FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2008

Problem Set 15

Problem 15.1

The figure shows a straight antenna of length $2a$ lying along the z-axis with its center at the origin. We assume that the charge of the antenna is at all times located at the endpoints. The current in the antenna (between the charged end points) is given by $I = I_0 \sin \omega t$ where ω and I_0 are constants. The antenna is electrical neutral at time $t = 0$. The field point is given by the position vector $\mathbf{r}$ and in spherical coordinates (r, θ, ϕ).

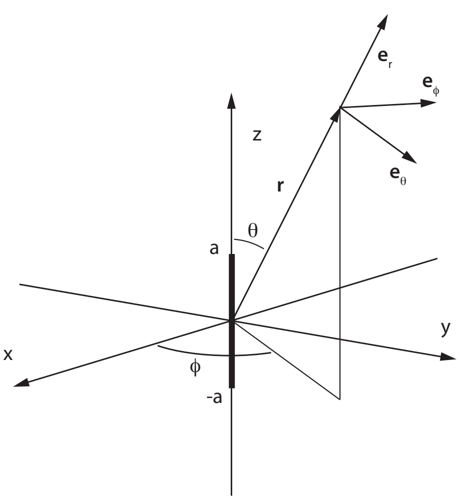

Figure 1:

a) Show that the antenna's electrical dipole moment at time t is given by $\mathbf{p}(t) = \frac{2aI_0}{\omega}(1 - \cos \omega t)\mathbf{k}$, where $\mathbf{k}$ is the unit vector in the z-direction.

We will now assume that the fields can be treated as electrical dipole radiation.

b) Find the components of the $\mathbf{B}$ and $\mathbf{E}$ fields in the directions $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ in the field point (r, θ, ϕ) at time t.

c) Show that the time average of the total radiated power in all directions kan be written as $\langle P \rangle = \frac{RI_0^2}{2}$ and find R (radiation resistance). What is the time average of the total power consumed by the antenna if it has an 'ordinary' resistance R_0 as well?

d) Find R for an antenna of length $2a = 5$ cm which is conducting a current with frequency $f = 150$ MHz. What is the time average of the total radiated power when $I_0 = 30$ A?

Problem 15.2
Exam 2007, Problem 3

In a circular loop of radius a an oscillating current of the form $I = I_0 \cos\omega t$ is running. The current loop lies in the x, y plane. We use the notation $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ for the Cartesian unit vectors in the directions x, y and z, in order to reserve the symbol $\mathbf{j}$ for the current density. The current loop is at all times charge neutral.

a) Explain why the electric dipole moment $\mathbf{p}$ of the current loop vanishes, and show that the magnetic dipole moment has the following time dependence, $\mathbf{m}(t) = m_0 \cos\omega t\, \mathbf{e}_z$, with m_0 as a constant. Find m_0 expressed in terms of a and I_0.

As a reminder, the general expressions for the radiation fields of a magnetic dipole are

$$\mathbf{E}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c r}\ddot{\mathbf{m}}_{ret} \times \mathbf{n}\,;, \quad \mathbf{B}(\mathbf{r},t) = -\frac{1}{c}\mathbf{E}(\mathbf{r},t) \times \mathbf{n} \tag{1}$$

with $\mathbf{m}_{ret} = \mathbf{m}(t - r/c)$ and $\mathbf{n} = \mathbf{r}/r$. In the following we assume that we study the fields far from the current loop (in the radiation zone) where the expressions (1) are valid.

b) Write down the expressions for the radiation fields for points on the x axis far from the current loop and show that that they have the form of electromagnetic waves that propagate in the direction away from the loop. What is the polarization of the waves?

c) Use the general expression for Poynting's vector $\mathbf{S}$ to find the radiated power per unit solid angle $\frac{dP}{d\Omega}$, in the x direction. What is the corresponding radiated power in the direction of the z axis?