UiO : Fysisk institutt
Det matematisk-naturvitenskapelige fakultet

Lecture 10

Recap

- **Hamilton's principle** or **the principle of least action** says that the action

$$S[q(t)] = \int_{t_1}^{t_2} L(q(t), \dot{q}(t), t)\,dt$$

as a function of the path q(t) is unchanged for small variations

$$q(t) \rightarrow q(t) + \delta q \quad \text{with} \quad \delta q(t_1) = \delta q(t_2) = 0$$

around the trajectory that fulfils the e.o.m., i.e.

$$\delta S = 0$$

Recap

- This can be shown to be equivalent to Lagrange's equations through

$$\delta S = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \right) \delta q \, dt = 0$$

- However, the derivation of this equation assumes nothing about mechanics and can be used to solve minimization problems for parametrized functions q(t) (**calculus of variations**), as long as the expression to be minimized depends on q and their derivatives.

Plan for today

- Relativity: fundamental principles. (Sections 4.1 and 4.2)
  - Some basic concepts
    - Galilean transformations
    - Lorentz transformations
    - Rapidity
  - Invariant distance & the metric
  - A first look at four-vectors for space-time (if time)

Summary

- To have the same velocity of light in all frames we introduce Lorentz transformations

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t = \gamma\left(t - \frac{v}{c^2}x\right)$$

- We define an invariant distance (metric)

$$\Delta s^2 \equiv c^2 \Delta t^2 - \Delta x^2$$

which is the same in all reference frames.