

i UNIVERSITETET I OSLO**Det matematisk-naturvitenskapelige fakultet**

Exam in: FYS3110, Quantum mechanics

Day of exam: Dec. 13., 2022

Exam hours: 15:00-19:00 (4 hours)

Permitted material: Approved calculator, K. Rottmann: "Matematisk formelsamling", One handwritten A4 page with your own notes (2 pages), Textbook: A. Berera and L. Del Debbio, "Quantum Mechanics", and the two printed lecture notes: "Atoms and solids" and "Zeeman effect and hyperfine splitting"

All problems count towards the final score. The maximum score for each subproblem is 6 points, except for the multiple-choice subproblems, where a correct answer gives 2 points, and wrong or no answer gives 0 points. The maximum score for the full problem set is 50 points.

In this exam, you are permitted to make handwritten calculations/drawings/sketches for tasks 3a), 3b), 4a) and 4b). You are to use the sketching paper handed to you in the exam room. You can use more than one sketching sheet per task. See instructions for filling out sketching sheets in the link below the task bar.

You may NOT hand in sketching sheets for any other tasks than 3a), 3b), 4a), and 4b). You will NOT be given extra time to fill out the "general information" on the sketching sheets (task codes, candidate number etc.).

1(a)

What is a quantum state? In your answer, explain how a quantum state is different from a classical state and how it is represented mathematically.

Fill in your answer here

Format

B

I

U

x_2

x^2

I_x











$\frac{1}{2} =$

$\therefore =$

Ω





Σ



Words: 0

Maximum marks: 6

1(b) Explain briefly in words what good states are, and why they are useful. Give an example.

Fill in your answer here

Format

B

I

U

x_2

x^2

I_x















Ω





Σ



Words: 0

Maximum marks: 6

2(a) Consider the Coulomb problem (hydrogen) bound states $|nlm\rangle$. Compute

$$\sum_{l',m'} \langle nl'm' | (L_x^2 + L_y^2 + 2L_z^2) | nlm \rangle$$

where L_x, L_y, L_z are the cartesian components of the orbital angular momentum operator.
The answer is

Select one alternative:

- ☐ $\hbar^2(l+1)\sqrt{(l+2)l}$
- ☐ $\hbar^2\sqrt{m(m+1)l(l+1)}$
- ☐ $\hbar^2\left(2m^2 + \sqrt{l(l+1) - m(m+1)}\sqrt{l(l+1) + m(m+1)}\right)$
- ☐ $\hbar^2(l(l+1) + m^2)$
- ☐ 0
- ☐ $\hbar^2(l^2 + 2m^2)$
- ☐ None of the above

Maximum marks: 2

2(b) An operator K is written as

$$K = e^{iaG}$$

where G is a linear hermitian operator and a is a real number. For general (real) values of a , the operator K is

Select one alternative:

- ☐ the identity
- ☐ linear and hermitian
- ☐ nonlinear and hermitian
- ☐ linear and unitary
- ☐ nonlinear and unitary
- ☐ linear, hermitian and unitary
- ☐ nonlinear, hermitian and unitary
- ☐ none of the above

Maximum marks: 2

2(c) What is the energy splitting induced when a proton with spin-1/2 at rest is placed in a magnetic field of 1T?

(Some properties of protons and electrons and constants: $g_p = 5.59$, $g_e = 2.0023$, $m_p = 1836m_e$, $\mu_B = \frac{e\hbar}{2m_e} = 5.788 \cdot 10^{-5} \text{eV/T}$.)

Select the nearest alternative:

- ☐ 10^{-8}eV
- ☐ 10^{-7}eV
- ☐ 10^{-6}eV
- ☐ 10^{-5}eV
- ☐ 10^{-4}eV
- ☐ 10^{-3}eV
- ☐ 10^{-2}eV
- ☐ 10^{-1}eV
- ☐ 1eV
- ☐ 10eV

Maximum marks: 2

- 2(d)** A d-electron ($l = 2$) is in a state where its orbital angular momentum z-component is known to be $-\hbar$ and its spin z-component is known to be $+\hbar/2$. What is the probability that its total angular momentum is measured to be $5/2$?

Select one alternative:

- ☐ 0
- ☐ 1/6
- ☐ 1/5
- ☐ 1/4
- ☐ 2/7
- ☐ 1/3
- ☐ 2/5
- ☐ 3/5
- ☐ 2/3
- ☐ 4/5
- ☐ 1
- ☐ None of the above

Maximum marks: 2

- 2(e)** Two indistinguishable spin-0 bosons have each an orbital angular momentum of $l = 1$. What is the set of possible values for the total angular momentum of these bosons?

Select one alternative:

- ☐ {0}
- ☐ {1}
- ☐ {2}
- ☐ {0, 1}
- ☐ {1, 2}
- ☐ {0, 2}
- ☐ {0, 1, 2}
- ☐ None of the above

Maximum marks: 2

2(f) What is the value of the integral

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta Y_1^{*0}(\theta, \phi) L_- Y_1^1(\theta, \phi).$$

Y_l^m are the usual spherical harmonics, and $L_- = L_x - iL_y$ where L_x and L_y are the x- and y-components of the orbital angular momentum operator respectively.

Select one alternative:

- ☐ $-2\pi\hbar$
- ☐ $-\sqrt{6}\hbar$
- ☐ $-\pi\hbar$
- ☐ $-\sqrt{2}\hbar$
- ☐ $-\hbar$
- ☐ 0
- ☐ $\hbar$
- ☐ $\sqrt{2}\hbar$
- ☐ $\pi\hbar$
- ☐ $\sqrt{6}\hbar$
- ☐ $2\pi\hbar$
- ☐ None of the above

Maximum marks: 2

2(g) A charged particle is moving in a 3D harmonic oscillator potential. When the energy of the particle is $\hbar\omega(2 + 3/2)$ there are 6 degenerate states.

What is the maximum ratio of lifetimes for these 6 states computed in the electric dipole approximation? Use results from first-order time-dependent perturbation theory in the electric dipole approximation.

(Hint: Maximum ratio of lifetimes= longest lifetime/shortest lifetime. The lifetime of a state $|i\rangle$ is the inverse sum of spontaneous transition rates out of state $|i\rangle$)

Select one alternative:

- ☐ 1
- ☐ $\sqrt{3/2}$
- ☐ $3/2$
- ☐ $\sqrt{2}$
- ☐ $\sqrt{3}$
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 6
- ☐ ∞
- ☐ none of the above

Maximum marks: 2

- 3(a)** A one dimensional harmonic oscillator with Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ is at time $t = 0$ in an equal-amplitude superposition of its two lowest energy eigenstates

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Then at time $t = 0$ the oscillator stiffness k is suddenly increased an amount α before it relaxes back to its original value. That is, the stiffness can be written as

$$k(t) = k + \alpha e^{-\gamma t}$$

for $t > 0$. α and γ are real positive numbers with units of force per length and inverse time respectively.

Use first order time-dependent perturbation theory to calculate the probability of finding the oscillator in the energy eigenstate $|2\rangle$ at time $t \rightarrow \infty$.

In this task you can hand in sketches. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Maximum marks: 6

- 3(b)** Use the variational principle with the normalized trial function

$$\psi(x) = \left(\frac{2\lambda}{\pi}\right)^{1/4} e^{-\lambda x^2}$$

to show that a particle moving in a one dimensional potential $V(x)$, which is *negative* for all x and where $V(|x| \rightarrow \infty) \rightarrow 0$, will have at least one bound state.

In this task you can hand in sketches. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Maximum marks: 6

- 4(a) Use time-independent perturbation theory to find an estimate correct to order a_0/ξ for the ground state energy of the hamiltonian

$$H = \frac{\vec{p}^2}{2m} - \frac{e^2}{4\pi\epsilon_0} \frac{e^{-r/\xi}}{r}$$

where ξ is a large real positive number with units of length. a_0 is the Bohr radius.

In this task you can hand in sketches. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Maximum marks: 6

- 4(b) A hamiltonian operator is written $H = A + bB$ where b is a real-valued parameter, and A and B are hermitian operators that are not dependent on the parameter b . This hamiltonian H has a non-degenerate ground state with an energy $E_{gs}(b)$ which is a function of b .

Show that $\left. \frac{\partial^2 E_{gs}}{\partial b^2} \right|_{b=b_0} \leq 0$ for all real values b_0 .

In this task you can hand in sketches. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Maximum marks: 6