FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2014

Problem Set 11

Problem 11.1

Two photons in the laboratory system have frequencies ν_1 and ν_2. The angle between the propagation directions is θ.

a) Write down the expressions for the total energy and momentum of the photons in the laboratory system.

b) Find the photons' frequency in the center of mass system.

c) Is it allways possible to find a center of mass system for the photons?

Problem 11.2

We send a photon towards an electron at rest.

a) What is the minimum energy of the photon required for the following process to take place

$$\gamma + e^- \to e^- + e^- + e^+ \tag{1}$$

The particles e^- and e^+ have the same mass m_e.

b) Show that the process

$$\gamma \to e^- + e^+ \tag{2}$$

is impossible.

Problem 11.3 (Exam 2013)

A straight rod is moving along the x-axis of an inertial reference frame S. The two endpoints A and B follow hyperbolic space-time trajectories, described the following time dependent x-coordinates in S,

$$x_A = c\sqrt{t^2 + c^2/a^2}\,, \quad x_B = c\sqrt{t^2 + c^2/b^2} \tag{3}$$

c is the speed of light, and a and b are positive constants, with $b < a$.

a) A second inertial frame S' moves along the x-axis with velocity v relative to S. The coordinates of the two reference frames are chosen to coincide at the space-time point $x = t = 0$.

Show that the motion of A and B, when expressed in terms of the coordinates of S', has precisely the same form as in S,

$$x'_A = c\sqrt{t'^2 + c^2/a^2}\,, \quad x'_B = c\sqrt{t'^2 + c^2/b^2} \tag{4}$$

(To demonstrate this it may be convenient to rewrite the above relations in terms of the squared coordinates x^2 and t^2.)

b) At time $t = 0$ the frame S is an instantaneous rest frame of both A and B. Show this and find the distance between A and B measured in S at this moment. The same results are valid for the reference frame S' at time $t' = 0$.

Based on this we may conclude that for any point on the space-time trajectory of A, the instantaneous inertial rest frame of A is a rest frame also for B. Furthermore the distance between A and B, when measured in the instantaneous inertial rest frame, is constant. Explain these conclusions.

c) Use the above results to show that the proper accelerations of the A and B are constants, and give the values of these.

d) At a given instant $t = 0$ a light signal with frequency ν_0 is sent from A and is subsequently received at B. What is the velocity of B (measured in S) when the signal is received, and what is the frequency of the signal, measured at B? (To answer the last question it may be convenient to use the relation between frequency and four-momentum for a photon sent from A to B.)