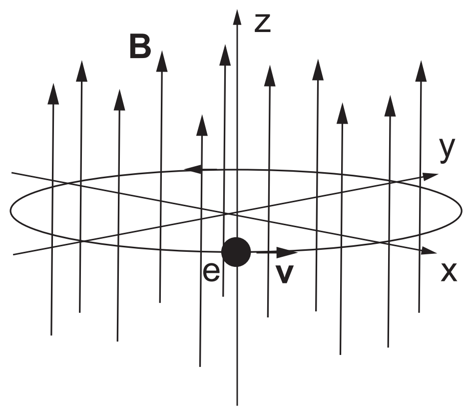
FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2014

Problem Set 12

Problem 12.1 (Exam 2004)



An electron moves in a circular orbit, under the influence of a constant magnetic field $\mathbf{B}$. We assume the $\mathbf{B}$ field to be oriented parallel to the z axis and the electron orbit to lie in the x, y-plane, with the origin in the centre of the circular orbit. The radius of the circle is $r = 3.0m$ and the velocity is $v = 0.9998c$, with $c = 3.0 \times 10^8 m/s$ as the speed of light. The electron mass is $m = 9.1 \times 10^{-31} kg$ and the charge is $e = -1.6 \times 10^{-19} C$.

a) Determine the relativistic γ-factor and the energy of the electron.

b) What is the acceleration of the electron as measured in the laboratory frame, and what is its proper acceleration (the acceleration in an instantaneous inertial rest frame)?

c) Find the strength of the magnetic field B in the lab frame. (Express it in the SI unit Tesla, with $1T = 1kgs^{-2}A^{-1} = 1kgs^{-1}C^{-1}$.)

d) Find the strength of the magnetic field B' and the electric field E' in the instantaneous inertial rest frame of the electron. What are the directions of these fields?

Problem 12.2

A thin straight conducting cable, oriented along the z axis in an inertial reference frame S, carries a constant current I. The cable is charge neutral.

a) Show, by use of Ampere's law, that the current produces a rotating magnetic field $\mathbf{B} = B(r)\mathbf{e}_\phi$, where (r, ϕ) are polar coordinates in the x, y plane and $\mathbf{e}_\phi$ is a unit vector in the direction of increasing ϕ. Determine the function $B(r)$.

Consider next the same situation in a reference frame S' that moves with velocity v along the z axis.

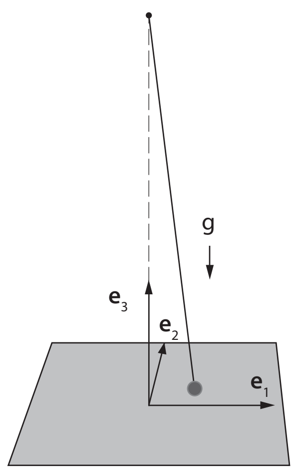
b) Use the fact that charge and current densities transform under Lorentz transformation as components of a current 4-vector to show that in S' the conducting cable will be charged. Determine the charge per unit length, λ' and the current I' in this reference frame.

c) Use Gauss' and Ampere's laws to determine the electric and magnetic fields, $\mathbf{E}'$ and $\mathbf{B}'$, as functions of the polar coordinates (r', ϕ') in reference frame S'.

d) Show that if the fields in S' are derived from the fields in S by use of the relativistic transformation formulas for $\mathbf{E}$ and $\mathbf{B}$, that gives the same results as found in c).

Problem 12.3 (Midterm Exam 2013)

In the lobby of the Physics building there is a Focault pendulum, which is the subject of our study. The idea is to use the Langrange method to study its motion, and in particular to derive the period of rotation of the pendulum due to the effect of earth's rotation. The length of the pendulum wire is $l = 14m$ and the mass of the brass sphere at the end of the wire is $m = 20kg$. Oslo is situated at the latitude $60°$ north.



We introduce three orthogonal unit vectors $\mathbf{e}_k, k = 1, 2, 3$, which are fixed relative to the building. These are defined with $\mathbf{e}_3$ pointing in the vertical direction, $\mathbf{e}_1$ pointing to the north, and $\mathbf{e}_2$ orthogonal to the two. The three unit vectors are used as the basis vectors of an earth-fixed reference frame S, with the origin of the reference frame taken as the equilibrium position of the pendulum sphere. In addition we introduce $\mathbf{k}$ as a unit vector in the plane spanned by $\mathbf{e}_1$ and $\mathbf{e}_3$, with direction parallel to the earth's rotational axis. The angle between $\mathbf{e}_3$ and $\mathbf{k}$ we refer to as θ.

The vertical direction is the direction opposite to the *effective* gravitational acceleration $\mathbf{g}$, which deviates from the true gravitational acceleration because of the centripetal acceleration of the Physics building, caused by earth's rotation. However, this effect is small and is not important for the present problem. We shall therefore simply consider the origin of the of the reference frame S to be non-accelerated, with $\mathbf{g}$ pointing towards the center of the earth. The fact that the reference frame S rotates with the angular velocity of the earth is however important for the effect to be studied. This rotation can be expressed in terms of the time derivatives of the basis vectors as

$$\dot{\mathbf{e}}_k = \boldsymbol{\omega} \times \mathbf{e}_k \,, \quad k = 1, 2, 3 \tag{1}$$

with $\boldsymbol{\omega} = \omega \mathbf{k}$ as the angular velocity of the earth.

The position vector of the pendulum sphere we express in the following as

$$\mathbf{r} = x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3 \tag{2}$$

where the coordinates satisfy the constraint $x^2 + y^2 + (l - z)^2 = l^2$, with l as the length of the pendulum wire.

a) Find the velocity vector $\dot{\mathbf{r}}$ expressed in terms of the basis vectors of S, and use this to derive the kinetic energy T, expressed in terms of the coordinates (x, y, z) and their time derivatives. Use the angle θ as a parameter in the expressions. Terms that are quadratic in ω are very small and may be dropped. Give a comment on why this is a good approximation. Also give the expression for the potential energy V.

b) Make use of x and y as the generalized coordinates of the pendulum, and assume small oscillations $x/l << 1$ and $y/l << 1$. Show that with this assumption the Lagrangian $L(x, y, \dot{x}, \dot{y}, t)$ is approximated by the following expression

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + m\omega\cos\theta(x\dot{y} - y\dot{x}) - \frac{1}{2l}mg(x^2 + y^2) \tag{3}$$

c) The form of the Lagrangian suggests that a change to polar coordinates may be convenient. Do that and derive the corresponding Lagrange's equations.

d) Show that the equations have a solution with constant angular velocity, and with oscillations in the radial variable. Find this solution and determine the angular velocity ω_ϕ as well as the circular frequency ω_p of the pendulum oscillations.

e) At 12 o'clock on a particular day, the pendulum is performing oscillations in the x, z-plane. At 12 o'clock the next day, the pendulum is swinging in a plane which is rotated by the angle $\Delta\phi$ relative to the original plane of oscillations. Determine the value of $\Delta\phi$.