} = 2^N \binom{N}{k} = 2^{-N} \frac{N!}{k!\,(N-k)!}$$

The probability of macrostate k in a two-state system with N part. (macro state)

$$\ln P(N,k) = -N\ln 2 + \ln N! - \ln k! - \ln(N-k)!$$

$$= -N\ln 2 + N\ln N - N + \ln\sqrt{2\pi N} - k\ln k + k - \ln\sqrt{2\pi k} - (N-k)\ln(N-k) + N - k - \ln\sqrt{2\pi(N-k)}$$

A $\quad$ B $\quad$ C



$$h = \frac{k - \bar{k}}{\bar{k}} \qquad \bar{\sigma} = \frac{N}{4}$$

$$\bar{k} = \frac{N}{2}$$

B: $N\ln N - \bar{k}(1+h)\ln\bar{k}(1+h) - \bar{k}(1-h)\ln\bar{k}(1-h)$

$$= N\ln N - \bar{k}(1+h)\left[\ln\bar{k} + \ln(1+h)\right] - \bar{k}(1-h)\left[\ln\bar{k} + \ln(1-h)\right]$$

$$\Rightarrow k = \bar{k}(1+h)$$

$$N-k = \bar{k}(1-h)$$

Taylor: $\ln(1+h) \simeq h - \frac{h^2}{2} + \ldots \quad \ln(1-h) \simeq -h - \frac{h^2}{2} + \ldots$

B: $N\ln N - \bar{k}\left[(1+h)\left(\ln\bar{k} + h - \frac{h^2}{2}\right) + (1-h)\left(\ln\bar{k} - h - \frac{h^2}{2}\right)\right]$

$$= N\ln N - \bar{k}\left[2\ln\bar{k} - h^2 + 2h^2\right] = N\ln N - \bar{k}\left[2\ln\bar{k} + h^2\right] \qquad \bar{k} = \frac{N}{2}$$

$$= N\ln N - \frac{N}{2}\left[2\ln\frac{N}{2} + \left(\frac{k-\bar{k}}{\bar{k}}\right)^2\right] \qquad h = \frac{k-\bar{k}}{\bar{k}}$$

$$\sigma^2 = \frac{N}{4}$$

$$= N\ln N - N\ln N + N\ln 2 - \bar{k}\left(\frac{k-\bar{k}}{\bar{k}}\right)^2$$

$$= +N\ln 2 - \frac{(k-\bar{k})^2}{\bar{k}} = N\ln 2 - \frac{(k-\bar{k})^2}{2\sigma^2}$$

C: $\ln\sqrt{2\pi N} - \ln\sqrt{2\pi k} - \ln\sqrt{2\pi(N-k)}$

$$= \frac{1}{2}\ln\left[\frac{2\pi N}{2\pi k \cdot 2\pi(N-k)}\right] = \frac{1}{2}\ln\left[\frac{N}{\bar{k}^2(1-h^2)2\pi}\right] \qquad h^2 \ll 1$$

$$= \frac{1}{2}\ln\left[\frac{N}{2\pi\left(\frac{N}{2}\right)^2}\right] = -\ln\sqrt{2\pi\sigma^2}$$

$$\ln P(N,k) = -N\ln 2 + N\ln 2 - \frac{(k-\bar{k})^2}{2\sigma^2} - \ln\sqrt{2\pi\sigma^2}$$

$$\underline{P(N,k) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\left(\frac{k-\bar{k}}{4\sigma}\right)^2}}$$